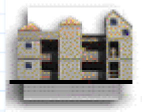


CHAPTER 6: Reinforced Concrete Flat Plates

6.1 Direct Design Moments and Flexural Reinforcement

Description

Flat plate construction consists of a cast-in-place concrete slab of uniform thickness, reinforced with top and bottom reinforcing bars in two directions. Flat plates are usually chosen for any type of building where a minimum floor to floor height is required. They are used for their uniform minimum depth construction and simple formwork, and are commonly used in multi-story apartment buildings.

This application calculates the service loads, the factored loads, the bending moments and the required flexural reinforcement for two-way reinforced concrete slab systems of uniform thickness (also referred to as "flat plates"), meeting the limitations of Section 13.6 (Direct Design Method) of ACI 318-89. **Section 6.2** covers the design of flat plates for shear.

In order to use the Direct Design Method for flat plates the following criteria must be met: There must be three continuous spans in each direction; the ratio of the longer to shorter span center-to-center of supports within a panel must be not greater than 2; successive span lengths center-to-center of supports in each direction shall not differ by more than one-third the longer span; columns may not be offset more than 10% of the span from either axis between centerlines of successive columns; all loads shall be due to gravity load only - uniformly distributed over the panel; and live load shall not exceed three times dead load.

The required input for this application includes the strengths of the concrete and the reinforcement, the unit weight of concrete, design live load per unit area, superimposed dead load per unit area, clear span lengths in the long and short direction, width of slab design strip transverse to clear spans, measured center-to-center of adjacent panels or from exterior edge of slab to center of panel, clear concrete cover of reinforcement, and estimated top and bottom bar sizes.

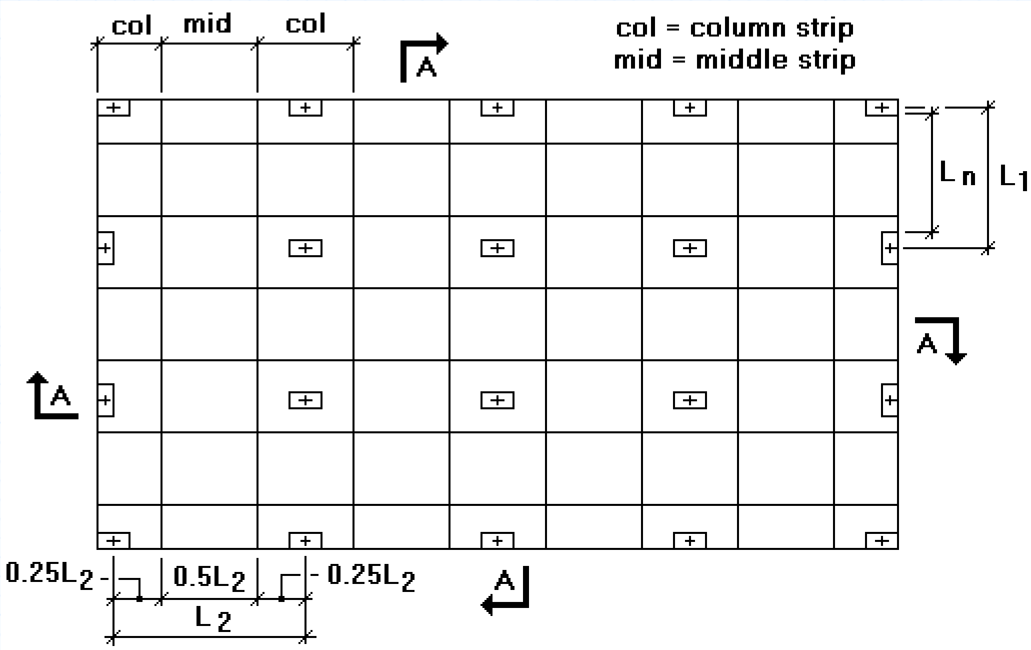
A summary of input and computed variables is given on pages 14-16.

Reference:

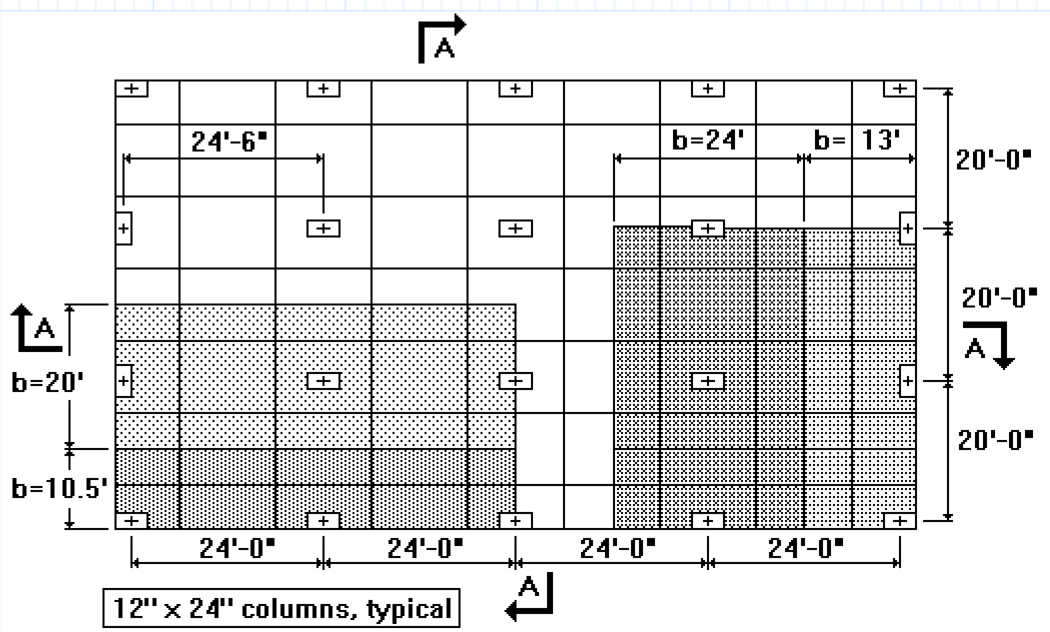
ACI 318-89 "Building Code Requirements for Reinforced Concrete." (Revised 1992)

Input

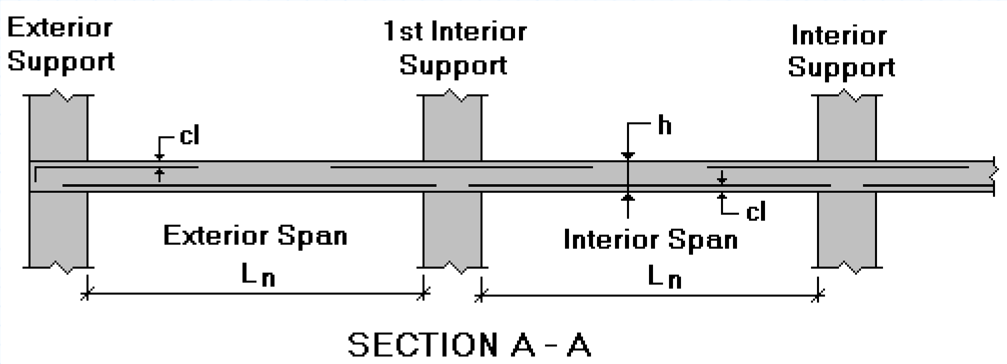
Notation



The following sketch shows the panel widths and spans used in the example.



	Full panel strip, longer span		Full panel strip, shorter span
	Half panel edge strip, longer span		Half panel edge strip, shorter span



Input Variables

Enter uniformly distributed service live load and superimposed dead load.

Service live load: $w_l := 50 \cdot psf$

Service dead load per unit area excluding slab weight: $w_{sd} := 10 \cdot psf$

Enter reinforcing bar size designation numbers (BarSize):

$$BarSize := \begin{bmatrix} 5 \\ 4 \\ 4 \\ 4 \end{bmatrix}$$

column strip top bars
middle strip top bars
column strip bottom bars
middle strip bottom bars

$$\begin{bmatrix} x0 \\ x1 \\ x2 \\ x3 \end{bmatrix} := BarSize$$

Enter b as a column vector, and Ln as a two column matrix with the number of rows equal to the number of design strips. The 1st column of the Ln matrix must be exterior spans, and second column must be interior spans. The panel widths and spans for each design strip must be in the same row.

Panel width and clear span for design strips with reinforcing bars in the outer layers (normally the longer span):

$$b_{long} := \begin{bmatrix} 20 \\ 10.5 \end{bmatrix} \cdot ft \qquad L_{n\_long} := \begin{bmatrix} 23 & 22 \\ 22 & 22 \end{bmatrix} \cdot ft$$

← Full width strip
← Half panel strip

Panel width and clear span for design strips with reinforcing bars in the inner layers (normally the shorter span):

$$b_{short} := \begin{bmatrix} 24 \\ 13 \end{bmatrix} \cdot ft \qquad L_{n\_short} := \begin{bmatrix} 19 & 19 \\ 18.5 & 18 \end{bmatrix} \cdot ft$$

← Full width strip
← Half panel strip

Computed Variables

The following variables are calculated in this document:

h	overall thickness of slab
w_u	total factored load per unit area
M_o	total factored static moment
k_{ext}	moment coefficients for exterior spans
k_{int}	moment coefficients for interior spans
M_{ext}	column and middle strip factored moments, exterior spans
M_{int}	column and middle strip factored moments, interior spans
ρ	required reinforcement ratios for each design section
A_s	required area of reinforcement at each design section

NumbBars   number of bars required at each design section

Material Properties and Constants

Enter values for f'_c, f_y, w_c, and w_{rc} if different from that shown.

