



## CHAPTER 7: Reinforced Concrete Column and Wall Footings

### 7.4 As Driven Pile Group Analysis

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#### Description

Pile footings are designed assuming the piles are driven or placed in their planned location. In practice, the location of the piles will deviate from the planned location, causing the centroid of the pile group to shift. The eccentricities between the load or column center and the actual centroid of the pile group induce moments, adding or subtracting load from the individual piles. It is therefore necessary to verify that the the loads on the individual piles do not exceed the capacity of an individual pile by a stipulated amount - commonly 10%. If the load on one or more piles is excessive, it is then necessary to add piles and redesign the pile footing.

This document locates the centroid of axially loaded, as driven pile groups, calculates the eccentricities between the column center and the centroid of the pile group, the moments of inertia of the pile group about the principal axes, and the load on each pile from the column service load and from the moments due to the eccentricities, assuming the pile footing takes all of the moment due to eccentricity.

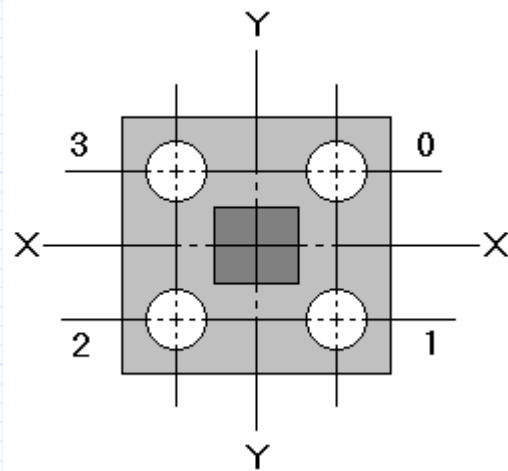
The four pile group shown below is for illustrative purposes - any number of piles may be entered. The required input includes the total service load on the pile group and the coordinates of the pile centers with respect to the center of the column.

A summary of input and calculated values is shown on page 6.

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## Input

### Notation



### Input Variables

Total service load on the pile group:

$$P := 240 \cdot \text{kip}$$

As driven coordinates of pile centers from column center:

$$x := [1.67 \quad 1.43 \quad -1.27 \quad -1.51]^T \cdot \text{ft}$$

$$y := [1.58 \quad -1.55 \quad -1.61 \quad 1.36]^T \cdot \text{ft}$$

The pile numbers should correspond to the index numbers of the pile coordinates  $x$  and  $y$  for identification. For example, the  $X$  and  $Y$  coordinates of pile No. 1 are:

$$x_1 = 1.43 \text{ ft} \quad y_1 = -1.55 \text{ ft}$$

## Computed Variables

- $e_x$  distance in the X direction from the column center to the centroid of the as driven pile group
- $e_y$  distance in the Y direction from the column center to the centroid of the as driven pile group
- $x'$  pile coordinate in the X direction from the centroid of the as driven pile group
- $y'$  pile coordinate in the Y direction from the centroid of the as driven pile group
- $I_x$  moment of inertia of the pile group about an axis through the centroid of the as driven pile group and parallel to the X axis through the column center
- $I_y$  moment of inertia of the pile group about an axis through the centroid of the as driven pile group and parallel to the Y axis through the column center
- $I_{xy}$  product of inertia of with respect to the centroid of the as driven pile group
- $\theta$  principal axes rotation from the X axis
- $I'_x$  moment of inertia of the pile group about the principal X axis
- $I'_y$  moment of inertia of the pile group about the principal Y axis
- $x_1$  pile coordinates in the direction of the principal X axis
- $y_1$  pile coordinates in the direction of the principal Y axis
- $Q$  Load on each pile of the as driven pile group

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## Calculations

Range variable  $i$  and total number of piles  $N$ :

$$i := 0 \dots \text{last}(y) \quad \text{last}(y) = 3 \quad N := \text{length}(y) \quad N = 4$$

