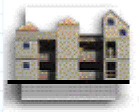


CHAPTER 5: Structural Steel Columns - ASD Design

5.2 Tubular Steel Columns

Description

As described in **Section 5.1**, the selection of steel columns, subject to axial load and bending, can sometimes involve a tedious, iterative process. The first step requires guessing at a suitable trial section or using an elaborate set of formulas to select a first trial selection for the steel columns. The second step requires checking the first trial section to ensure that it satisfies the AISC Specification (specifically, the AISC interaction equations). If the first trial selection is not satisfactory, then the process must be repeated with a second trial section. In some cases additional iterations are required to ensure that the lowest weight section is obtained.

This application allows the user to rapidly check trial sections by entering only the width, depth and tube wall thickness of trial sections. All necessary section properties and parameters required are then computed by the application. If the trial section is not satisfactory, the user may repeat the process until a satisfactory section is obtained.

This application computes section properties, compact section criteria, lateral bracing lengths, and slenderness ratios of square or rectangular tubular column sections, actual and allowable axial and bending stresses, amplification factors and moment modifiers, and the beam-column interaction equations of the AISC Specification for a column with axial load and end moments about one or both axes. This application does not cover sections with "slender compression elements" as defined in the AISC Specification.

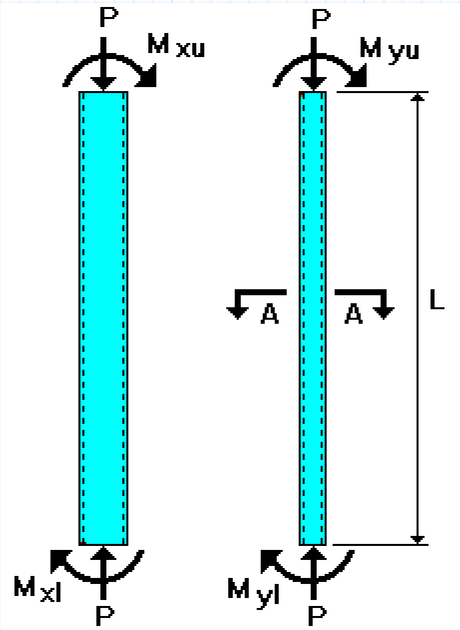
The required input includes axial load and end moments about one or both axes, column length, effective length factors, yield strength, flag denoting if column is free to sway about one or both axis directions, and the depth, width and thickness of the section.

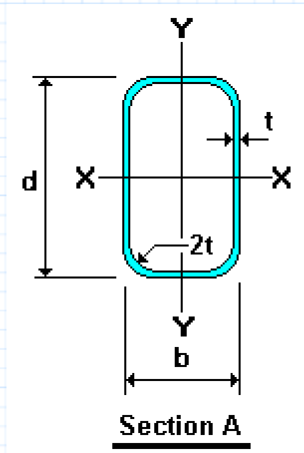
A summary listing input, the solution of the interaction equations, and a listing of all computed values is shown on pages 11-13.

Reference: AISC "Specification for Structural Steel Buildings — Allowable Stress Design and Plastic Design with Commentary." June 1, 1989

Input

Notation





Input Variables

Yield strength of steel:	$F_y := 46\ ksi$
Column axial load:	$P := 200\ kip$
Unbraced column length:	$L := 11.5\ ft$
End moment about the X axis, at the upper end of the column:	$M_{xu} := 120\ kip \cdot ft$
End moment about the X axis, at the lower end of the column:	$M_{xl} := -120\ kip \cdot ft$
End moment about the Y axis, at the upper end of the column:	$M_{yu} := 40\ kip \cdot ft$

End moment about the Y axis, at the lower end of the column: $M_{yl} := -40\ kip \cdot ft$

Note ⇒ Positive values of M indicate clockwise moments and negative values indicate counter-clockwise moments.