Specified compressive strength of concrete: $f'_c := 4 \cdot \mathbf{ksi}$

Specified yield strength of reinforcement (f_y may not exceed 60 ksi, ACI 318, 11.5.2): $f_y := 60 \cdot \mathbf{ksi}$

Unit weight of concrete: $w_c := 145 \cdot \mathbf{pcf}$

Weight of reinforced concrete: $w_{rc} := 150 \cdot \mathbf{pcf}$

Modulus of elasticity of reinforcement (ACI 318, 8.5.2): $E_s := 29000 \cdot \mathbf{ksi}$

Strain in concrete at compression failure (ACI 318, 10.3.2): $\varepsilon_c := 0.003$

Strength reduction factor for flexure (ACI 318, 9.3.2.1): $\phi_f := 0.9$

Clear concrete cover of longitudinal reinforcement: $cl := \frac{3}{4} \cdot in$

Multiple for rounding slab thickness: $SzF := \frac{1}{2} \cdot in$

Ratio of live load to dead load: $R := 1$

Combined load factor for dead + live load: $F := \frac{1.4 + 1.7 \cdot R}{1 + R} = 1.55$

Reinforcing bar number designations, diameters, and areas:

$$No := [0\ \ 1\ \ 2\ \ 3\ \ 4\ \ 5\ \ 6\ \ 7\ \ 8\ \ 9\ \ 10\ \ 11\ \ 12\ \ 13\ \ 14\ \ 15\ \ 16\ \ 17\ \ 18]^T$$

$$d_b := [0\ \ 0\ \ 0\ \ 0.375\ \ 0.5\ \ 0.625\ \ 0.75\ \ 0.875\ \ 1.00\ \ 1.128\ \ 1.27\ \ 1.41\ \ 0\ \ 0\ \ 1.693\ \ 0\ \ 0\ \ 0\ \ 2.257]^T \cdot in$$

$$A_b := [0\ \ 0\ \ 0\ \ 0.11\ \ 0.20\ \ 0.31\ \ 0.44\ \ 0.60\ \ 0.79\ \ 1.00\ \ 1.27\ \ 1.56\ \ 0\ \ 0\ \ 2.25\ \ 0\ \ 0\ \ 0\ \ 4.00]^T \cdot in^2$$

Bar numbers, diameters and areas are in the vector rows (or columns in the transposed vectors shown) corresponding to the bar numbers. Individual bar numbers, diameters, areas and development lengths and splices of a specific bar can be referred to by using the vector subscripts as shown in the example below.

Example: $No_5 = 5$ $\quad d_{b_5} = 0.625\ in$ $\quad A_{b_5} = 0.31\ in^2$

The following values are computed from the entered material properties.

Modulus of elasticity of concrete for values of wc between 90 pcf and 155 pcf (ACI 318, 8.5.1):

$$E_c := \left(\frac{w_c}{pcf}\right)^{1.5} \cdot 33 \cdot \sqrt{\frac{f'_c}{psi}} \cdot psi = 3644\ ksi$$

Strain in reinforcement at yield stress:

$$\varepsilon_y := \frac{f_y}{E_s} = 0.002$$

Factor used to calculate depth of equivalent rectangular stress block (ACI 318, 10.2.7.3):

$$\beta_1 := \mathbf{if}\left(\left(f'_c \ge 4 \cdot ksi\right) \cdot \left(f'_c \le 8 \cdot ksi\right), 0.85 - 0.05 \cdot \frac{f'_c - 4 \cdot ksi}{ksi}, \mathbf{if}\left(\left(f'_c \le 4 \cdot ksi\right), 0.85, 0.65\right)\right)$$

$$\beta_1 = 0.85$$

Reinforcement ratio producing balanced strain conditions (ACI 318, 10.3.2):

$$\rho_b := \frac{\beta_1 \cdot 0.85 \cdot f'_c}{f_y} \cdot \frac{E_s \cdot \varepsilon_c}{E_s \cdot \varepsilon_c + f_y} = 2.851\%$$

Maximum reinforcement ratio (ACI 318, 10.3.3):

$$\rho_{max} := \frac{3}{4} \cdot \rho_b = 2.138\%$$

Minimum reinforcement ratio for beams (ACI 318, 10.5.1, Eq. (10-3)):

$$\rho_{min} := \frac{200}{f_y} \cdot \frac{\mathbf{lbf}}{\mathbf{in}^2} = 0.333\%$$

Shrinkage and temperature reinforcement ratio (ACI 318, 7.12.2.1):

$$\rho_{temp} := \begin{array}{l} \text{if } f_y \le 50\ \mathbf{ksi} \\ \quad 0.002 \\ \text{else if } f_y \le 60\ \mathbf{ksi} \\ \quad 0.002 - \dfrac{f_y}{60 \cdot \mathbf{ksi}} \cdot 0.0002 \\ \text{else if } \dfrac{0.0018 \cdot 60 \cdot \mathbf{ksi}}{f_y} \ge 0.0014 \\ \quad \dfrac{0.0018 \cdot 60 \cdot \mathbf{ksi}}{f_y} \\ \text{else} \\ \quad 0.0014 \end{array} = 0.18\%$$

