CHAPTER 7
BASIC TOOLS FOR TOLERANCE ANALYSIS OF MECHANICAL ASSEMBLIES

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7.1 INTRODUCTION

As manufacturing companies pursue higher quality products, they spend much of their efforts monitoring and controlling variation. Dimensional variation in production parts accumulate or stack up statistically and propagate through an assembly kinematically, causing critical features of the final product to vary. Such variation can cause costly problems during assembly, requiring extensive rework or scrapped parts. It can also cause unsatisfactory performance of the finished product, drastically increasing warranty costs and creating dissatisfied customers.

One of the effective tools for variation management is tolerance analysis. This is a quantitative tool for predicting the accumulation of variation in an assembly by performing a stack-up analysis. It involves the following steps:

1. Identifying the dimensions which chain together to control a critical assembly dimension or feature.
2. The mean, or average, assembly dimension is determined by summing the mean of the dimensions in the chain.
3. The variation in the assembly dimension is estimated by summing the corresponding component variations. This process is called a "stack-up."
4. The predicted assembly variation is compared to the engineering limits to estimate the number of rejects, or nonconforming assemblies.
5. Design or production changes may be made after evaluating the results of the analysis.

If the parts are production parts, actual measured data may be used. This is preferred. However, if the parts are not yet in production, measured data is not available. In that case, the engineer searches for data on similar parts and processes. That failing, he or she may substitute the tolerance on each dimension in place of its variation, assuming that quality controls will keep the individual part variations within tolerance. This substitution is so common in the design stage that the process is generally called *tolerance analysis*.

The four most popular models for tolerance stack-up are shown in Table 7.1. Each has its own advantages and limitations.

TABLE 7.1 Models for Tolerance Stack-Up Analysis in Assemblies

Model	Stack formula	Predicts	Application
Worst Case (WC)	$\sigma_{ASM} = \sum \lvert T_i \rvert$ Not statistical	Extreme limits of variation No rejects permitted	Critical systems Most costly
Statistical (RSS)	$\sigma_{ASM} = \sqrt{\sum \left(\frac{T_i}{3}\right)^2}$	Probable variation Percent rejects	Reasonable estimate Some rejects allowed Less costly
Six Sigma (6s)	$\sigma_{ASM} = \sqrt{\sum \left(\frac{T_i}{3C_P(1-k)}\right)^2}$	Long-term variation Percent rejects	Drift in mean over time is expected High quality levels desired
Measured Data (Meas)	$\sigma_{ASM} = \sqrt{\sum \sigma_i^2}$	Variation using existing part measurements Percent rejects	After parts are made What-if? studies

7.2 COMPARISON OF STACK-UP MODELS

The two most common stack-up models are:

Worst Case (WC). Computes the extreme limits by summing absolute values of the tolerances, to obtain the worst combination of over and undersize parts. If the worst case is within assembly tolerance limits, there will be no rejected assemblies. For given assembly limits, WC will require the tightest part tolerances. Thus, it is the most costly.

Statistical (RSS). Adds variations by root-sum-squares (RSS). Since it considers the statistical probabilities of the possible combinations, the predicted limits are more reasonable. RSS predicts the statistical distribution of the assembly feature, from which percent rejects can be estimated. It can also account for static mean shifts.

As an example, suppose we had an assembly of nine components of equal precision, such that the same tolerance T_i may be assumed for each. The predicted assembly variation would be:

$$\text{WC:}\quad T_{ASM} = \sum \lvert T_i \rvert = 9 \times 0.01 = \pm 0.09$$

$$\text{RSS:}\quad T_{ASM} = \sqrt{\sum T_i^2} = \sqrt{9 \times 0.01^2} = \pm 0.03$$

(± denotes a symmetric range of variation)

Clearly, WC predicts much more variation than RSS. The difference is even greater as the number of component dimensions in the chain increases.

Now, suppose $T_{ASM} = 0.09$ is specified as a design requirement. The stack-up analysis is reversed. The required component tolerances are determined from the assembly tolerance.

$$\text{WC:}\quad T_i = \frac{T_{ASM}}{9} = \frac{0.09}{9} = \pm 0.01$$

$$\text{RSS:}\quad T_i = \frac{T_{ASM}}{\sqrt{9}} = \frac{0.09}{3} = \pm 0.03$$

Here, WC requires much tighter tolerances than RSS to meet an assembly requirement.

7.3 USING STATISTICS TO PREDICT REJECTS

All manufacturing processes produce random variations in each dimension. If you measured each part and kept track of how many are produced at each size, you could make a *frequency plot,* as shown in Fig. 7.1.

