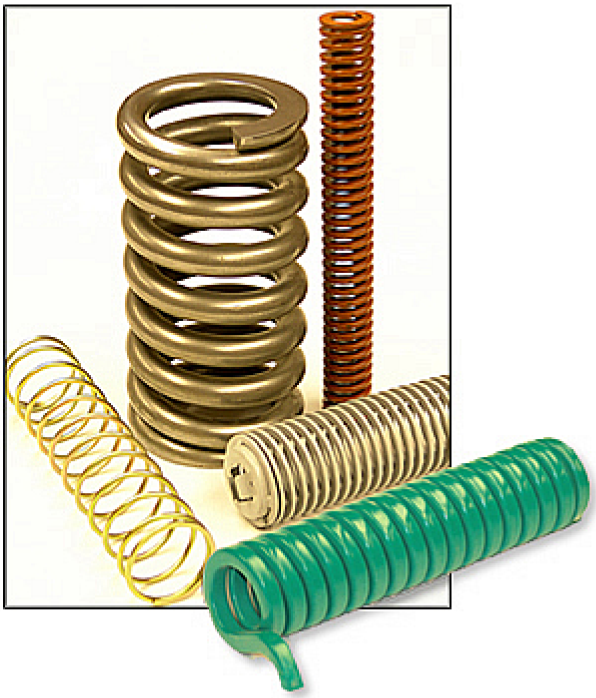
Mechanical Spring

Pegas Mekanikal

pg 500

Objective

Selection of spring by load and stress analysis.



Application

Vehicle suspension, weighing system, energy storage system, door system etc

Kinds of Springs

Helical spring, Spiral spring, Leaf spring, Torsional spring

Important topics

Helical Tension & Compression spring (Pg)

<u>For Helical springs</u>

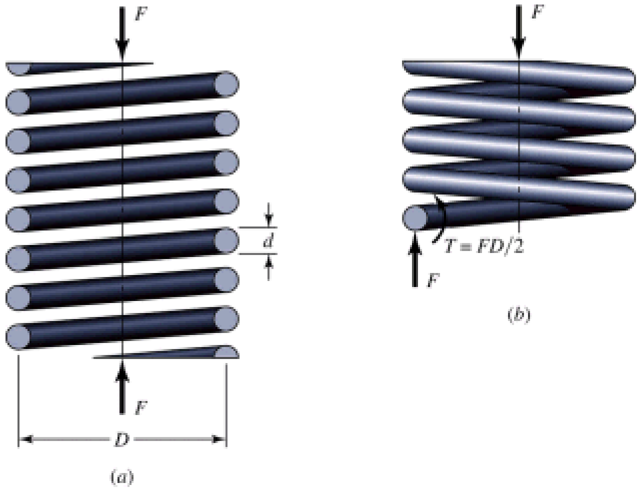
The basic specs

D - diameter of wire
d - mean diameter of spring
OD - outside diameter of spring, $OD = D + d$
ID - inside diameter , $ID = D - d$
Nt - number of coils
Na - number of active coils
Lf - free length
Lo - operating length of string
Li - installed length
Ls - solid length
p - pitch

Stresses and Deflection in Helical Springs

Loads that cause failure in spring

1. torsional load
2. direct shear load
3. bending load
4. buckling load
5. variable load and fatigue (not in scope)



Shear stress is caused by transverse shear loads and torsional shear loads.

Transverse Shear Stress $\tau_p = \frac{F}{A}$ where $A = \frac{\pi d^2}{4}$

Torsional Load $\tau_s = \frac{T\,r}{J}$ where $J = \frac{\pi d^4}{32}$ & $T = \frac{F\,D}{2}$ & $r = \frac{d}{2}$

Therefore, overall shear stress in spring : $\tau = \frac{T\,r}{J} + \frac{F}{A} = \frac{8F\,D}{\pi d^3} + \frac{4F}{\pi d^2}$

Spring Index, $C = \frac{D}{d}$

Correction factor for Shear stress, $K_S = \frac{2C + 1}{2C}$

Wahl factor, $K_W = \frac{4\,C - 1}{4\,C - 4} + \frac{0.615}{C}$

Bergstrassor factor, $K_B = \frac{4\,C + 2}{4\,C - 3}$

Therefore the curvature correction factor, $K_C = \frac{K_B}{K_S}$ which handle curvature effect only

The Wahl and Bergstrasser factors consider the effect of both curvature of wire and shear stress. Their values differ only by 1%. Shigley recommends using K_B .