Preferred reinforcement ratio:

$$\rho := \frac{1}{2} \cdot \rho_{max} = 1.069\%$$

Flexural coefficient K, for rectangular beams or slabs, as a function of ρ (ACI 318, 10.2):
(Moment capacity $\phi M_n = K(\rho)F$, where $F = bd^2$)

$$K(\rho) := \phi_f \cdot \rho \cdot \left(1 - \frac{\rho \cdot f_y}{2 \cdot 0.85 \cdot f'_c}\right) \cdot f_y$$

Calculations

Design strip panel widths and spans combined:

$$b := \text{augment}\left(b_{long}^{T}, b_{short}^{T}\right)^{T}$$

$$L_n := \text{augment}\left(L_{n\_long}^{T}, L_{n\_short}^{T}\right)^{T}$$

$$b^{T} = \begin{bmatrix} 20 & 10.5 & 24 & 13 \end{bmatrix} \, ft$$

$$L_n^{T} = \begin{bmatrix} 23 & 22 & 19 & 18.5 \\ 22 & 22 & 19 & 18 \end{bmatrix} \, ft$$

Minimum slab thickness (unless deflections are checked)

The revised 1992 edition of ACI 318-89 presents minimum thickness requirements for flat plates in tabular form, however the formulas of the 1989 edition are valid and preferable for mathematical expression, and therefore used for this application.

Longest clear span and minimum h, for interior spans:

$$\max\left(L_n^{\langle 1 \rangle}\right) = 22 \, ft$$

$$Int\_h_{min} := \frac{800 \cdot psi + 0.005 \cdot f_y}{36000 \cdot psi} \cdot \max\left(L_n^{\langle 1 \rangle}\right) = 8.067 \, in$$

Longest clear span and minimum h, for exterior spans:

$$\max\left(L_n^{\langle 0 \rangle}\right) = 23 \, ft$$

$$Ext\_h_{min} := 1.10 \cdot \left(\frac{800 \cdot psi + 0.005 \cdot f_y}{36000 \cdot psi} \cdot \max\left(L_n^{\langle 0 \rangle}\right)\right) = 9.277 \, in$$

Since slab thickness is customarily uniform for flat plates, the minimum thickness will be the larger thickness required for either the interior or exterior span or the absolute code minimum of 5 inches:

$$h_e := Ext\_h_{min} \qquad h_i := Int\_h_{min}$$

$$h_{min} := \textbf{if}\left(\left(h_e > h_i\right) \cdot \left(h_e > 5 \, in\right), h_e, \textbf{if}\left(\left(h_i > h_e\right) \cdot \left(h_e > 5 \, in\right), h_e, 5 \, in\right)\right) = 9.277 \, in$$

Slab thickness h defined as h_{min} rounded up to the nearest multiple of the rounding factor SzF:

$$h := SzF \cdot \text{ceil}\left(\frac{h_{min}}{SzF}\right) = 9.5 \, in$$

Factored design load w_u:

$$w_u := 1.7 \cdot w_l + 1.4 \cdot \left(w_{sd} + h \cdot w_{rc}\right) = 265.25 \, psf$$

Moment coefficients for exterior spans:

$$k_{ext} := \begin{bmatrix} (0.26 \cdot 100)\% \\ (0.26 \cdot 0)\% \\ (0.52 \cdot 60)\% \\ (0.52 \cdot 40)\% \\ (0.70 \cdot 75)\% \\ (0.70 \cdot 25)\% \end{bmatrix}$$

1st row ← column strip exterior support negative moment coefficient
2nd row ← middle strip exterior support negative moment coefficient
3rd row ← column strip exterior span positive moment coefficient
4th row ← middle strip exterior span positive moment coefficient
5th row ← column strip 1st interior support negative moment coefficient
6th row ← middle strip 1st interior support negative moment coefficient

Moment coefficients for interior spans:

$$k_{int} := \begin{bmatrix} (0.65 \cdot 75)\% \\ (0.65 \cdot 25)\% \\ (0.35 \cdot 60)\% \\ (0.35 \cdot 40)\% \end{bmatrix}$$

1st row ← column strip interior negative moment coefficient
2nd row ← middle strip interior negative moment coefficient
3rd row ← column strip interior positive moment coefficient
4th row ← middle strip interior positive moment coefficient

The column strip is assigned 100% of the exterior negative moment. The exterior middle strip will be provided with minimum shrinkage and temperature reinforcement.