Generally, most of the parts will be clustered about the mean or average value, causing the plot to bulge in the middle. The further you get from the mean, the fewer parts will be produced, causing the frequency plot to decrease to zero at the extremes.

A common statistical model used to describe random variations is shown in the figure. It is called a *normal,* or *Gaussian,* distribution. The *mean* μ marks the highest point on the curve and tells how close the process is to the target dimension. The spread of the distribution is expressed by its *standard deviation* σ, which indicates the precision or process capability.

UL and *LL* mark the *upper and lower limits* of size, as set by the design requirements. If *UL* and *LL* correspond to the $\pm 3\sigma$ process capability, as shown, a few parts will be rejected (about 3 per 1000).

Any normal distribution may be converted to a *standard normal,* which has a mean of zero and σ of 1.0. Instead of plotting the frequency versus size, it is plotted in terms of the number of standard deviations from the mean. Standard tables then permit you to determine the fraction of assemblies which will fail to meet the engineering limits. This is accomplished as follows:

1. Perform a tolerance stack-up analysis to calculate the mean and standard deviation of the assembly dimension X, which has design requirements X_{UL} and X_{LL}.
2. Calculate the number of standard deviations from the mean to each limit:

$$Z_{UL} = \frac{X_{UL} - \bar{X}}{\sigma_X} \qquad Z_{LL} = \frac{X_{LL} - \bar{X}}{\sigma_X}$$

   where $\bar{X}$ and σ_X are the mean and standard deviation of the assembly dimension X, and $\bar{Z} = 0$ and $\sigma_Z = 1.0$ are the mean and standard deviation of the transformed distribution curve.
3. Using standard normal tables, look up the fraction of assemblies lying between Z_{LL} and Z_{UL} (the area under the curve). This is the predicted *yield,* or fraction of assemblies which will meet the requirements. The fraction lying outside the limits is (1.0 − yield). These are the predicted *rejects,* usually expressed in parts per million (ppm).

Note: Standard tables list only positive Z, since the normal distribution is symmetric.

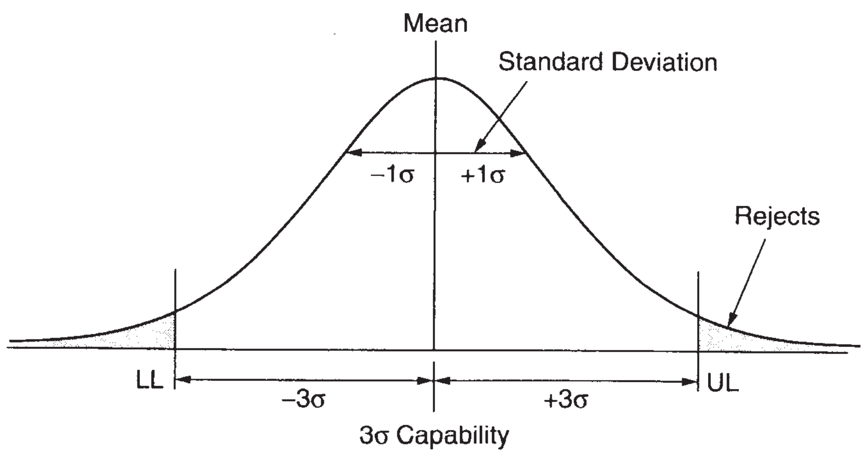


FIGURE 7.1 Frequency plot of size distribution for a process with random error.

TABLE 7.2 Comparative Quality Level vs. Number of Standard Deviations in Z_{LL} and Z_{UL}

Z_{LL} and Z_{UL}	Yield fraction	Rejects per million	Quality level
$\pm 2\sigma$	0.9545	45500	Unacceptable
$\pm 3\sigma$	0.9973	2700	Moderate
$\pm 4\sigma$	0.9999366	63.4	High
$\pm 5\sigma$	0.999999426	0.57	Very high
$\pm 6\sigma$	0.999999998	0.002	Extremely high

Expressing the values Z_{LL} and Z_{UL} in standard deviations provides a nondimensional measure of the quality level of an assembly process. A comparison of the relative quality in terms of the number of σ is presented in Table 7.2.