Hence Torsional and direct shear stress $\tau = K_B \frac{8F\,D}{\pi d^3}$

Deflection of Helical Spring

Torsional deflection, $\theta = \dfrac{T\,L}{G\,J}$

Force, $F = k\,y$

$$k = \frac{d^4 G}{8D^3 N_a} \qquad \text{or}$$

$$y = \frac{8F\,D^3 N_a}{d^4 G} = \frac{8F\,C^3 N_a}{d\,G} \qquad [10\text{-}8]$$

See Table 10-5 to get value of Modulus of material.

k - spring constant

G - modulus of rigidity

Na - number of active coils

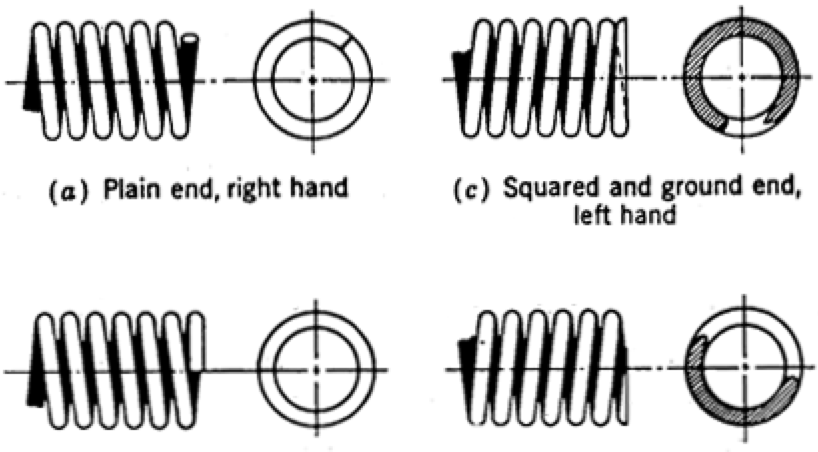
y - linear deflection

Compression Spring

STANDARD Compression spring

Types of Ends

Refer to Figure 10-2, pg 503



(a) Plain end, right hand

(c) Squared and ground end, left hand

(b) Squared or closed end, right hand

(d) Plain end, ground, left hand

Number of active coils

(a) $N_a = N_t$

(b) or (c), $N_a = N_t - 2$

(d) $N_a = N_t - 1$

Non-STANDARD spring design

The Table 10-1 and Figures 10-2 cannot be used when designing non-standard spring. Manual evalution based on observation and measurement is used to obtained the specifications for the spring.

Buckling - Stability of Compression Spring

The spring will buckle if the force is too large and the deflection is higher than the critical deflection. See pg 745, Topic Buckling

Stability of Compression Spring

The spring will buckle if the force is too large and the deflection is higher than the critical deflection. This can be determined by End Condition Constants, α (refer to Table 10-2, pg 504.)

For absolute stability : $L_o < \frac{\pi D}{\alpha} \cdot \sqrt{\frac{2(E-G)}{2G+E}}$

For all steel, the spring is stable (no buckling) if $L_o < 2.63 \frac{D}{\alpha}$

For stability of square and ground ends steel spring ==> $\alpha = 0.5$ **and** $L_o < 5.26D$

Spring material Pg 505

Tensile strength : $S_{ut} = \frac{A}{d^m}$

Get A & m from Table-10-4. unit: d [mm, inch] , A [MPa.mm]

Shear yield strength of spring in torsion : from Table 10-6

$$S_{sy} = \begin{pmatrix} 0.45\ S_{ut} \\ 0.50\ S_{ut} \\ 0.35\ S_{ut} \end{pmatrix}$$

- cold drawn carbon steel, hard drawn steel
- hardened & tempered carbon and alloy steel
- austenitic stainless steel and non-ferrous alloy

SEE Sample 1: EXAMPLE 10-1 pg 509 : Analyzing standard spring

SEE Sample 2: Example - Analyzing Non-standard spring

SOLVE Sample 3: QUESTION 10-4 pg 543 : Analyzing standard spring

SOLVE Sample 4: QUESTION 10-18 pg 544 : Analyzing non-standard spring

Helical compression spring design for static service

Spring deflect non-linearly when compressed to solid length. To avoid the non-linear behaviour, the maximum operating load should be limited to $F_{max} \le \frac{7}{8} F_s$

where the shear force at solid length $F_s = (1 + \xi) F_{max}$ and deflection $y_s = (1 + \xi) y_{max}$

