3.5 Reinforced Concrete Section Properties

Description

This application calculates gross section moment of inertia neglecting reinforcement, moment of inertia of the cracked section transformed to concrete, and effective moment of inertia for T-beams, rectangular beams, or slabs, in accordance with Section 9.5.2.3 of ACI 318.

The user must enter the strengths of the concrete and the reinforcement, the unit weight of concrete, strength reduction factor for lightweight concrete, beam dimensions and slab thickness, maximum moments in the member at the stage deflection is computed, cross-sectional areas of reinforcement and distances from the extreme compression fiber to the centroid of the tension and compression reinforcement.

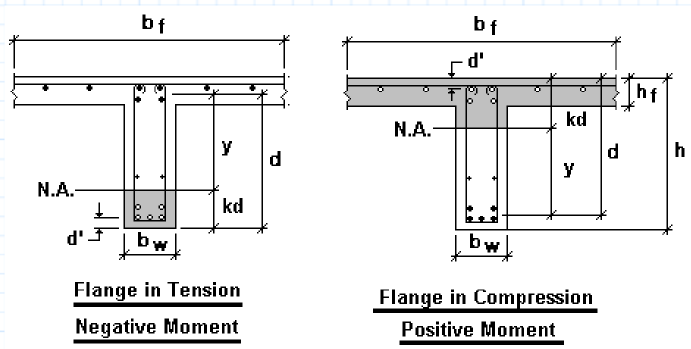
A summary of input and computed values is shown on pages 10-12.

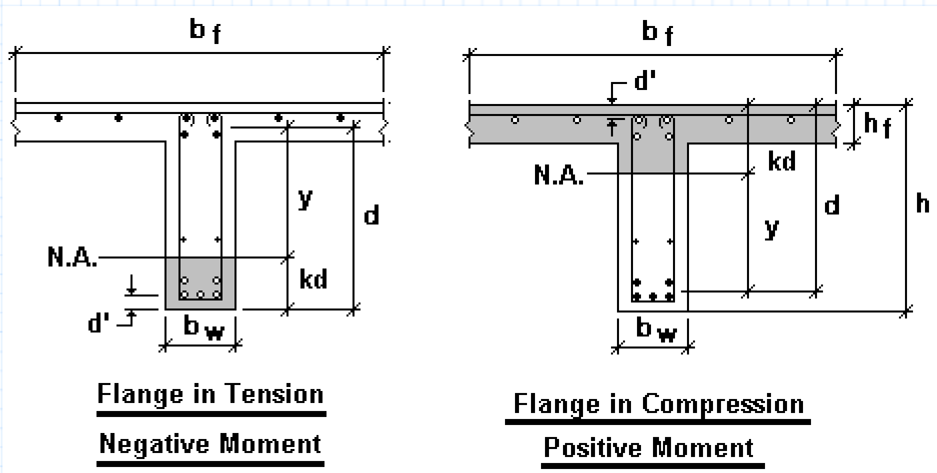
Reference: ACI 318-89 "Building Code Requirements for Reinforced Concrete." (Revised 1992)

Input

Notation

Bending Moment Diagram for Beam with Uniformly Distributed Load





Input Variables

Enter flange width, flange thickness, beam web width and overall member thickness (for slabs enter the same width for the flange and the beam web).

Flange width:	$b_f := 72\ in$
Overall thickness of member:	$h := 20\ in$
Beam web width:	$b_w := 10\ in$
Flange thickness:	$h_f := 4.0\ in$

Enter maximum moments, effective depths and tension reinforcement areas (the first and third elements for the left and right negative moment sections, and the second for the positive moment section):

Maximum moments in member at stage deflection is computed: $M_a := \begin{bmatrix} 28 & 56 & 78 \end{bmatrix}^{\mathrm{T}}\ kip \cdot ft$

Tension Reinforcement

Distances from the extreme compression fiber to the centroid of the tension reinforcement: $d := [17.5 \;\; 17.5 \;\; 17.5]^{T}$ *in*

Cross-sectional areas of tension reinforcement: $A_s := [0.80 \;\; 1.32 \;\; 2.22]^{T}$ in^2

Compression Reinforcement (if used)

Continuous top bars, or bottom bars lapped at supports, may be included as compression reinforcement in calculating cracked section properties. Enter 0 for each element of d' and A's where compression reinforcement is not used.