7.4 PERCENT CONTRIBUTION

Another valuable, yet simple, evaluation tool is the *percent contribution*. By calculating the percent contribution that each variation contributes to the resultant assembly variation, designers and production personnel can decide where to concentrate their quality improvement efforts. The contribution is just the ratio of a component standard deviation to the total assembly standard deviation:

$$\text{WC: } \%\text{Cont} = 100\frac{T_i}{T_{ASM}} \qquad \text{RSS: } \%\text{Cont} = 100\frac{\sigma_i^2}{\sigma_{ASM}^2}$$

7.5 EXAMPLE 1—CYLINDRICAL FIT

A clearance must be maintained between the rotating shaft and bushing shown in Fig. 7.2. The minimum clearance must not be less than 0.002 in. The max is not specified. Nominal dimension and tolerance for each part are given in Table 7.3, below:

The first step involves converting the given dimensions and tolerances to centered dimensions and symmetric tolerances. This is a requirement for statistical tolerance analysis. The resulting centered

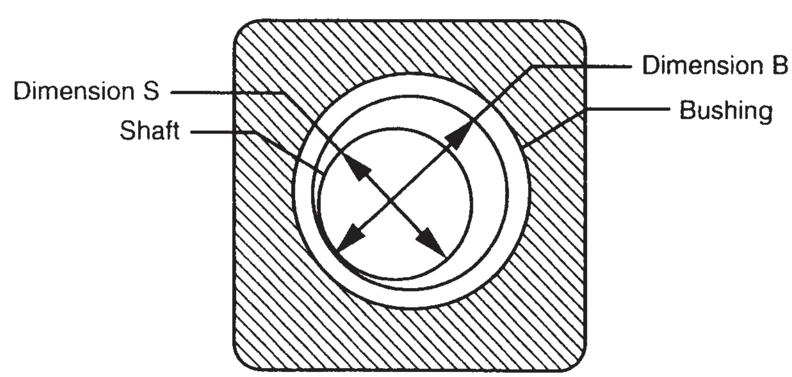


FIGURE 7.2 Shaft and bushing cylindrical fit.

TABLE 7.3 Dimensions and Tolerances—Cylindrical Fit Assembly

Part	Nominal dimension	*LL* tolerance	*UL* tolerance	Centered dimension	Plus/minus tolerance
Bushing *B*	0.75	−0	+.0020	0.7510	±0.0010
Shaft *S*	0.75	−0.0028	−0.0016	0.7478	±0.0006
Clearance *C*				0.0032	±0.0016 WC ±0.00117RSS

dimensions and symmetric tolerances are listed in the last two columns. If you calculate the maximum and minimum dimensions for both cases, you will see that they are equivalent.

The next step is to calculate the mean clearance and variation about the mean. The variation has been calculated both by WC and RSS stackup, for comparison.

$$\text{Mean clearance:}\quad \bar{C} = \bar{B} - \bar{S} = 0.7510 - 0.7478 = 0.0032 \text{ in}$$

(the bar denotes the mean or average value)

$$\text{WC variation:}\quad T_C = |T_B| + |T_S| = 0.0010 + 0.0006 = 0.0016 \text{ in}$$

$$\text{RSS variation:}\quad T_C = \sqrt{T_B^2 + T_S^2} = \sqrt{0.0010^2 + 0.0006^2} = 0.00117 \text{ in}$$

Note that even though C is the difference between B and S, the tolerances are summed. Component tolerances are always summed. You can think of the absolute value canceling the negative sign for WC and the square of the tolerance canceling for RSS.

The predicted range of the clearance is $C = 0.0032 \pm 0.00117$ in (RSS),

or, $C_{max} = 0.0044$, $C_{min} = 0.00203$ in

Note that C_{max} and C_{min} are not absolute limits. They represent the $\pm 3\sigma$ limits of the variation. It is the overall process capability of this assembly process, calculated from the process capabilities of each of the component dimensions in the chain. The tails of the distribution actually extend beyond these limits.

So, how many assemblies will have a clearance less than 0.002 in? To answer this question, we must first calculate Z_{LL} in terms of dimensionless σ units. The corresponding yield is obtained by table lookup in a math table or by using a spreadsheet, such as Microsoft Excel:

$$\sigma_C = \frac{T_C}{3} = 0.00039 \text{ in}$$

$$Z_{LL} = \frac{LL - \bar{C}}{\sigma_C} = \frac{0.002 - 0.0032}{0.00039} = -3.087\sigma$$

The results from Excel are:

Yield = NORMSDIST(Z_{LL}) = 0.998989   Reject fraction = 1.0 − Yield = 0.001011

or, 99.8989 percent good assemblies, 1011 ppm (parts per million) rejects.

Only Z_{LL} was needed, since there was no upper limit specified.