The range variable i is the index range for the design strips (matrix rows) entered, and the range variable j1 is the index range for exterior and interior spans (matrix columns):

$i := 0 .. \text{rows}(b) - 1$ $\qquad$ $j1 := 0 .. 1$

Design strips (rows):

$\text{rows}(b) = 4$

Static moments Mo:

$$M_{o_{i,j1}} := \frac{w_u \cdot b_i \cdot \left(L_{n_{i,j1}}\right)^2}{8} \qquad M_o = \begin{bmatrix} 350.8 & 321 \\ 168.5 & 168.5 \\ 287.3 & 287.3 \\ 147.5 & 139.7 \end{bmatrix} kip \cdot ft$$

The range variable j2 is the index range for the six design moments for each exterior span, and the range variable j3 is the index range for the four design moments for each interior span:

$j2 := 0 .. 5$ $\qquad$ $j3 := 0 .. 3$

Exterior span column and middle strip moments:

$$M_{ext_{i,j2}} := k_{ext_{j2}} \cdot M_{o_{i,0}} \qquad M_{ext} = \begin{bmatrix} 91.2 & 0 & 109.4 & 73 & 184.2 & 61.4 \\ 43.8 & 0 & 52.6 & 35 & 88.5 & 29.5 \\ 74.7 & 0 & 89.6 & 59.8 & 150.8 & 50.3 \\ 38.4 & 0 & 46 & 30.7 & 77.4 & 25.8 \end{bmatrix} kip \cdot ft$$

Interior span column and middle strip moments:

$$M_{int_{i,j3}} := k_{int_{j3}} \cdot M_{o_{i,1}}$$

$$M_{int} = \begin{bmatrix} 156.5 & 52.2 & 67.4 & 44.9 \\ 82.1 & 27.4 & 35.4 & 23.6 \\ 140 & 46.7 & 60.3 & 40.2 \\ 68.1 & 22.7 & 29.3 & 19.6 \end{bmatrix} kip \cdot ft$$

Column and middle strip moments combined for exterior and interior spans:

$$M_u := \text{augment}\left(M_{ext}, M_{int}\right)$$

$$M_u = \begin{bmatrix} 91.2 & 0 & 109.4 & 73 & 184.2 & 61.4 & 156.5 & 52.2 & 67.4 & 44.9 \\ 43.8 & 0 & 52.6 & 35 & 88.5 & 29.5 & 82.1 & 27.4 & 35.4 & 23.6 \\ 74.7 & 0 & 89.6 & 59.8 & 150.8 & 50.3 & 140 & 46.7 & 60.3 & 40.2 \\ 38.4 & 0 & 46 & 30.7 & 77.4 & 25.8 & 68.1 & 22.7 & 29.3 & 19.6 \end{bmatrix} kip \cdot ft$$

Reinforcing bar number designations, diameters, and single bar areas for each design section:

$j := 0 \,..\, 9$ $\qquad A_i := 1$

$$BarNo_{i,j} := \left(\begin{bmatrix} No_{x0} & No_{x1} & No_{x2} & No_{x3} & No_{x0} & No_{x1} & No_{x0} & No_{x1} & No_{x2} & No_{x3} \end{bmatrix}^{T}\right)_j \cdot A_i$$

$$BarDia_{i,j} := \left(\begin{bmatrix} d_{b_{x0}} & d_{b_{x1}} & d_{b_{x2}} & d_{b_{x3}} & d_{b_{x0}} & d_{b_{x1}} & d_{b_{x0}} & d_{b_{x1}} & d_{b_{x2}} & d_{b_{x3}} \end{bmatrix}^{T}\right)_j \cdot A_i$$

$$BarArea_{i,j} := \left(\begin{bmatrix} A_{b_{x0}} & A_{b_{x1}} & A_{b_{x2}} & A_{b_{x3}} & A_{b_{x0}} & A_{b_{x1}} & A_{b_{x0}} & A_{b_{x1}} & A_{b_{x2}} & A_{b_{x3}} \end{bmatrix}^{T}\right)_j \cdot A_i$$

BarDia and BarArea are the bar diameters and bar areas for the standard bar sizes (BarNo), displayed below:

$$BarNo = \begin{bmatrix} 5 & 4 & 4 & 4 & 5 & 4 & 5 & 4 & 4 & 4 \\ 5 & 4 & 4 & 4 & 5 & 4 & 5 & 4 & 4 & 4 \\ 5 & 4 & 4 & 4 & 5 & 4 & 5 & 4 & 4 & 4 \\ 5 & 4 & 4 & 4 & 5 & 4 & 5 & 4 & 4 & 4 \end{bmatrix}$$

Range variables i1 and i2, the index ranges for the "long" and "short" design spans:

$i1 := 0 \,..\, \text{last}\left(b_{long}\right)$ $\qquad i2 := \text{rows}\left(b_{long}\right) \,..\, \text{last}\left(b\right)$

$\text{last}\left(b_{long}\right) = 1$ $\qquad \text{rows}\left(b_{long}\right) = 2$ $\qquad \text{last}\left(b\right) = 3$

Effective slab depths for the "long" and "short" design spans:

$d_{i1,j} := h - cl - \frac{1}{2} \cdot BarDia_{i1,j}$ $\qquad d_{i2,j} := h - cl - \frac{3}{2} \cdot BarDia_{i2,j}$

$$d = \begin{bmatrix} 8.438 & 8.5 & 8.5 & 8.5 & 8.438 & 8.5 & 8.438 & 8.5 & 8.5 & 8.5 \\ 8.438 & 8.5 & 8.5 & 8.5 & 8.438 & 8.5 & 8.438 & 8.5 & 8.5 & 8.5 \\ 7.813 & 8 & 8 & 8 & 7.813 & 8 & 7.813 & 8 & 8 & 8 \end{bmatrix} in$$

$$\begin{matrix} 7.813 & 8 & 8 & 8 & 7.813 & 8 & 7.813 & 8 & 8 & 8 \\ 7.813 & 8 & 8 & 8 & 7.813 & 8 & 7.813 & 8 & 8 & 8 \end{matrix}\Bigg]$$

Theoretical reinforcement ratios required for flexure ρ':

$$\rho'_{i,j} := \left(1 - \left(\sqrt{1 - \frac{2 \cdot M_{u_{i,j}}}{\phi_f \cdot \frac{b_i}{2} \cdot \left(d_{i,j}\right)^2 \cdot 0.85 \cdot f'_c}}\right)\right) \cdot \frac{0.85 \cdot f'_c}{f_y}$$

$$\rho' = \begin{bmatrix} 0.242\% & 0 & 0.288\% & 0.19\% & 0.501\% & 0.16\% & 0.423\% & 0.135\% & 0.175\% & 0.116\% \\ 0.221\% & 0 & 0.263\% & 0.174\% & 0.457\% & 0.146\% & 0.423\% & 0.135\% & 0.175\% & 0.116\% \\ 0.192\% & 0 & 0.22\% & 0.146\% & 0.395\% & 0.123\% & 0.366\% & 0.114\% & 0.147\% & 0.098\% \\ 0.182\% & 0 & 0.209\% & 0.138\% & 0.374\% & 0.116\% & 0.327\% & 0.102\% & 0.132\% & 0.088\% \end{bmatrix}$$

Maximum calculated reinforcement ratio max(ρ') and maximum useable reinforcement ratio ρ_{max}:

$$\max\left(\rho'\right) = 0.501\% \qquad \rho_{max} = 2.138\%$$

Slab thickness must be increased if max(ρ') exceeds ρ_{max}.

Design reinforcement ratios with minimum shrinkage and temperature reinforcement substituted for values of $\rho < \rho_{temp}$ * h/d:

$$\rho_{i,j} := \text{if}\left(\frac{\rho_{temp} \cdot h}{d_{i,j}} > \rho'_{i,j}, \frac{\rho_{temp} \cdot h}{d_{i,j}}, \rho'_{i,j}\right)$$

$$\rho = \begin{bmatrix} 0.242\% & 0.201\% & 0.288\% & 0.201\% & 0.501\% & 0.201\% & 0.423\% & 0.201\% & 0.201\% & 0.201\% \\ 0.221\% & 0.201\% & 0.263\% & 0.201\% & 0.457\% & 0.201\% & 0.423\% & 0.201\% & 0.201\% & 0.201\% \\ 0.219\% & 0.214\% & 0.22\% & 0.214\% & 0.395\% & 0.214\% & 0.366\% & 0.214\% & 0.214\% & 0.214\% \\ 0.219\% & 0.214\% & 0.214\% & 0.214\% & 0.374\% & 0.214\% & 0.327\% & 0.214\% & 0.214\% & 0.214\% \end{bmatrix}$$

Required reinforcement areas A_s:

$$A_{s_{i,j}} := \rho_{i,j} \cdot \frac{b_i}{2} \cdot d_{i,j}$$

$$A_s = \begin{bmatrix} 2.45 & 2.05 & 2.94 & 2.05 & 5.07 & 2.05 & 4.28 & 2.05 & 2.05 & 2.05 \\ 1.18 & 1.08 & 1.41 & 1.08 & 2.43 & 1.08 & 2.25 & 1.08 & 1.08 & 1.08 \\ 2.46 & 2.46 & 2.54 & 2.46 & 4.44 & 2.46 & 4.12 & 2.46 & 2.46 & 2.46 \\ 1.33 & 1.33 & 1.33 & 1.33 & 2.28 & 1.33 & 1.99 & 1.33 & 1.33 & 1.33 \end{bmatrix} in^2$$

Theoretical number and spacing of bars with minimum required area of reinforcement:

$$NumbBars\_1 := \overrightarrow{\frac{A_s}{BarArea}}$$

$$BarSpacing\_1_{i,j} := \frac{b_i}{2 \cdot NumbBars\_1_{i,j}}$$

Maximum permissible bar spacing, the smaller of 3 x h or 18 inches:

$$MaxSpacing := \textbf{if}\,(3 \cdot h > 18 \cdot in, 18 \cdot in, 3 \cdot h) \qquad MaxSpacing = 18\ in$$

