CHAPTER 6: Reinforced Concrete Flat Plates

6.2 Design of Flat Plates for Shear

Description

Design for shear is the most critical element in the design of flat plates since shear failures may occur without sufficient deflection of the structure to warn of impending failure. This application computes the section properties of the peripheral shear area around square or rectangular, interior, exterior or corner columns, useable shear stress at factored load, and the shear stresses at the four corners of the shear area due to axial load and bending moments about one or both axes, in accordance with the Strength Design Method of ACI 318-89. The flexural design of flat plates is covered in **Section 6.1**.

The required input for this application includes the strength of the concrete, the unit weight of concrete, effective slab depths, plan dimensions of the column, free edge slab extensions (if applicable), and the total factored load and moments transferred between the slab and the column.

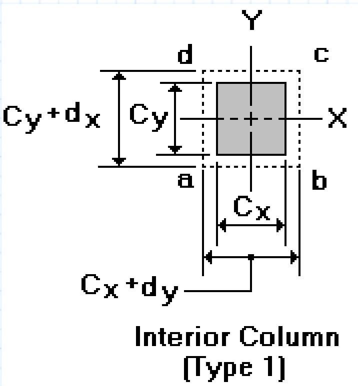
A summary of input and calculated values is shown on pages 9-10.

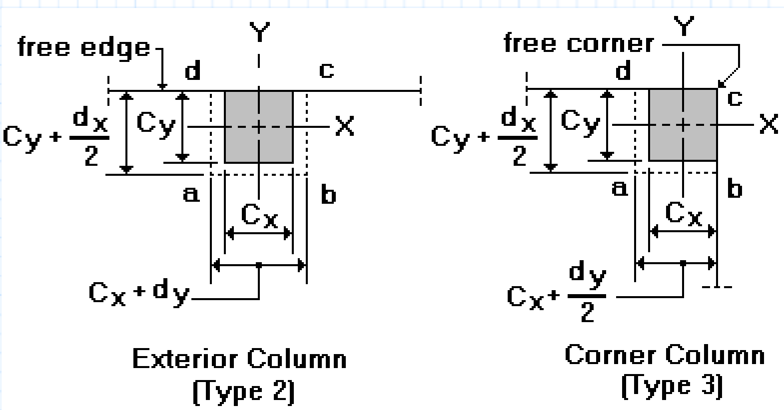
Reference:

ACI 318-89 "Building Code Requirements for Reinforced Concrete." (Revised 1992)

Input

Notation





Input Variables

Factored load from slab: $V_u := 47.1 \cdot kip$

Total moment about the X axis transferred between the slab and column: $M_x := 88.1 \cdot kip \cdot ft$

Total moment about the Y axis transferred between the slab and column: $M_y := 0 \cdot kip \cdot ft$

Column plan dimension parallel to the X axis: $C_x := 18\ in$

Column plan dimension parallel to the Y axis: $C_y := 18\ in$

Effective slab depth for moments about the X axis: $d_x := 6.5\ in$

Effective slab depth for moments about the Y axis: $d_y := 6.25\ in$

Free edge slab extension in the X direction for corner columns if applicable: $x_o := 0\ in$

Free edge slab extension in the Y direction for exterior or corner columns if applicable: $y_o := 0\ in$

Type of critical shear section, designated by T = 1 for interior, 2 for exterior, or 3 for corner: $T := 2$

Computed Variables

x_{ad} distance from the centroid of the critical shear section to line ad

x_{bc} distance from the centroid of the critical shear section to line bc

y_{ab} distance from the neutral axis of the critical shear section to line ab

y_{cd} distance from the neutral axis of the critical shear section to line cd

e_x eccentricity from the Y axis through the column center to the neutral axis of the shear section

e_y eccentricity from the X axis through the column center to the neutral axis of the shear section

v_{ua} shear stress at point a

v_{ub} shear stress at point b

v_{uc} shear stress at point c

v_{ud} shear stress at point d

v_{cp} nominal permissible two way shear stress at factored load

Notes

1) The ACI 318 Commentary implies that it is acceptable to use an average effective slab depth in calculating the section properties of the critical shear section. However, this application provides for entry of effective depths for both the x and y directions.

2) Dimensions x_o and y_o provide for small free edge slab extensions beyond the exterior faces of exterior or corner columns. Longer slab extensions sufficient to ensure development of a four sided critical shear section should be treated as interior columns.