Eccentricity from column center to the centroid of the as driven pile group:

$$e_x := \frac{\sum x}{N} \quad e_x = 0.96 \text{ in} \quad e_y := \frac{\sum y}{N} \quad e_y = -0.66 \text{ in}$$

Pile coordinates from the centroid of the as driven pile group:

$$x' := x - e_x \quad x'^T = [1.59 \quad 1.35 \quad -1.35 \quad -1.59] \text{ ft}$$
$$y' := y - e_y \quad y'^T = [1.635 \quad -1.495 \quad -1.555 \quad 1.415] \text{ ft}$$

Moments of inertia of the pile group about axes through the centroid of the pile group and parallel to the X and Y axes of the column:

$$I_x := \sum_i (y'_i)^2 \quad I_x = 9.329 \text{ ft}^2$$

$$I_y := \sum_i (x'_i)^2 \quad I_y = 8.701 \text{ ft}^2$$

Product of inertia with respect to the centroid of the as driven pile group (the product of inertia will be 0 if there is symmetry about one or both axes):

$$I_{xy} := \sum_i (x'_i \cdot y'_i) \quad I_{xy} = 0.431 \text{ ft}^2$$

Principal axes rotation  $\theta$  from the X axis (+ is clockwise, - is counter clockwise):

$$\theta := \frac{1}{2} \cdot \text{atan} \left( \frac{2 \cdot I_{xy}}{I_y - I_x} \right) \quad \theta = -26.971 \text{ deg}$$

Moments of inertia about the principal X and Y axes:

$$I_{x'} := I_x \cdot \cos(\theta)^2 + I_y \cdot \sin(\theta)^2 - 2 \cdot I_{xy} \cdot \sin(\theta) \cdot \cos(\theta) \quad I_{x'} = 9.548 \text{ ft}^2$$

$$I_{y'} := I_x \cdot \sin(\theta)^2 + I_y \cdot \cos(\theta)^2 + 2 \cdot I_{xy} \cdot \sin(\theta) \cdot \cos(\theta) \quad I_{y'} = 8.482 \text{ ft}^2$$

Pile locations in coordinates of the principal axes:

$$x1 := x' \cdot \cos(\theta) + y' \cdot \sin(\theta)$$

$$x1^T = [0.676 \quad 1.881 \quad -0.498 \quad -2.059] \text{ ft}$$

$$y1 := y' \cdot \cos(\theta) - x' \cdot \sin(\theta)$$

$$y1^T = [2.178 \quad -0.72 \quad -1.998 \quad 0.54] \text{ ft}$$

Eccentricities from column center to pile group centroid in coordinates of the principal axes:

$$e_{x1} := e_x \cdot \cos(\theta) + e_y \cdot \sin(\theta) \quad e_{x1} = 1.155 \text{ in}$$

$$e_{y1} := e_y \cdot \cos(\theta) - e_x \cdot \sin(\theta) \quad e_{y1} = -0.153 \text{ in}$$

Loads on each pile due to column service load and eccentricities:

$$Q := \frac{P}{N} - x1 \cdot \frac{P \cdot e_{x1}}{I_{y'}} - y1 \cdot \frac{P \cdot e_{y1}}{I_{x'}}$$

$$Q^T = [58.9 \quad 54.6 \quad 60.7 \quad 65.8] \text{ kip}$$

$$\sum Q = 240 \text{ kip}$$

$$\max(Q) = 65.8 \text{ kip}$$

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## Summary

### *Input*

Column service load:

$$P = 240 \text{ kip}$$

As driven coordinates of pile centers from column center:

$$x^T = [1.67 \quad 1.43 \quad -1.27 \quad -1.51] \text{ ft}$$

$$y^T = [1.58 \quad -1.55 \quad -1.61 \quad 1.36] \text{ ft}$$

### *Computed Variables*

Load on each pile of planned pile group:

$$\frac{P}{N} = 60 \text{ kip}$$

Load on each pile of as driven pile group:

$$Q^T = [58.858 \quad 54.646 \quad 60.716 \quad 65.78] \text{ kip}$$

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