Effective length factor relative to the X axis: $K_x := 1.0$

Effective length factor relative to the Y axis: $K_y := 1.0$

Variable "X_Sway" defined as 0 if sidesway is prevented, and 1 if sidesway is permitted: $X\_Sway := 0$

Variable "Y_Sway" defined as 0 if sidesway is prevented, and 1 if sidesway is permitted: $Y\_Sway := 0$

Section Dimensions and Wall Thickness

Enter estimated values for the depth, width and tube wall thickness.

Section depth: $d := 16\ in$

Section width: $b := 12\ in$

Tube wall thickness: $t := 0.500\ in$

Computed Section Properties

A	cross section area of member
W	weight of section per unit length
I_x	moment of inertia about the X axis
r_x	radius of gyration about the X axis
S_x	section modulus about the X axis
I_y	moment of inertia about the Y axis
r_y	radius of gyration about the Y axis
S_y	section modulus about the Y axis

Computed Variables

f_a	axial compressive stress
f_{b_xu}	bending stress about the X axis at the upper end of the column
f_{b_xl}	bending stress about the X axis at the lower end of the column
f_{b_yu}	bending stress about the Y axis at the upper end of the column
f_{b_yl}	bending stress about the Y axis at the lower end of the column
F_a	allowable compressive axial stress (AISC Specification, Eqs. (E2-1) and (E2-2))
F_{bx}	allowable bending stress about the X axis (AISC Specification, Eqs. (F1-1), (F1-2), (F1-3) and (F1-5))
F_{by}	allowable bending stress about the Y axis (AISC Specification, Identity (F2-2) and Eq. (F2-3))
F'_{ex}	Euler's stress about the X axis divided by a factor of safety (AISC Specification, Sect. H-1)
F'_{ey}	Euler's stress about the Y axis divided by a factor of safety (AISC Specification, Sect. H-1)
C_b	moment modifier factor (AISC Specification, Sect. F1-3)
L_c	Lateral bracing length (AISC Specification, Eq. (F3-2))
CF	a variable defined within this application to reflect compact section criteria of AISC Specification, Table B5.1. (Section is compact if CF = 2, non-compact if CF = 1, and a slender compression element section not covered by this application if CF = 0.)

Calculations

This application is limited to steel with a yield strength less than or equal to 65 ksi:

$F_y := \text{if}\left(F_y > 65 \cdot ksi, 65 \cdot ksi, F_y\right)$ $\qquad F_y = 46\ ksi$

Section Properties

Cross section area:

$A := 2 \cdot t \cdot (b + d - 8 \cdot t) + 3 \cdot \pi \cdot t^2$ $\qquad A = 26.356\ in^2$

Section weight per unit length:

$W := A \cdot 3.4 \cdot \frac{lb}{ft \cdot in^2}$ $\qquad W = 89.611\ \frac{lb}{ft}$

Moment of inertia about the X axis:

$$I_x := \frac{t \cdot (d - 4\ t)^3}{6} + \frac{t \cdot (b - 4\ t) \cdot (d - t)^2}{2} + \frac{(b - 4\ t) \cdot t^3}{6} + 15\ t^4 \cdot \left(\frac{\pi}{8} - \frac{8}{9\ \pi}\right) + 3\ \pi \cdot t^2 \cdot \left(\frac{d}{2} - \frac{2}{3}\ t \cdot \left(3 - \frac{14}{3\ \pi}\right)\right)^2$$

$I_x = 961.967\ in^4$

Section modulus about the X axis:

$S_x := \frac{2\ I_x}{d}$ $\qquad S_x = 120.246\ in^3$

Radius of gyration about the X axis:

$r_x := \sqrt{\frac{I_x}{A}}$ $\qquad r_x = 6.041\ in$

Moment of inertia about the Y axis:

$$I_y := \frac{t \cdot (b - 4\ t)^3}{6} + \frac{t \cdot (d - 4\ t) \cdot (b - t)^2}{2} + \frac{(d - 4\ t) \cdot t^3}{6} + 15\ t^4 \cdot \left(\frac{\pi}{8} - \frac{8}{9\ \pi}\right) + 3\ \pi \cdot t^2 \cdot \left(\frac{b}{2} - \frac{2}{3}\ t \cdot \left(3 - \frac{14}{3\ \pi}\right)\right)^2$$