Distances from the extreme compression fiber to the centroid of the compression reinforcement: $d' := [2.25 \;\; 2.5 \;\; 2.25]^{T}$ *in*

Cross-sectional areas of compression reinforcement: $A'_s := [0.40 \;\; 0.88 \;\; .62]^{T}$ in^2

Computed Variables

f_r	modulus of rupture of concrete (ACI 318, 9.5.2.3)
M_{cr}	cracking moment as defined in ACI 318, 9.5.2.3
y	distance from the tension reinforcement to the neutral axis of the cracked section
y_t	distance from the neutral axis of the gross section to the top of the section
y_b	distance from the neutral axis of the gross section to the bottom of the section
kd	distance from the extreme fiber in compression to the neutral axis of the cracked section
I_g	moment of inertia of gross concrete section about centroidal axis, neglecting reinforcement
I_{cr}	moment of inertia of the cracked section transformed to concrete
I_e	effective moment of inertia for computation of deflection (ACI 318, 9.5.2.3)

Material Properties and Constants

Enter concrete and reinforcement strengths, unit weight of concrete and the strength reduction factor for lightweight concrete:

Specified compressive strength of concrete: $f'_c := 4$ *ksi*

Unit weight of concrete: $w_c := 145$ *pcf*

Specified yield strength of non-prestressed reinforcement: $f_y := 60$ *ksi*

Strength reduction factor for lightweight concrete applied to f_r (0.75 for all-lightweight and 0.85 for sand-lightweight concrete (ACI 318, 9.5.2.3 (b)): $k_v := 1$

Modulus of elasticity of reinforcement (ACI 318, 8.5.2): $E_s := 29000$ *ksi*

Modulus of elasticity of concrete for unit weights between 90 pcf and 155 pcf
(ACI 318, 8.5.1):

$$E_c := \left(\frac{w_c}{pcf}\right)^{1.5} \cdot 33 \cdot \sqrt{\frac{f'_c}{psi}} \cdot psi \qquad E_c = 3644\ ksi$$

Modular ratio of elasticity:

$$n := \frac{E_s}{E_c} = 7.958$$

If preferred, the modular ratio of elasticity may be rounded to an even number at this point:

Calculations

The specified yield strength of the reinforcement may not exceed 80 ksi in accordance with ACI 318, Section 9.4:

$$f_y := \text{if}\left(f_y \le 80 \cdot ksi, f_y, 80 \cdot ksi\right) \qquad f_y = 60\ ksi$$

Distance from the neutral axis of the gross section to the top of the section:

$$y_t := \frac{\frac{1}{2} \cdot \left(b_w \cdot h^2 + \left(b_f - b_w\right) \cdot h_f^{\,2}\right)}{b_w \cdot h + \left(b_f - b_w\right) \cdot h_f} \qquad y_t = 5.571\ in$$

Distance from the neutral axis of the gross section to the bottom of the section:

$$y_b := h - y_t \qquad y_b = 14.429\ in$$

Gross section moment of inertia:

$$I_g := \frac{1}{12} \cdot \left(b_w \cdot h^3 + \left(b_f - b_w\right) \cdot h_f^{\,3}\right) + b_w \cdot h \cdot \left(\frac{h}{2} - y_t\right)^2 + \left(b_f - b_w\right) \cdot h_f \cdot \left(\frac{h_f}{2} - y_t\right)^2$$

$$I_g = 14083.05\ in^4$$

Neutral axis location for positive moment section

Sum of moments about the neutral axis as functions of kd:

1) Neutral axis within the slab

$$f1(kd) := b_f \cdot \frac{kd^2}{2} - n \cdot A_{s_1} \cdot \left(d_1 - kd\right) + (n-1) \cdot A'_{s_1} \cdot \left(kd - d'_1\right)$$

The neutral axis for flanged beams may be above the level of the top reinforcement. In that case the effective area of of top reinforcement will be underestimated by 1/n x A's, but the top reinforcement will be in tension and will have negligible effect on the location of kd.

2) Neutral axis within the beam web

$$f2(kd) := \left(b_w \cdot \frac{kd^2}{2} + \left(b_f - b_w\right) \cdot h_f \cdot \left(kd - \frac{h_f}{2}\right)\right) - n \cdot A_{s_1} \cdot \left(d_1 - kd\right) + (n-1) \cdot A'_{s_1} \cdot \left(kd - d'_1\right)$$

Initialize vector kd:

$i := 0..2$ $\qquad\qquad kd_i := 0\ in$

Guess value of kd':

$kd' := h_f$

$kd' := \mathbf{root}(f1(kd'), kd')$ $\qquad\qquad kd' = 2.132\ in$

$kd_1 := \mathbf{if}(kd' \le h_f, kd', \mathbf{root}(f2(kd'), kd'))$ $\qquad\qquad kd_1 = 2.132\ in$

Subscript 1 refers to the positive moment section - the second element in a three element vector.

