The Extreme value Type I Distribution, a.k.a. the Fisher-Tippett distribution Type I distribution, a.k.a. the double exponential distribution.

$$f(x) = \frac{1}{a}\cdot \exp\left(\pm\left(\frac{x-b}{a}\right) - \exp\left(\pm\left(\frac{x-b}{a}\right)\right)\right)$$

where $-\infty < b < \infty$, $-\infty < x < \infty$, and $a > 0$

In this problem the parameters of the Type I EV distribution will be computed from the data. The cumulative distrbution and the empirical distribution are then plotted on the same axis and compared.

Corney_Creek_Annual_Flow :=

	2	3	4	5
0	"1956-02-06"	NaN	$1.9\cdot10^3$	NaN
1	"1957-04-29"	NaN	$4.66\cdot10^3$	NaN
2	"1958-04-28"	NaN	$2.14\cdot10^4$	NaN
3	"1959-06-09"	NaN	$3.91\cdot10^3$	NaN
4	"1960-03-05"	NaN	$2.15\cdot10^3$	NaN
5	"1961-07-17"	NaN	$7.49\cdot10^3$	NaN
6	"1962-05-02"	NaN	$3.55\cdot10^3$	NaN
7	"1963-07-18"	NaN	852	NaN
8	"1964-04-27"	NaN	$3.02\cdot10^3$	NaN
9	"1965-04-01"	NaN	$1.88\cdot10^3$	NaN
10	"1966-05-03"	NaN	$1.5\cdot10^3$	NaN
11	"1967-05-08"	NaN	$1.3\cdot10^3$	NaN
12	"1968-01-10"	NaN	$2.21\cdot10^3$	NaN
13	"1969-02-03"	NaN	$3.6\cdot10^3$	NaN
14	"1970-03-05"	NaN	$4.3\cdot10^3$	NaN
15	"1971-04-24"	NaN	$1.06\cdot10^3$	...

$$i := 0 .. 46$$

$$Annual\_flow_i := Corney\_Creek\_Annual\_Flow_{i,4}\cdot\frac{ft^3}{sec}$$

$$mean(Annual\_flow) = 5.432\times10^3\cdot\frac{ft^3}{sec}$$

$$\text{stdev}(\text{Annual\_flow}) = 5.325 \times 10^{3} \cdot \frac{ft^{3}}{sec}$$

$$\text{sorted\_flow} := \text{sort}(\text{Annual\_flow})$$

$$\text{rank} := \text{Rank}(\text{sorted\_flow})$$

$$\text{prob}_i := \frac{\text{rank}_i}{\max(\text{rank}) + 1}$$

The annual maximum discharges in a river show a mean of $\mu_X := \text{mean}(\text{Annual\_flow})$ and a standard deviation of $\sigma_X := \text{stdev}(\text{Annual\_flow})$.

Discussion : The Extreme Value Type I (Gumbel) requires that the parent distribution be unbounded in the direction of the extreme value. Specifically, Type I distributions require that the parent distribution falls off in an exponentail manner, such that the upper tail of the cumulative distribution can be expressed in the form:

$P_X(x) = 1 - e^{-g(x)}$ **where g(x) is an increasing function of x**

In using the Type I distribution with the normal distribution as the parent the probability density function for the Extreme value Type I distribution is given by:

$$f(x) = \frac{1}{a} \cdot \exp\left(\pm\left(\frac{x-b}{a}\right) - \exp\left(\pm\left(\frac{x-b}{a}\right)\right)\right)$$

where $-\infty < b < \infty, \; -\infty < x < \infty,$ **and** $a > 0$

That is we assume that the distribution of flows within a year can be described by a normal distribution. This being the case, the distribution of the maximum flow for each year is given by the distribution above. a and b are referred to as "scale parameters" and b is actually the mode of the distribution.

The mean, standard deviation and skew can be computed from a and b and when seeking maximum values are:

$$\mu_X = b + .577a \qquad \sigma^2 = 1.645 \cdot a^2 \qquad g_X = 1.1396$$

The mean, standard devation, and skew for minimum values are:

$$\mu_X = b - .577a \qquad \sigma^2 = 1.645 \cdot a^2 \qquad g_X = -1.1396$$

By using the transformation: $y = \frac{x - b}{a}$ **we can write:** $f(y) = \exp(\pm y - \exp(\pm y))$

which yields the following cumulative distrbutions for the maximum yearly and minimum yearly flows.

$$\text{maxima}: F(y) = \exp(-\exp(-y))$$

$$\text{minima}: F(y) = 1 - \exp(-\exp(y))$$

In order to solve the problem at hand we must first find the values of a and b.

$$\mu_X = 5.432 \times 10^3 \cdot \frac{ft^3}{sec} \qquad \sigma_X = 5.325 \times 10^3 \cdot \frac{ft^3}{sec}$$

$$a := 5 \cdot \frac{m^3}{sec} \qquad b := 4 \cdot \frac{m^3}{sec}$$

Given

$$\mu_X = b + .577a$$

$$\sigma_X = \sqrt{1.645 \cdot a^2}$$

$$\begin{pmatrix} a_{soln} \\ b_{soln} \end{pmatrix} := \text{Find}(a, b)$$

$$\begin{pmatrix} a_{soln} \\ b_{soln} \end{pmatrix} = \begin{pmatrix} 4.152 \times 10^3 \\ 3.037 \times 10^3 \end{pmatrix} \cdot \frac{ft^3}{sec}$$