$I_y = 617.752\ in^4$

Section modulus about the X axis:

$S_y := \frac{2\ I_y}{b} = 102.959\ in^3$

Radius of gyration about the X axis:

$$r_y := \sqrt{\frac{I_y}{A}} = 4.841\ in$$

Axial and Bending Stresses

Axial stress:

$$f_a := \frac{P}{A} = 7.588\ ksi$$

Bending stress due to moment about the X axis at the upper end of the column:

$$f_{b\_xu} := \frac{M_{xu}}{S_x} = 11.975\ ksi$$

Bending stress due to moment about the X axis at the lower end of the column:

$$f_{b\_xl} := \frac{M_{xl}}{S_x} = -11.975\ ksi$$

Bending stress due to moment about the Y axis at the upper end of the column:

$$f_{b\_yu} := \frac{M_{yu}}{S_y} = 4.662\ ksi$$

Bending stress due to moment about the Y axis at the lower end of the column:

$$f_{b\_yl} := \frac{M_{yl}}{S_y} = -4.662\ ksi$$

Slenderness ratios about the X and Y axes:

$$\frac{K_x \cdot L}{r_x} = 22.8 \qquad \frac{K_y \cdot L}{r_y} = 28.5$$

Larger slenderness ratio:

$$SR := \textbf{if}\left(\frac{K_x \cdot L}{r_x} > \frac{K_y \cdot L}{r_y}, \frac{K_x \cdot L}{r_x}, \frac{K_y \cdot L}{r_y}\right) = 28.504$$

If SR is >200 a larger section or bracing is required.

Modulus of elasticity of steel:

$$E := 29000 \cdot ksi$$

Column slenderness ratio separating elastic and inelastic buckling (see AISC Specification, Eq. (E2-1)):

$$C_c := \sqrt{\frac{2 \cdot \pi^2 \cdot E}{F_y}} = 111.554$$

Compact Section Criteria

Section is compact if CF = 2, non-compact if CF = 1, and a slender compression element section not covered by this application if CF = 0 (AISC Specification, Sect. B5 and Table B5.1).

$$\frac{b - 3\ t}{t} = 21$$

$$CF := \text{if}\left(\frac{b-3\ t}{t} \le \frac{190}{\sqrt{F_y \cdot ksi^{-1}}}, 2, \text{if}\left(\frac{b-3 \cdot t}{t} \le \frac{238}{\sqrt{F_y \cdot ksi^{-1}}}, 1, 0\right)\right) = 2$$

An inside bending radius of 1.5t is assumed in this equation to allow for tubes produced with a radius smaller than the 2t value used to calculate section properties.

Allowable axial stress (AISC Specification, Eqs. (E2-1) and (E2-2)) with SR substituted for Kl/r. If SR is greater than 200 or if the section is a "slender compression element" this equation will return F_a equal to 0:

$$F_a := \text{if}\left(\left(\left(SR \ge C_c\right) \cdot \left(SR \le 200\right)\right), \frac{12 \cdot \pi^2 \cdot E}{23 \cdot SR^2}, \text{if}\left(\left(SR > 200\right) + \left(CF = 0\right), 0 \cdot ksi, \frac{\left(1 - \frac{SR^2}{2 \cdot C_c^{\ 2}}\right) \cdot F_y}{\frac{5}{3} + \frac{3 \cdot SR}{8 \cdot C_c} - \frac{SR^3}{8 \cdot C_c^{\ 3}}}\right)\right)$$

$$F_a = 25.3\ ksi$$

Ratio of the smaller end moment M_{xa}, to the larger end moment M_{xb} about the X axis:

$$M_{xa} := \text{if}\left(\left|M_{xu}\right| < \left|M_{xl}\right|, M_{xu}, M_{xl}\right)$$