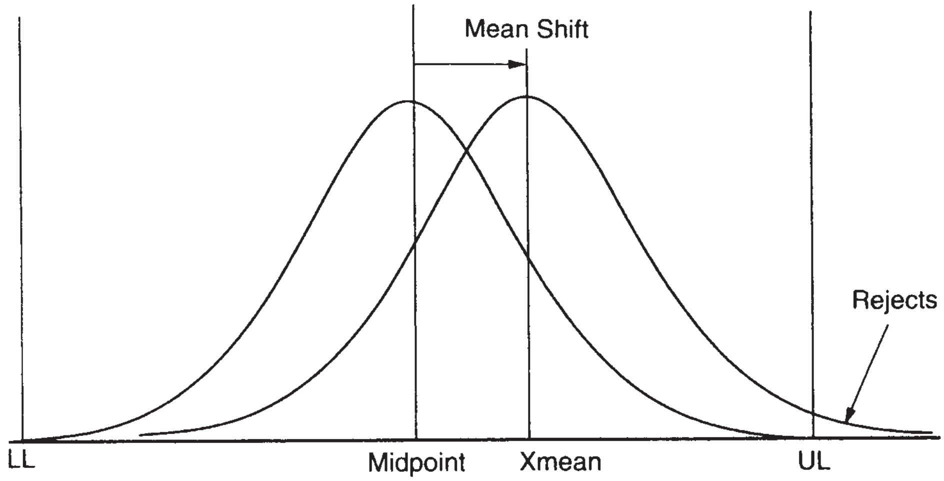


FIGURE 7.3 Normal distribution with a mean shift causes an increase in rejects.

A Z_{LL} magnitude of 3.087σ indicates a moderate quality level is predicted, provided the specified tolerances truly represent the $\pm 3\sigma$ process variations. Figure 7.3 is a plot showing a normal distribution with a mean positive shift. Note the increase in rejects due to the shift.

7.6 HOW TO ACCOUNT FOR MEAN SHIFTS

It is common practice in statistical tolerance analysis to assume that the mean of the distribution is stationary, located at the midpoint between the *LL* and *UL*. This is generally not true. All processes shift with time due to numerous causes, such as tool wear, thermal expansion, drift in the electronic control systems, operator errors, and the like. Other errors cause shifts of a fixed amount, including fixture errors, setup errors, setup differences from batch to batch, material properties differences, etc. A shift in the nominal dimension of any part in the chain can throw the whole assembly off center by a corresponding amount.

When the mean of the distribution shifts off center, it can cause serious problems. More of the tail of the distribution is shoved beyond the limit, increasing the number of rejects. The slope of the curve steepens as you move the mean toward the limit, so the rejects can increase dramatically. Mean shifts can become the dominant source of rejects. No company can afford to ignore them.

There are two kinds of mean shifts that must be considered: static and dynamic. *Static mean shifts* occur once, and affect every part produced thereafter with a fixed error. They cause a fixed shift in the mean of the distribution. *Dynamic mean shifts* occur gradually over time. They may drift in one direction, or back and forth. Over time, large-scale production requires multiple setups, multicavity molds, multiple suppliers, etc. The net result of each dynamic error source is to degrade the distribution, increasing its spread. Thus, more of the tails will be thrust beyond the limits.

To model the effect of static mean shifts, one simply alters the mean value of one or more of the component dimensions. If you have data of actual mean shifts, that is even better. When you calculate the distance from the mean to *LL* and *UL* in σ units, you can calculate the rejects at each limit. That gives you a handle on the problem.

Modeling dynamic mean shifts requires altering the tolerance stackup model. Instead of estimating the standard deviation σ_i of the dimensional tolerances from $T_i = 3\sigma_i$, as in conventional RSS tolerance analysis, a modified form is used to account for higher quality level processes:

$$T_i = 3\,Cp_i\,\sigma_i \qquad \text{where } Cp \text{ is the } \textit{process capability index}$$