Material Properties

Enter values for f'_c, w_c, and k_v if different from that shown.

Specified compressive strength of concrete: $f'_c := 4 \cdot ksi$

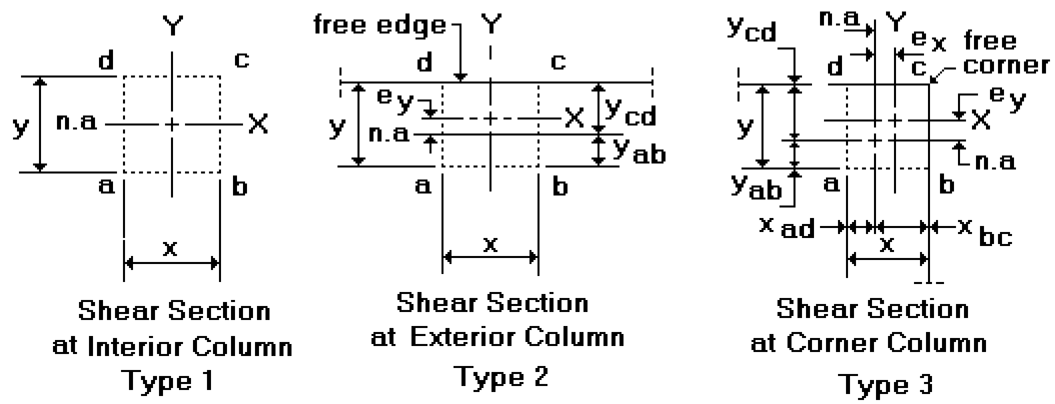
Unit weight of concrete: $w_c := 145 \cdot pcf$

Shear strength reduction factor for lightweight concrete k_v = 1 for normal weight, 0.75 for all-lightweight and 0.85 for sand lightweight concrete (ACI 318, 11.2.1.2.): $k_v := 1$

Limit the value of f`'c for computing shear to 10 ksi by substituting f'_{c_max} for f'_c in formulas for computing shear (ACI 318, 11.1.2):

$f'_{c\_max} := \text{if}\left(f'_c > 10 \cdot ksi, 10 \cdot ksi, f'_c\right)$ $f'_{c\_max} = 4\ ksi$

Calculations



Plan dimensions of the critical shear area (ACI 318, 11.12.1.2):

$$x := \text{if}\left((T=1)+(T=2), C_x + d_y, C_x + \frac{d_y}{2} + x_o\right) = 24.25\ in$$

$$y := \text{if}\left(T=1, C_y + d_x, C_y + \frac{d_x}{2} + y_o\right) = 21.25\ in$$

Shear section area (ACI 318, 11.12.2.1 and Fig. R11.12.6.2 of ACI Commentary):

$$A_c := \text{if}\left(T=1, 2\cdot x\cdot d_x + 2\cdot y\cdot d_y, \text{if}\left(T=2, x\cdot d_x + 2\cdot y\cdot d_y, x\cdot d_x + y\cdot d_x\right)\right) = 423.25\ in^2$$

Distance from the centroid of the critical shear section to line ab:

$$y_{ab} := \text{if}\left(T=1, \frac{y}{2}, \text{if}\left(T=2, \frac{d_y\cdot y^2}{A_c}, \frac{d_y\cdot y^2}{2\cdot A_c}\right)\right) = 6.668\ in$$

Distance from the centroid of the critical shear section to line cd:

$$y_{cd} := \text{if}\left(T=1, \frac{y}{2}, y - y_{ab}\right) = 14.582\ in$$

Eccentricity from the Y axis through the column center to the neutral axis of the shear section:

$$e_y := \frac{C_y + d_x}{2} - y_{ab} = 5.582\ in$$

Distance from the centroid of the critical shear section to line ad:

$$x_{ad} := \text{if}\left((T=1)+(T=2), \frac{x}{2}, \frac{d_x\cdot x^2}{2\cdot A_c}\right) = 12.125\ in$$

Distance from the centroid of the critical shear section to line bc:

$$x_{bc} := \text{if}\left((T=1)+(T=2), \frac{x}{2}, x - x_{ad}\right) = 12.125\ in$$

Eccentricity from the X axis through the column center to the neutral axis of the shear section:

$$e_x := \text{if}\left((T=1)+(T=2), 0 \cdot in, \frac{C_x + d_y}{2} - x_{ad}\right) = 0\ in$$