$$M_{xb} := \text{if}\left(M_{xa} = M_{xu}, M_{xl}, M_{xu}\right)$$

$$\frac{M_{xa}}{M_{xb}} = -1$$

Ratio of the smaller end moment M_{ya}, to the larger end moment M_{yb} about the Y axis:

$$M_{ya} := \text{if}\left(\left|M_{yu}\right| < \left|M_{yl}\right|, M_{yu}, M_{yl}\right)$$

$$M_{yb} := \text{if}\left(M_{ya} = M_{yu}, M_{yl}, M_{yu}\right)$$

$$\frac{M_{ya}}{M_{yb}} = -1$$

Moment modifier factor Cb (AISC Specification, Sect. F1-3):

$$C_b := 1.75 + 1.05 \cdot \frac{M_{xa}}{M_{xb}} + 0.3 \cdot \left(\frac{M_{xa}}{M_{xb}}\right)^2$$

$$C_b := \text{if}\left(\left(X\_Sway = 0\right) \cdot \left(f_a > 0 \cdot ksi\right), 1.0, \text{if}\left(C_b < 2.3, C_b, 2.3\right)\right)$$

$$C_b = 1$$

If the ends of the member are braced to prevent sidesway in the X direction and the axial stress is greater than 0 ksi, C_b equals 1.0.

Lateral bracing length L_c (AISC Specification, Eq. (F3-2)):

$$L_c := \text{if}\left(\left(1950 + 1200 \cdot \frac{M_{xa}}{M_{xb}}\right) > 1200, \left(1950 + 1200 \cdot \frac{M_{xa}}{M_{xb}}\right) \cdot \frac{b \cdot ksi}{F_y}, 1200 \cdot \frac{b \cdot ksi}{F_y}\right) = 26.087\ ft$$

Allowable bending stress about strong or weak axis, for compact or non-compact sections (AISC Specification, Eqs. (F3-1), (F3-2), and (F3-3)). If the section contains a "slender compression element" this equation returns Fb equal to 0:

$$F_b := \text{if}\left(CF = 0, 0 \cdot ksi, \text{if}\left(\left(CF = 2\right) \cdot \left(L \le L_c\right) \cdot \left(d \le 6 \cdot b\right), 0.66 \cdot F_y, 0.60 \cdot F_y\right)\right) = 30.36\ ksi$$

Euler's stress for the X and Y directions, divided by a factor or safety and using the actual unbraced length and corresponding radius of gyration in the plain of bending (AISC Specification, Sect. H-1):

$$F'_{ex} := \frac{12 \cdot \pi^2 \cdot E}{23 \cdot \left(\frac{K_x \cdot L}{r_x}\right)^2} = 286.201\ ksi$$

$$F'_{ey} := \frac{12 \cdot \pi^2 \cdot E}{23 \cdot \left(\frac{K_y \cdot L}{r_y}\right)^2} = 183.791\ ksi$$

Moment modifier coefficients Cmx and Cmy (AISC Specification, Sect. H-1):

$$C_{mx} := \text{if}\left(X\_Sway = 0, 0.6 - 0.4 \cdot \left(\frac{M_{xa}}{M_{xb}}\right), 0.85\right) = 1$$

$$C_{my} := \text{if}\left(Y\_Sway = 0, 0.6 - 0.4 \cdot \left(\frac{M_{ya}}{M_{yb}}\right), 0.85\right) = 1$$

AISC Specification, Eqs. (H1-1) and (H1-2) for combined axial compression and bending, expressed as functions of bending stresses about the X and Y axes:

$$H1\_1\left(f_{bx}, f_{by}\right) := \frac{f_a}{F_a} + \frac{C_{mx} \cdot f_{bx}}{\left(1 - \frac{f_a}{F'_{ex}}\right) \cdot F_b} + \frac{C_{my} \cdot f_{by}}{\left(1 - \frac{f_a}{F'_{ey}}\right) \cdot F_b}$$

$$H1\_2\left(f_{bx}, f_{by}\right) := \frac{f_a}{0.60 \cdot F_y} + \frac{f_{bx}}{F_b} + \frac{f_{by}}{F_b}$$