Theoretical bar spacing with spacing no greater than the maximum spacing:

$$BarSpacing\_2_{i,j} := \textbf{if}\left(\frac{b_i}{2 \cdot NumbBars\_1_{i,j}} < MaxSpacing, \frac{b_i}{2 \cdot NumbBars\_1_{i,j}}, MaxSpacing\right)$$

Required number of reinforcing bars per design section rounded up to the nearest integer:

$$NumbBars_{i,j} := \text{ceil}\left(\frac{b_i}{2 \cdot BarSpacing\_2_{i,j}}\right)$$

$$NumbBars = \begin{bmatrix} 8 & 11 & 15 & 11 & 17 & 11 & 14 & 11 & 11 & 11 \\ 4 & 6 & 8 & 6 & 8 & 6 & 8 & 6 & 6 & 6 \\ 8 & 13 & 13 & 13 & 15 & 13 & 14 & 13 & 13 & 13 \\ 5 & 7 & 7 & 7 & 8 & 7 & 7 & 7 & 7 & 7 \end{bmatrix}$$

The required number of bars for middle strips of interior panels are the sum of the two half middle strips on each side of the panel centerline. If the adjacent panel widths differ, the number of bars for the shared middle strip must be adjusted accordingly.

Calculated bar spacing with actual number of bars used:

$$BarSpacing_{i,j} := \frac{b_i}{2 \cdot NumbBars_{i,j}}$$

$$BarSpacing = \begin{bmatrix} 15 & 10.9 & 8 & 10.9 & 7.1 & 10.9 & 8.6 & 10.9 & 10.9 & 10.9 \\ 15.8 & 10.5 & 7.9 & 10.5 & 7.9 & 10.5 & 7.9 & 10.5 & 10.5 & 10.5 \end{bmatrix}\ in$$

$$BarSpacing = \begin{bmatrix} \text{[illegible]} \\ 18 & 11.1 & 11.1 & 11.1 & 9.6 & 11.1 & 10.3 & 11.1 & 11.1 & 11.1 \\ 15.6 & 11.1 & 11.1 & 11.1 & 9.8 & 11.1 & 11.1 & 11.1 & 11.1 & 11.1 \end{bmatrix} \; in$$

Separate column and middle strips:

$$NB := NumbBars \qquad Bar := BarNo$$

$$NB_c := \text{augment}\left(\text{augment}\left(\text{augment}\left(NB^{\langle 0\rangle}, NB^{\langle 2\rangle}\right), \text{augment}\left(NB^{\langle 4\rangle}, NB^{\langle 6\rangle}\right)\right), NB^{\langle 8\rangle}\right)$$

$$NB_m := \text{augment}\left(\text{augment}\left(\text{augment}\left(NB^{\langle 1\rangle}, NB^{\langle 3\rangle}\right), \text{augment}\left(NB^{\langle 5\rangle}, NB^{\langle 7\rangle}\right)\right), NB^{\langle 9\rangle}\right)$$

$$Bar_c := \text{augment}\left(\text{augment}\left(\text{augment}\left(Bar^{\langle 0\rangle}, Bar^{\langle 2\rangle}\right), \text{augment}\left(Bar^{\langle 4\rangle}, Bar^{\langle 6\rangle}\right)\right), Bar^{\langle 8\rangle}\right)$$

$$Bar_m := \text{augment}\left(\text{augment}\left(\text{augment}\left(Bar^{\langle 1\rangle}, Bar^{\langle 3\rangle}\right), \text{augment}\left(Bar^{\langle 5\rangle}, Bar^{\langle 7\rangle}\right)\right), Bar^{\langle 9\rangle}\right)$$

Separate "long" and "short" spans:

$$j4 := 0 .. 4 \qquad i3 := 0 .. \text{rows}\left(b_{short}\right) - 1$$

$$NB_{cl_{i1,j4}} := NB_{c_{i1,j4}} \qquad NB_{ml_{i1,j4}} := NB_{m_{i1,j4}}$$

$$Bar_{cl_{i1,j4}} := Bar_{c_{i1,j4}} \qquad Bar_{ml_{i1,j4}} := Bar_{m_{i1,j4}}$$

$$NB_{cs_{i3,j4}} := NB_{c_{i3+2,j4}} \qquad NB_{ms_{i3,j4}} := NB_{m_{i3+2,j4}}$$

$$Bar_{cs_{i3,j4}} := Bar_{c_{i3+2,j4}} \qquad Bar_{ms_{i3,j4}} := Bar_{m_{i3+2,j4}}$$

Rename matrices containing number and size of reinforcing bars:

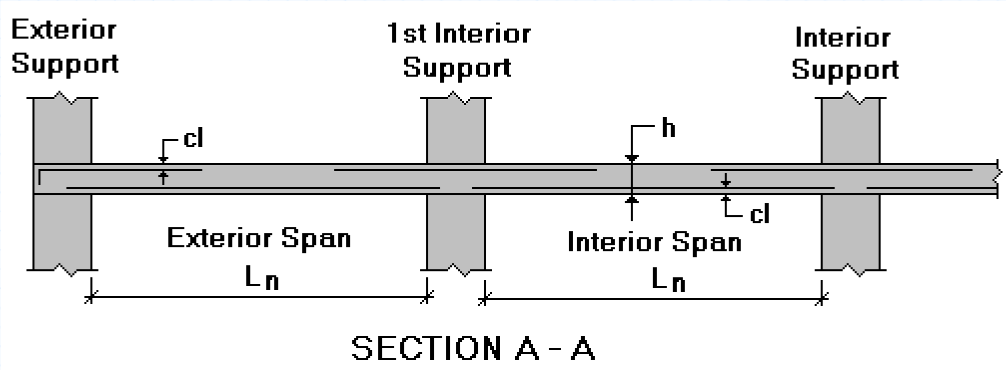
$$NumbBars_{col\_lg} := NB_{cl} \qquad NumbBars_{mid\_lg} := NB_{ml}$$

$$NumbBars_{col\_sh} := NB_{cs} \qquad NumbBars_{mid\_sh} := NB_{ms}$$

$$BarNo_{col\_lg} := Bar_{cl} \qquad BarNo_{mid\_lg} := Bar_{ml}$$

$$BarNo_{col\_sh} := Bar_{cs} \qquad BarNo_{mid\_sh} := Bar_{ms}$$

Summary



Specified compressive strength of concrete: $f'_c = 4\ ksi$

Specified yield strength of reinforcement: $f_y = 60\ ksi$

Unit weight of concrete: $w_c = 145\ pcf$

Unit weight of reinforced concrete: $w_{rc} = 150\ pcf$

Service live load: $w_l = 50\ psf$

Superimposed dead load: $w_{sd} = 10\ psf$

Clear cover of reinforcement: $cl = 0.75\ in$

Sizing factor for rounding slab thickness: $SzF = 0.5\ in$

$b_{long} = \begin{bmatrix} 20 \\ 10.5 \end{bmatrix} ft$ $L_{n\_long} = \begin{bmatrix} 23 & 22 \\ 22 & 22 \end{bmatrix} ft$

Panel width and span for design strips with reinforcing bars in the inner layers (normally the shorter span):

$b_{short} = \begin{bmatrix} 24 \\ 13 \end{bmatrix} ft$ $L_{n\_short} = \begin{bmatrix} 19 & 19 \\ 18.5 & 18 \end{bmatrix} ft$

Slab thickness: $h = 9.5\ in$

Maximum reinforcement ratio used: $\max(\rho) = 0.501\%$

Total factored load per unit area: $w_u = 265.25\ psf$

Maximum permissible reinforcement ratio: $\rho_{max} = 2.138\%$

The number and size of reinforcing bars are shown below in 5 column matrices.

Column 0 - negative (top) reinforcement exterior support
Column 1 - positive (bottom) reinforcement exterior span
Column 2 - negative (top) reinforcement 1st interior support
Column 3 - positive (bottom) reinforcement exterior span
Column 4 - negative (top) reinforcement interior support

"Long" Spans

Column strip, exterior and interior spans:

$$NumbBars_{col\_lg} = \begin{bmatrix} 8 & 15 & 17 & 14 & 11 \\ 4 & 8 & 8 & 8 & 6 \end{bmatrix}$$

$$BarNo_{col\_lg} = \begin{bmatrix} 5 & 4 & 5 & 5 & 4 \\ 5 & 4 & 5 & 5 & 4 \end{bmatrix}$$

Middle strip, exterior and interior spans:

$$NumbBars_{mid\_lg} = \begin{bmatrix} 11 & 11 & 11 & 11 & 11 \\ 6 & 6 & 6 & 6 & 6 \end{bmatrix}$$

$$BarNo_{mid\_lg} = \begin{bmatrix} 4 & 4 & 4 & 4 & 4 \\ 4 & 4 & 4 & 4 & 4 \end{bmatrix}$$

"Short" Spans

Column strip, exterior and interior spans:

$$NumbBars_{col\_sh} = \begin{bmatrix} 8 & 13 & 15 & 14 & 13 \\ 5 & 7 & 8 & 7 & 7 \end{bmatrix}$$

$$BarNo_{col\_sh} = \begin{bmatrix} 5 & 4 & 5 & 5 & 4 \\ 5 & 4 & 5 & 5 & 4 \end{bmatrix}$$

Middle strip, exterior and interior spans:

$$NumbBars_{mid\_sh} = \begin{bmatrix} 13 & 13 & 13 & 13 & 13 \\ 7 & 7 & 7 & 7 & 7 \end{bmatrix}$$

$$BarNo_{mid\_sh} = \begin{bmatrix} 4 & 4 & 4 & 4 & 4 \\ 4 & 4 & 4 & 4 & 4 \end{bmatrix}$$