Properties of shear section analogous to the polar moment of inertia, for moments about X and Y axes:

$$J1_x := \frac{d_y \cdot y^3 + y \cdot d_y^3}{6} + \frac{x \cdot d_x \cdot y^2}{2}$$

$$J2_x := \frac{y \cdot d_y^3 + d_y \cdot y^3}{6} + 2 \cdot d_y \cdot y \cdot \left(\frac{y}{2} - y_{ab}\right)^2 + d_x \cdot x \cdot y_{ab}^2$$

$$J_{cx} := \text{if}\left(T=1, J1_x, \text{if}\left(T=2, J2_x, \frac{y \cdot d_y^3 + d_y \cdot y^3}{12} + d_y \cdot y \cdot \left(\frac{y}{2} - y_{ab}\right)^2\right)\right) = 22028\ in^4$$

$$J1_y := \frac{d_x \cdot x^3 + x \cdot d_x^3}{6} + \frac{y \cdot d_y \cdot x^2}{2} \qquad J2_y := \frac{d_x \cdot x^3 + x \cdot d_x^3}{12} + \frac{y \cdot d_y \cdot x^2}{2}$$

$$J_{cy} := \text{if}\left(T=1, J1_y, \text{if}\left(T=2, J2_y, \frac{x \cdot d_x^3 + d_x \cdot x^3}{12} + d_x \cdot x \cdot \left(\frac{x}{2} - x_{ad}\right)^2\right)\right) = 47330\ in^4$$

Moment transferred by eccentricity of shear about the centroid of the shear section (ACI 318, 11.12.6):

$$M_{xt} := M_x - V_u \cdot e_y = 66.191\ kip \cdot ft$$

$$M_{yt} := M_y - V_u \cdot e_x = 0\ kip \cdot ft$$

$$\gamma_{vx} := \left(1 - \frac{1}{1 + \frac{2}{3} \cdot \sqrt{\frac{y}{x}}}\right) = 0.384 \qquad \gamma_{vx} \cdot M_{xt} = 25.435\ kip \cdot ft$$

$$\gamma_{vy} := \left(1 - \frac{1}{1 + \frac{2}{3} \cdot \sqrt{\frac{x}{y}}}\right) = 0.416 \qquad \gamma_{vy} \cdot M_{yt} = 0\ kip \cdot ft$$

Ratio of long side to short side of column:

$$\beta_c := \textbf{if}\left(\left(C_x \geq C_y\right), \frac{C_x}{C_y}, \frac{C_y}{C_x}\right) = 1$$

Nominal "two way" concrete shear strength per unit area in slabs and footings. d is the average of the effective depths in the X and Y directions. α_s is equal to 40 for interior columns, 30 for edge columns, and 20 for corner columns. b_o is the critical shear perimeter for slabs and footings (ACI 318, 11.12.2.1, Eqs. (11-36), (11-37) and (11-38)):

$$\alpha_s := \textbf{if}\left(T = 1, 40, \textbf{if}\left(T = 2, 30, 20\right)\right) = 30$$

$$d := \frac{1}{2} \cdot \left(d_x + d_y\right) = 6.375 \ \boldsymbol{in}$$

$$b_o := 2 \cdot \left(C_x + C_y + 2 \cdot d\right) = 8.125 \ \boldsymbol{ft}$$

$$v_{cp} := min\left(\begin{bmatrix} 2 + \dfrac{4}{\beta_c} \\ \dfrac{\alpha_s \cdot d}{b_o} + 2 \\ 4 \end{bmatrix}\right) \cdot k_v \cdot \sqrt{\frac{f'_{c\_max}}{\boldsymbol{psi}}} \cdot \boldsymbol{psi} = 250.55 \ \boldsymbol{psi}$$