At upper end of column:

$$H1\_1\left(\left|f_{b\_xu}\right|, \left|f_{b\_yu}\right|\right) = 0.866$$

$$H1\_2\left(\left|f_{b\_xu}\right|, \left|f_{b\_yu}\right|\right) = 0.823$$

At lower end of column:

$$H1\_1\left(\left|f_{b\_xl}\right|, \left|f_{b\_yl}\right|\right) = 0.866$$

$$H1\_2\left(\left|f_{b\_xl}\right|, \left|f_{b\_yl}\right|\right) = 0.823$$

Maximum value of H1_1 or H1_2:

$$HEQ := \begin{bmatrix} H1\_1\left(f_{b\_xu}, f_{b\_yu}\right) & H1\_1\left(f_{b\_xl}, f_{b\_yl}\right) \\ H1\_2\left(f_{b\_xu}, f_{b\_yu}\right) & H1\_2\left(f_{b\_xl}, f_{b\_yl}\right) \end{bmatrix}$$

$$\max(HEQ) = 0.866$$

If the maximum value of interaction equations H1_1 or H1_2 is less than or equal 1.0 the section is adequate. If the section is too small or too large, a new trial section may be entered.

Summary

Tube size:

Section depth: $d = 16\ in$

Section width: $b = 12\ in$

Tube wall thickness: $t = 0.5\ in$

Input

Yield strength of steel:	$F_y = 46\ ksi$
Column axial load:	$P = 200\ kip$
Unbraced column length:	$L = 11.5\ ft$
End moment about the X axis, at the upper end of the column:	$M_{xu} = 120\ kip \cdot ft$
End moment about the X axis, at the lower end of the column:	$M_{xl} = -120\ kip \cdot ft$
End moment about the Y axis, at the upper end of the column:	$M_{yu} = 40\ kip \cdot ft$
End moment about the Y axis, at the lower end of the column:	$M_{yl} = -40\ kip \cdot ft$
Effective length factor relative to the X axis:	$K_x = 1$
Effective length factor relative to the Y axis:	$K_y = 1$
Variable "X_Sway" defined as 0 if sidesway is prevented, and 1 if sidesway is permitted:	$X\_Sway = 0$
Variable "Y_Sway" defined as 0 if sidesway is prevented, and 1 if sidesway is permitted:	$Y\_Sway = 0$
Maximum value of H1_1 or H1_2:	$\max(HEQ) = 0.866$

Note ⇒ If the maximum value of H1_1 or H1_2 is significantly larger than 1.0, a stronger section must be used.

Estimated trial tube wall thickness for follow-up estimates, assuming no change in b_f or d. Suggested value of t may be too thin or unavailable:

$\max(HEQ) \cdot t = 0.433\ in$

Intermediate Computed Values:
(Refer to the document for definitions)

$A=26.356\ in^2$

$I_x=961.967\ in^4$

$r_x=6.041\ in$

$S_x=120.246\ in^3$

$I_y=617.752\ in^4$

$r_y=4.841\ in$

$S_y=102.959\ in^3$

$L_c=26.087\ ft$

$CF=2$

$f_a=7.588\ ksi$

$f_{b\_xu}=11.975\ ksi$

$f_{b\_xl}=-11.975\ ksi$

$f_{b\_yu}=4.662\ ksi$

$f_{b\_yl}=-4.662\ ksi$

$\frac{K_x \cdot L}{r_x}=22.842$

$\frac{K_y \cdot L}{r_y}=28.504$

$F_a=25.277\ ksi$

$F_b=30.36\ ksi$

$F'_{ex}=286.201\ ksi$

$F'_{ey}=183.791\ ksi$

$C_c=111.554$

$C_b=1$

$C_{mx}=1$

$C_{my}=1$