$$C_p = \frac{UL - LL}{6\sigma}$$

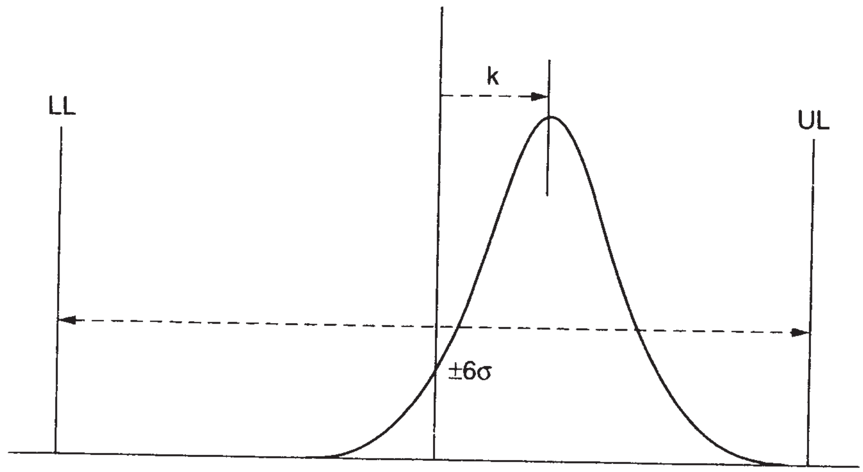


FIGURE 7.4 The six sigma model uses a drift factor k and $\pm 6\sigma$ limits to simulate high quality levels.

If the UL and LL correspond to $\pm 3\sigma$ of the process, then the difference $UL - LL = 6\sigma$, and Cp will be 1.0. Thus, a Cp of 1.0 corresponds to a "moderate quality level" of $\pm 3\sigma$. If the tolerances correspond to $\pm 6\sigma$, $UL - LL = 12\sigma$, and $Cp = 2.0$, corresponding to an "extremely high quality level" of $\pm 6\sigma$.

The Six Sigma model for tolerance stack-up accounts for both high quality and dynamic mean shift by altering the stack-up equation to include the Cp and a drift factor k for each dimension in the chain.

$$\sigma_{ASM} = \sqrt{\sum \left(\frac{T_i}{3C_{Pi}(1-k_i)} \right)^2}$$

As Cp increases, the contribution of that dimension decreases, causing σ_{ASM} to decrease.

The drift factor k measures how much the mean of a distribution has been observed to drift during production. Factor k is a fraction, between 0 and 1.0. Figure 7.4 shows that k corresponds to the shift in the mean as a percent of the tolerance. If there is no data, it is usually set to $k = 0.25$. The effects of these modifications are demonstrated by a comprehensive example.

7.7 EXAMPLE 2—AXIAL SHAFT AND BEARING STACK

The shaft and bearing assembly shown in Fig. 7.5 requires clearance between the shoulder and inner bearing race (see inset) to allow for thermal expansion during operation. Dimensions A through G stack up to control the clearance U. They form a chain of dimensions, indicated by vectors added tip-to-tail in the figure. The chain is 1-D, but the vectors are offset vertically for clarity. The vector chain passes from mating-part to mating-part as it crosses each pair of mating surfaces. Note that all the vectors acting to the right are positive and to the left are negative. By starting the chain on the left side of the clearance and ending at the right, a positive sum indicates a clearance and a negative sum, an interference.

Each dimension is subject to variation. Variations accumulate through the chain, causing the clearance to vary as the resultant of the sum of variations. The nominal and process tolerance limits for each one are listed in Table 7.4 with labels corresponding to the figure.

The design requirement for the clearance U is given below. The upper and lower limits of clearance U are determined by the designer, from performance requirements. Such assembly

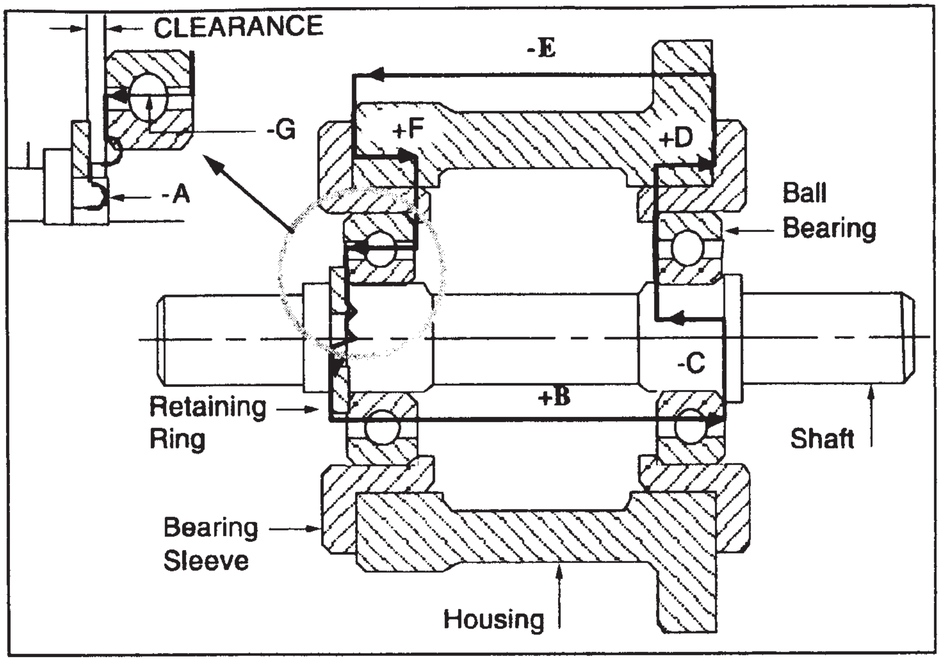


FIGURE 7.5 Example Problem 2: Shaft and bearing assembly. (Fortini, 1967)

requirements are called *key characteristics*. They represent critical assembly features, which affect performance.