Shear Stresses:

$$\begin{bmatrix} v_{ua} \\ v_{ub} \\ v_{uc} \\ v_{ud} \end{bmatrix} := \begin{bmatrix} \dfrac{V_u}{A_c} + \dfrac{\gamma_{vx} \cdot M_{xt} \cdot y_{ab}}{J_{cx}} + \dfrac{\gamma_{vy} \cdot M_{yt} \cdot x_{ad}}{J_{cy}} \\ \dfrac{V_u}{A_c} + \dfrac{\gamma_{vx} \cdot M_{xt} \cdot y_{ab}}{J_{cx}} - \dfrac{\gamma_{vy} \cdot M_{yt} \cdot x_{bc}}{J_{cy}} \\ \textbf{if}\left(T = 3, 0 \cdot \boldsymbol{psi}, \dfrac{V_u}{A_c} - \dfrac{\gamma_{vx} \cdot M_{xt} \cdot y_{cd}}{J_{cx}} + \dfrac{\gamma_{vy} \cdot M_{yt} \cdot x_{ad}}{J_{cy}}\right) \\ \dfrac{V_u}{A_c} - \dfrac{\gamma_{vx} \cdot M_{xt} \cdot y_{cd}}{J_{cx}} - \dfrac{\gamma_{vy} \cdot M_{yt} \cdot x_{ad}}{J_{cy}} \end{bmatrix} = \begin{bmatrix} 204 \\ 204 \\ -91 \\ -91 \end{bmatrix} \boldsymbol{psi}$$

Maximum absolute value of shear stress at factored load:

$$v_u := \begin{bmatrix} \left|v_{ua}\right| & \left|v_{ub}\right| & \left|v_{uc}\right| & \left|v_{ud}\right| \end{bmatrix}^{\mathrm{T}}$$

$$\max\left(v_u\right) = 204 \ psi$$

Strength reduction factor for shear (ACI 318, 9.3.2.3):

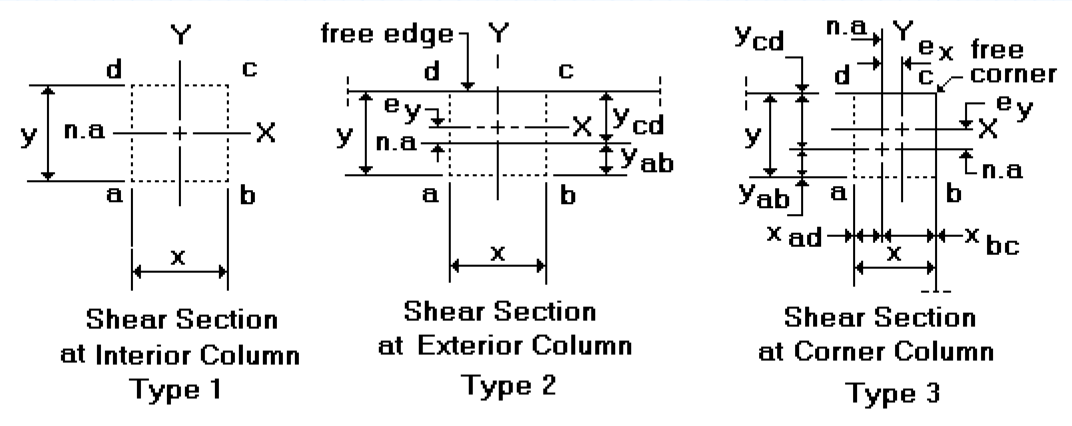
$\phi_v := 0.85$

Useable unit shear strength at factored load:

$\phi_v \cdot v_{cp} = 213\ psi$

If the maximum shear stress is greater than the useable shear, a thicker slab, or a larger column is required.

Summary



Input Variables

Concrete strength: $f'_c = 4\ ksi$

Unit weight of concrete: $w_c = 145\ pcf$

Factored load from slab: $V_u = 47.1\ kip$

Type of Shear Section: $T = 2$

Total moment about the X axis transferred between the slab and column: $M_x = 88.1\ kip \cdot ft$

Total moment about the Y axis transferred between the slab and column: $M_y = 0\ kip \cdot ft$

Effective slab depth for moments about the X axis: $d_x = 6.5\ in$

Effective slab depth for moments about the Y axis: $d_y = 6.25\ in$

Column plan dimension $C_x = 18\ in$

Column plan dimension parallel to the Y axis: $C_y = 18\ in$

Free edge slab extension in the X direction for corner columns: $x_o = 0\ in$

Free edge slab extension in the Y direction for corner columns: $y_o = 0\ in$

Shear stresses:

$v_{ua} = 204\ psi$ $v_{ub} = 204\ psi$

$v_{uc} = -91\ psi$ $v_{ud} = -91\ psi$

Useable shear stress at factored load: $\phi_v \cdot v_{cp} = 213\ psi$