Neutral axis locations for negative moment sections

Sum of moments about the neutral axis as functions of kd:

1) Neutral axis within the slab (this case will govern for a very shallow beam web projection relative to the slab thickness)

Left end negative moment section,

$$f3(kd) := b_w \cdot \frac{kd^2}{2} + (b_f - b_w) \cdot \frac{(kd + h_f - h)^2}{2} - n \cdot A_{s_0} \cdot (d_0 - kd) + (n-1) \cdot A'_{s_0} \cdot (kd - d'_0)$$

Right end negative moment section,

$$f4(kd) := b_w \cdot \frac{kd^2}{2} + (b_f - b_w) \cdot \frac{(kd + h_f - h)^2}{2} - n \cdot A_{s_2} \cdot (d_2 - kd) + (n-1) \cdot A'_{s_2} \cdot (kd - d'_2)$$

Subscripts 0 and 2 refer to the left and right end sections - the first and third elements in a three element vector.

2) Neutral axis within the beam web

Left end negative moment section,

$$f5(kd) := b_w \cdot \frac{kd^2}{2} - n \cdot A_{s_0} \cdot (d_0 - kd) + (n-1) \cdot A'_{s_0} \cdot (kd - d'_0)$$

Right end negative moment section,

$$f6(kd) := b_w \cdot \frac{kd^2}{2} - n \cdot A_{s_2} \cdot (d_2 - kd) + (n-1) \cdot A'_{s_2} \cdot (kd - d'_2)$$

Guess value of kd':

kd for section at left end: $kd' := \frac{h}{3}$

$$kd_0 := \mathbf{if}\left(kd_0 \ge \left(h - h_f\right), \mathbf{root}\left(f3\left(kd'\right), kd'\right), \mathbf{root}\left(f5\left(kd'\right), kd'\right)\right) = 4.022\ in$$

Guess value of kd':

kd for section at right end: $kd' := \frac{h}{3}$

$$kd_2 := \mathbf{if}\left(kd_2 \ge \left(h - h_f\right), \mathbf{root}\left(f4\left(kd'\right), kd'\right), \mathbf{root}\left(f6\left(kd'\right), kd'\right)\right)$$

$$kd_2 = 6.085\ in$$

Cracked section moment of inertia for positive moment section:

$$I_{cr_1} := n \cdot A_{s_1} \cdot \left(d_1 - kd_1\right)^2 + (n-1) \cdot A'_{s_1} \cdot \left(kd_1 - d'_1\right)^2 + \begin{Vmatrix} \text{if } kd_1 \le h_f \\ \quad \begin{Vmatrix} \frac{1}{3}\ b_f \cdot \left(kd_1\right)^3 \end{Vmatrix} \\ \text{else} \\ \quad \begin{Vmatrix} \frac{1}{3}\ b_w \cdot \left(kd_1\right)^3 + \frac{1}{12}\ \left(b_f - b_w\right) \cdot h_f^{\ 3} + \left(b_f - b_w\right) \cdot h_f \cdot \left(kd_1 - \frac{h_f}{2}\right. \end{Vmatrix} \end{Vmatrix}$$

$$I_{cr_1} = 2714.32\ in^4$$

Cracked section moment of inertia for negative moment sections:

Left end section,

$$I_{cr_0} := \frac{1}{3}\ b_w \cdot \left(kd_0\right)^3 + n \cdot A_{s_0} \cdot \left(d_0 - kd_0\right)^2 + (n-1) \cdot A'_{s_0} \cdot \left(kd_0 - d'_0\right)^2 + \left(kd_0 > \left(h - h_f\right)\right) \cdot \frac{1}{3} \cdot \left(b_f - b_w\right) \cdot \left(kd_0 - h + h_f\right)^3$$

$$I_{cr_0} = 1382.1\ in^4$$

Right end section,

$$I_{cr_2} := \frac{1}{3}\ b_w \cdot \left(kd_2\right)^3 + n \cdot A_{s_2} \cdot \left(d_2 - kd_2\right)^2 + (n-1) \cdot A'_{s_2} \cdot \left(kd_2 - d'_2\right)^2 + \left(kd_2 > \left(h - h_f\right)\right) \cdot \frac{1}{3} \cdot \left(b_f - b_w\right) \cdot \left(kd_2 - h + h_f\right)^3$$

$$I_{cr_2} = 3116.49\ in^4$$

Note: Modulus of rupture (if available, the value of ft / 6.7 may be substituted for f'c and kv defined as 1, in accordance with ACI 318, Section 9.5.3.3 (a)):