$$\text{Design requirement: Clearance } (U) = 0.020 \pm 0.015 \text{ in}$$

Initial design tolerances for dimensions *B*, *D*, *E*, and *F* were selected from a tolerance chart, which describes the "natural variation" of the processes by which parts are made (Trucks 1987). It is a bar chart, indicating the range of variation achievable by each process. Also note that the range of variation depends on the nominal size of the part dimension. The tolerances for *B*, *D*, *E*, and *F* were chosen from the middle of the range of the turning process, corresponding to the nominal size of each. These values are used as a first estimate, since no parts have been made. As the variation analysis progresses, the designer may elect to modify them to meet the design requirements. The bearings, and retaining ring, however, are vendor-supplied. The dimensions and tolerances for *A*, *C*, and *G* are therefore fixed, not subject to modification.

The next step is to calculate the mean clearance and variation about the mean. The variation has been calculated both by WC and RSS stackup, for comparison.

TABLE 7.4 Nominal Dimensions and Tolerances for the Example Problem 2

Part	Dimension	Nominal in	Tolerance in	Process limits Min Tol	Process limits Max Tol
Retaining ring	A*	−.0505	±.0015*	*	*
Shaft	B	8.000	±.008	±0.003	±0.020
Bearing	C*	−.5090	±.0025*	*	*
Bearing sleeve	D	.400	±.002	±0.0008	±0.005
Housing	E	−7.705	±.006	±0.0025	±0.0150
Bearing sleeve	F	.400	±.002	±0.0008	±0.005
Bearing	G*	−.5090	±.0025*	*	*

* Vendor-supplied part

Mean clearance:

$$
\begin{aligned}
\overline{U} &= -\overline{A} + \overline{B} - \overline{C} + \overline{D} - \overline{E} + \overline{F} - \overline{G} \\
&= -0.0505 + 8.000 - 0.509 + 0.400 - 7.705 + 0.400 - 0.509 \\
&= 0.0265
\end{aligned}
$$

WC variation:

$$
\begin{aligned}
T_U &= |T_A| + |T_B| + |T_C| + |T_D| + |T_E| + |T_F| + |T_G| \\
&= 0.0015 + 0.008 + 0.0025 + 0.002 + 0.006 + 0.002 + 0.0025 \\
&= 0.0245
\end{aligned}
$$

RSS variation:

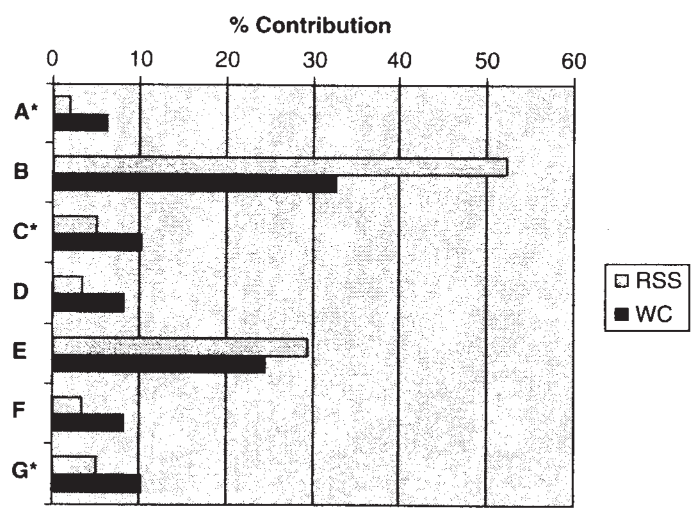
$$
\begin{aligned}
T_U &= \sqrt{T_A^2 + T_B^2 + T_C^2 + T_D^2 + T_E^2 + T_F^2 + T_G^2} \\
&= \sqrt{0.0015^2 + 0.008^2 + 0.0025^2 + 0.002^2 + 0.006^2 + 0.002^2 + 0.0025^2} \\
&= 0.01108
\end{aligned}
$$