These are the values of a and b for the cumulative distribution of annual maximum values

$$Y = \frac{X - b}{a} = \frac{X - b_{soln}}{a_{soln}}$$

for a 100 year event the cumulative probability is 0.99 thus we can write:

$$.99 = \exp\left(-\exp\left(-y_{100}\right)\right)$$

$$0.99 = e^{-e^{-y_{100}}}$$

$$y_{100} := 4.60$$

$$y_{100} = \frac{x_{100} - b_{soln}}{a_{soln}}$$

$$x_{100} := y_{100} \cdot a_{soln} + b_{soln} = 2.214 \times 10^{4} \cdot \frac{ft^3}{sec}$$

There is a 1% chance that the annual maximum flow in a given year will be greater than $x_{100} = 2.214 \times 10^{4} \cdot \frac{ft^3}{sec}$

for a 50 year event the cumulative probability is 0.98, thus we can write:

$$.98 = \exp\left(-\exp\left(-y_{50}\right)\right)$$

$$0.98 = e^{-e^{-y_{50}}}$$

$$y_{50} := 3.90$$

$$y_{50} = \frac{x_{50} - b_{soln}}{a_{soln}}$$

$$x_{50} := b_{soln} + a_{soln} \cdot y_{50} = 1.923 \times 10^{4} \cdot \frac{ft^3}{sec}$$

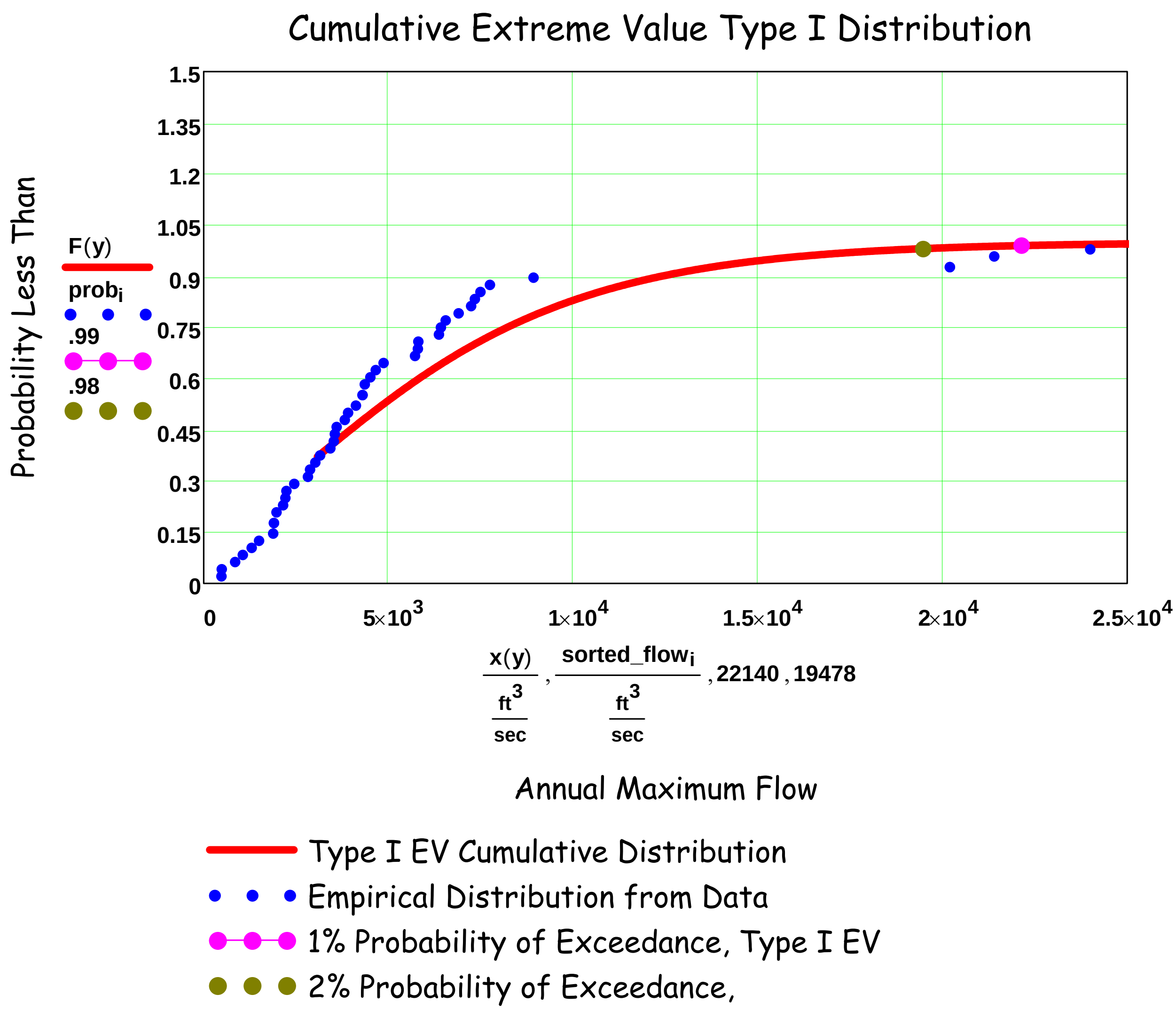
There is a 2% chance in any given year that the maximum flow for that year will be less than $x_{50} = 1.923 \times 10^{4} \cdot \frac{ft^3}{sec}$