$$f_r := k_v \cdot 7.5 \cdot \sqrt{\frac{f'_c}{psi}} \cdot psi \qquad k_v = 1$$

$$f_r = 474.342\ psi$$

Cracking moment for positive moment section:

$$M_{cr_1} := \frac{f_r \cdot I_g}{y_b} \qquad M_{cr_1} = 38.582\ kip \cdot ft$$

Cracking moments for negative moment sections:

$$M_{cr_0} := \frac{f_r \cdot I_g}{y_t} \qquad M_{cr_0} = 99.917\ kip \cdot ft$$

$$M_{cr_2} := \frac{f_r \cdot I_g}{y_t} \qquad M_{cr_2} = 99.917\ kip \cdot ft$$

Effective moments of inertia (ACI 318, Eq. (9-7)), modified to limit Ie to Ig:

$$i := 0..2$$

$$I_{e_i} := \mathbf{if}\left(M_{cr_i} > M_{a_i}, I_g, \left(\frac{M_{cr_i}}{M_{a_i}}\right)^3 \cdot I_g + \left(1 - \left(\frac{M_{cr_i}}{M_{a_i}}\right)^3\right) \cdot I_{cr_i}\right)$$

$$I_e = \begin{bmatrix} 14083.05 \\ 6432.22 \\ 14083.05 \end{bmatrix} in^4$$

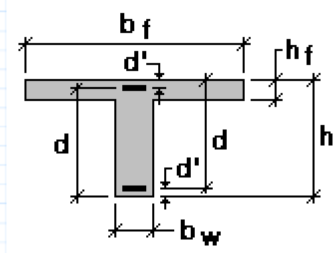
Average of the moments of inertia at the negative and positive moment sections:

$$Avg\_I_e := \frac{1}{2} \cdot \left(\frac{I_{e_0} + I_{e_2}}{2} + I_{e_1}\right) \qquad Avg\_I_e = 10257.63\ in^4$$

The average effective moment of inertia may be used for computation of deflections given in ACI 318, Section 9.5.2.4.

Summary

Input



Specified compressive strength of concrete: $f'_c = 4\ ksi$

Specified yield strength of non-prestressed reinforcement: $f_y = 60\ ksi$

Unit weight of concrete: $w_c = 145\ pcf$

Modulus of elasticity of reinforcement: $E_s = 29000\ ksi$

Flange width: $b_f = 72\ in$

Overall thickness of member: $h = 20\ in$

Strength reduction factor for lightweight concrete applied to fr (ACI 318, 9.5.2.3): $k_v = 1$

Beam web width: $b_w = 10\ in$

Flange thickness: $h_f = 4\ in$

Maximum moments in member at stage deflection is computed:

$$M_a^{\mathrm{T}} = [28\ \ 56\ \ 78]\ kip \cdot ft$$

Depths from the extreme compression fiber to the centroid of the tension reinforcement:

$$d^{\mathrm{T}} = [17.5\ \ 17.5\ \ 17.5]\ in$$

Cross-sectional areas of tension reinforcement:

$$A_s^{\mathrm{T}} = [0.8\ \ 1.32\ \ 2.22]\ in^2$$

Depths from the extreme compression fiber to the centroid of the compression reinforcement:

$$d'^{\mathrm{T}} = [2.25\ \ 2.5\ \ 2.25]\ in$$

Cross-sectional areas of compression reinforcement:

$$A'_s{}^{\mathrm{T}} = [0.4\ \ 0.88\ \ 0.62]\ in^2$$

Computed Variables

Modulus of elasticity of concrete: $E_c = 3644\ ksi$

Distance from the neutral axis to the top of the gross section: $y_t = 5.571\ in$

Gross moment of inertia neglecting reinforcement: $I_g = 14083.048\ in^4$

Cracked section moments of inertia: $I_{cr} = \begin{bmatrix} 1382.105 \\ 2714.318 \\ 3116.49 \end{bmatrix} in^4$

Effective moments of inertia: $I_e = \begin{bmatrix} 14083.048 \\ 6432.219 \\ 14083.048 \end{bmatrix} in^4$

Modular ratio of elasticity: $n = 7.958$

Distance from the neutral axis to the top of the gross section: $y_b = 14.429\ in$

Note: 1st row of the I_{cr} and I_e vectors is the left-end negative moment section, 2nd row is the mid-span positive moment section, and the third row is right-end negative moment section.

Average effective moment of inertia for computation of deflections: $Avg\_I_e = 10257.633\ in^4$