Parts-per-million rejects:

$$
Z_{UL} = \frac{U_{UL} - \overline{U}}{\sigma_U} = \frac{0.035 - 0.0265}{0.00369} = 2.30\sigma \quad \Rightarrow \quad 10{,}679 \quad PPM\_\text{Rejects}
$$

$$
Z_{LL} = \frac{U_{LL} - \overline{U}}{\sigma_U} = \frac{0.005 - 0.0265}{0.00369} = -5.82\sigma \quad \Rightarrow \quad 0.0030 \quad PPM\_\text{Rejects}
$$

Percent Contribution. The percent contribution has been calculated for all seven dimensions, for both WC and RSS. A plot of the results is shown in Fig. 7.6. RSS is greater because it is the square of the ratio of the variation.



% Contribution	
WC	**RSS**
6.12	1.83
32.65	52.14
10.20	5.09
8.16	3.26
24.49	29.33
8.16	3.26
10.20	5.09

FIGURE 7.6 Percent contribution chart for Example 2.

7.8 CENTERING

The example problem discovered a mean shift of 0.0065 in from the target value, 0.020 in, midway between *LL* and *UL*. The analysis illustrates the effect of the mean shift—a large increase in rejects at the upper limit and reduced rejects at the lower limit. To correct the problem, we must modify one or more nominal values of the dimensions *B*, *D*, *E*, or *F*, since *A*, *C*, and *G* are fixed.

Correcting the problem is more challenging. Simply changing a callout on a drawing to center the mean will not make it happen. The mean value of a single dimension is the average of many produced parts. Machinists cannot tell what the mean is until they have made many parts. They can try to compensate, but it is difficult to know what to change. They must account for tool wear, temperature changes, set up errors, etc. The cause of the problem must be identified and corrected. It may require tooling modifications, changes in the processes, careful monitoring of the target value, a temperature-controlled workplace, adaptive machine controls, etc. Multicavity molds may have to be qualified cavity-by-cavity and modified if needed. It may require careful evaluation of all the dimensions in the chain to see which is most cost effective to modify.

In this case, we have chosen to increase dimension *E* by 0.0065 in, to a value of 7.7115 in. The results are:

Mean	σ_{ASM}	Z_{LL}	Rejects	Z_{UL}	Rejects
0.020 in	0.01108 in	–4.06	24 ppm	4.06	24 ppm

This would be a good solution, if we could successfully hold that mean value by better fixturing, more frequent tool sharpening, statistical process control, and the like.

7.9 ADJUSTING THE VARIANCE

Suppose the mean of the process cannot be controlled sufficiently. In that case, we may choose to adjust the tolerance of one or more dimensions. The largest contributors are dimensions *B* on the shaft and *E* on the housing. We reduce them both to 0.004 in with the results:

Mean	σ_{ASM}	Z_{LL}	Rejects	Z_{UL}	Rejects
0.0265 in	0.00247 in	–8.72	0 ppm	3.45	284 ppm

This corresponds to an effective quality level of $\pm 3.63\sigma$, that is, for a two-tailed, centered distribution having the same number of total rejects (142 at each limit).

7.10 MIXING NORMAL AND UNIFORM DISTRIBUTIONS

Suppose the shaft (Part **B**) is jobbed out to a new shop, with which we have no previous experience. We are uncertain how much variation to expect. How shall we account for this uncertainty? We could do a worst case analysis, but that would penalize the entire assembly for just one part of unknown quality. We could instead resort to a uniform distribution, applied to dimension *B*, leaving the others as normal.

The *uniform* distribution is sometimes called the "equal likelihood" distribution. It is rectangular in shape. There are no tails, as with the normal. Every size between the upper and lower tolerance limits has an equal probability of occurring. The uniform distribution is conservative. It predicts greater variation than the normal, but not as great as worst case.

For a uniform distribution, the tolerance limits are not $\pm 3\sigma$, as they are for the normal. They are equal to $\pm\sqrt{3}\sigma$. Thus, the stackup equation becomes:

$$\sigma_{ASM} = \sqrt{\sum\left(\frac{T_i}{3}\right)^2 + \sum\left(\frac{T_j}{\sqrt{3}}\right)^2}$$

where the first summation is the squares of the σ_i for the normal distributions and the second sum is for the uniform distributions.