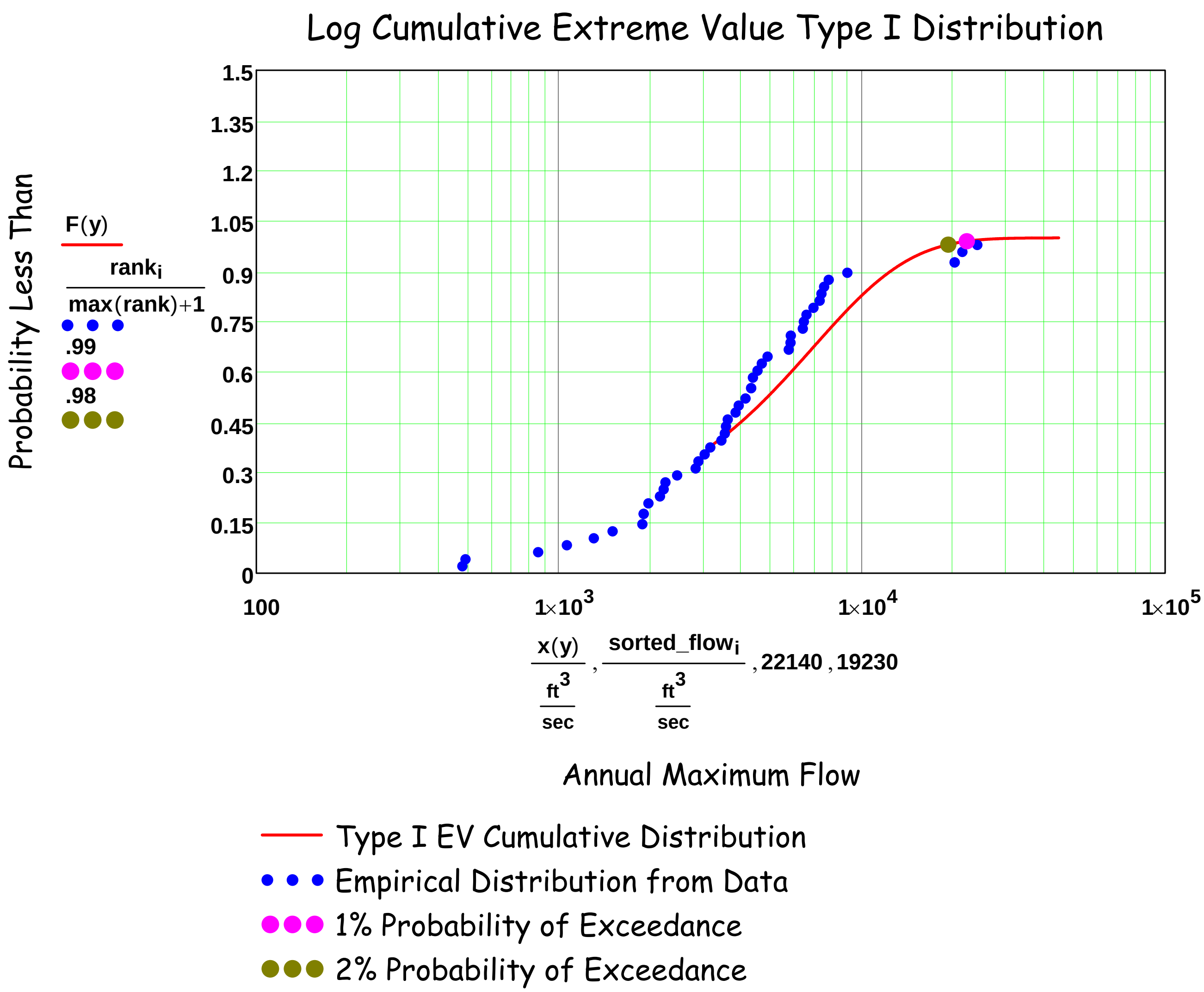
Plot the cumulative distribution for the annual maximum flows:

$$\text{maxima : } F(y) := \exp(-\exp(-y))$$

$$y := .01, .02 .. 10$$

$$x(y) := \left(b_{soln} + a_{soln} \cdot y\right)$$





Comments : Note that most of the data occurs for smaller values. There are only one or two very large values. This is usually the case when dealing with extremes. For smaller flows (smaller return periods) the empirical distribution predicts significantly larger flows. However, for really large flows the theoretical and empirical distributions are relatively close.