Based on the above factor, the following design condition is recommended :

$4 \le C \le 12$

$3 \le N_a \le 15$

$\xi \ge 0.15$

$n_s \ge 1.2$ (Safety factor at solid length)

As-wound : $D = C \cdot d$

where $\alpha = \frac{S_{sy}}{n_s}$, $\beta = \frac{8(1+\xi)F_{max}}{\pi d^2}$

and $C = \frac{2\alpha - \beta}{4\beta} + \sqrt{\left(\frac{2\alpha - \beta}{4\beta}\right)^2 - \frac{3\alpha}{4\beta}}$

Over-a-rod : $D = d_{rod} + d + allowance$

In-a-hole : $D = d_{hole} - d - allowance$

SEE Sample 5: EXAMPLE 10-2 pg 512 : Analyzing standard spring for static loading

Critical Frequency of Helical Springs

When the spring is loaded and suddenly released, a certain vibration-like action occurs until it is damped out. This is called Spring-surge. Unfavored reaction to this surge is the spring may actually jump out of contact with its support-plate causing extensive damage to the plate and spring. To avoid this, designer must ensure that the natural frequency of the spring is not closed to the frequency of the applied force to avoid resonance.

For spring with both end in contact with the plates, the natural freq $f = \frac{1}{2}\sqrt{\frac{k\,g}{W}}$

For spring with one end free, the natural freq $f = \frac{1}{4}\sqrt{\frac{k\,g}{W}}$

The weight of the active part of the spring is $W = \frac{\pi^2 d^2 D N_a \gamma}{4}$ where γ is the specific weight.

Zimmerli Endurance Strength Components for Helical compression spring under fatigue loading

According to Zimmerli, size, material and tensile strength have no effect on the endurance limits (infinite life only) of spring steels in sizes under 10mm.

Shot peening can be used to improve fatigue strength of dynamically loaded springs by 20% or more. Base on test for peened and unpeened spring, the following value is obtained and applied in spring fatigue analysis. For spring steel, the corresponding endurance strength components for infinite life is

1. Unpeened $S_{sa} = 241MPa$, $S_{sm} = 379MPa$

   Peened $S_{sa} = 398MPa$, $S_{sm} = 534MPa$

Based on Eqn 6-53 pg 309, $S_{su} = 0.67S_{ut}$ where $S_{ut} = \dfrac{A}{d^m}$

Gerber's fatigue criterion

$$S_{se} = \frac{S_{sa}}{1 - \left(\dfrac{S_{sm}}{S_{su}}\right)^2}$$

$$\frac{\eta\,\tau_a}{S_{se}} + \left(\frac{\eta\,\tau_m}{S_{su}}\right)^2 = 1$$

Modified Goodman fatigue criterion

$$S_{se} = \frac{S_{sa}}{1 - \dfrac{S_{sm}}{S_{su}}}$$

$$\frac{\tau_a}{S_{se}} + \frac{\tau_m}{S_{su}} = \frac{1}{\eta}$$

See Sample 6: Example 10-4 pg 519 - Analyzing STANDARD spring with fatigue loading.

Sine Failure Criterion

Unlike rotating shaft which support both tensile and compression loads continuously, springs are usually designed for either compression or tension load. With preload the load curve is as Fig 6-23d on pg 293. This criterion shall assume that the midrange stresses can be ignored in the fatigue analysis (like reverse-loading).

Hence $S_{sm} = 0$ and $\tau_m = 0$ and hence $S_{se} = S_{sa}$

Helical compression spring under fatigue Loads

Alternating load, $$F_a = \frac{F_{max} - F_{min}}{2}$$

Mean load, $$F_m = \frac{F_{max} + F_{min}}{2}$$

For pre-loaded spring:
$F_{min} = F(preload)$

For torsional and shear stress

$$\tau_a = K_B \frac{8F_a D}{\pi d^3}$$

K_B - Bergstrasser factor

$$\tau_m = K_B \frac{8F_m D}{\pi d^3}$$

Spring Marin Factors