For the example problem, dimension B has upper and lower limits of 8.008 and 7.992 in, respectively, corresponding to the $\pm\sqrt{3}\sigma$ limits. We assume the assembly distribution has been centered and the only change is that B is uniform rather than normal. Substituting the tolerance for B in the second summation and the tolerance for each of the other dimensions in the first summation, the results are:

Mean	σ_{ASM}	Z_{LL}	Rejects	Z_{UL}	Rejects
0.020 in	0.00528 in	−2.84	2243 ppm	2.84	2243 ppm

The predicted rejects assume that the resulting distribution of assembly clearance U is normal. This is generally true if there are five or more dimensions in the stack. Even if all of the component dimensions were uniform, the resultant would still approximate a normal distribution. However, if one non-normal dimension has a much larger variation than the sum of all the others in the stack, the assembly distribution would be non-normal.

7.11 SIX SIGMA ANALYSIS

Six Sigma analysis accounts for long-term drift in the mean, or dynamic mean shift, in manufactured parts. It uses the process capability index Cp and drift factor k to simulate the long term spreading of the distribution, as mentioned earlier. In the following, Six Sigma is applied to two models for example Problem 2. The first uses $Cp = 1.0$ for comparison directly with RSS, corresponding to a $\pm 3\sigma$ quality level, with and without drift correction. The second case uses $Cp = 2.0$ for comparison of $\pm 6\sigma$ quality levels with $\pm 3\sigma$. The results are presented in Table 7.5 alongside WC and RSS results for comparison. All centered cases used the modified nominals to center the distribution mean.

TABLE 7. 5 Comparison of Tolerance Analysis Models for Example 2

Model	Mean in	σ_{ASM} in	Z_{LL}/Z_{UL} σ	Rejects ppm	Quality σ
Centered					
WC	0.020	0.00820*	N/A	N/A	N/A
RSS—Uniform	0.020	0.00640	±2.34	19027	2.34
RSS—Normal	0.020	0.00369	±4.06	48	4.06
6Sigma—Cp = 1	0.020	0.00492	±3.05	2316	3.05
6Sigma—Cp = 2	0.020	0.00246	±6.1	0.0011	6.10
Mean Shift					
RSS—Uniform	0.0265	0.00640	−3.36/1.33	92341	1.68
RSS—Normal	0.0265	0.00369	−5.82/2.30	10679	2.55
6Sigma—Cp = 1	0.0265	0.00492	−4.37/1.73	42162	2.03
6Sigma—Cp = 2	0.0265	0.00246	−8.73/3.45	278	3.64

*WC has no σ. This is calculated from $T_{ASM}/3$ for comparison with RSS methods.

Noncentered cases used a mean shift of 0.0065 in. The RSS—uniform results were not presented before, as this case applied uniform distributions to all seven dimensions for comparison to WC.

7.12 REMARKS

The foregoing discussion has presented techniques for predicting tolerance stacking, or the accumulation of variation, in mechanical assembly processes. There is quite a wide range of results, depending on the assumptions, available data, and the quality goals involved. As with any analytical modeling, it is wise to verify the results by measurements. When production data become available, values of the mean and standard deviation of the measured dimensions may be substituted into the RSS stack equation. This will give real-world data to benchmark against.

In 1-D stacks, the means do add linearly and standard deviations do add by root-sum-squares, as long as the variations are independent (not correlated). There are tests for correlation, which may be applied. Verification will build confidence in the methods. Experience will improve your assembly modeling skills and help you decide which analytical models are most appropriate for given applications.

There are many topics which have been omitted from this introduction, including:

1. Modeling variable clearances, such as the clearance around a bolt or shaft, which can introduce variation into a chain of dimensions as an input rather than a resultant assembly gap.
2. Treating errors due to human assembly operations, such as positioning parts in a slip-joint before tightening the bolts.
3. Available standards for tolerancing, such as cylindrical fits, or standard parts, like fasteners.
4. How to apply GD&T to tolerance stacks.
5. Tolerance allocation algorithms, which assist in assigning tolerances systematically.
6. When and how to use Monte Carlo Simulation, design of experiments, response surface methodology, and method of system moments for advanced applications.
7. How to treat non-normal distributions, such as skewed distributions.
8. Methods for modeling 2-D and 3-D assembly stacks.
9. CAD-based tolerance analysis tools.

The results here presented were obtained using an Excel spreadsheet called CATS 1-D, which is available as a free download, along with documents, from the ADCATS web site, listed in the References.

For further reading, see below. Additional papers which discuss many of these topics are available on the ADCATS web site.

REFERENCES

Trucks, H. E., *Designing for Economical Production*, 2nd ed., Society of Manufacturing Engineers, Dearborn, Michigan, 1987.

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FURTHER READING

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