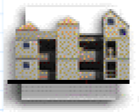

CHAPTER 10: Reinforced Concrete Material Properties, Development, and Splice Lengths

10.1 Material Properties, Development, and Splice Lengths

Description

This application computes factors and properties and defines constants required to compute shear strength, flexural strength, minimum thickness, and the development and splice lengths of reinforcing bars, in accordance with ACI 318-89.

The user must enter the compressive strength of concrete, yield strength of reinforcement, unit weight of concrete, strength reduction factor for lightweight concrete and the factor for increasing development and splice lengths for lightweight concrete.

Development and splice lengths in this document are limited to bars which meet the requirements of ACI 318 Section 12.2.3.1 for no increase in basic development length to account for bar cover, bar spacing and enclosing transverse reinforcement. The factors of Sections 12.2.3.4 and 12.2.3.5 for reducing basic development length are not applied. Development lengths that are determined by bar cover, bar spacing, and confinement reinforcement cannot be tabulated and must be calculated on a case by case basis.

A summary of input and calculated values is shown on pages 10-12.

Reference:
ACI 318-89 "Building Code Requirements for Reinforced Concrete." (Revised 1992).

Input

Variables

Specified compressive strength of concrete: $f'_c := 4 \cdot ksi$

Specified yield strength of reinforcement
(f_y may not exceed 60 ksi, ACI 318 11.5.2): $f_y := 60 \cdot ksi$

Unit weight of concrete: $w_c := 145 \cdot pcf$

Weight of reinforced concrete: $w_{rc} := 150 \cdot pcf$

Shear strength reduction factor for lightweight concrete
k_v = 1 for normal weight, 0.75 for all-lightweight and
0.85 for sand-lightweight concrete (ACI 318, 11.2.1.2.): $k_v := 1$

Weight factor for increasing development and splice
lengths k_w = 1 for normal weight and 1.3 for
lightweight aggregate concrete (ACI 318, 12.2.4.2): $k_w := 1$

Limit the value of f'_c for computing shear and
development lengths to 10 ksi by substituting f'_{c_max}
for f'_c in formulas for computing shear (ACI 318,
11.1.2, 12.1.2):

$f'_{c\_max} := \text{if}\left(f'_c > 10\ ksi, 10\ ksi, f'_c\right)$

$f'_{c\_max} = 4\ ksi$

Input
Constants

Modulus of elasticity of
reinforcement (ACI 318, 8.5.2): $E_s := 29000\ ksi$

Strain in concrete at compression
failure (ACI 318, 10.3.2): $\varepsilon_c := 0.003$

Strength reduction factor for flexure
(ACI 318, 9.3.2.1): $\phi_f := 0.9$

Strength reduction factor for shear
(ACI 318, 9.3.2.3): $\phi_v := 0.85$

Reinforcing bar number designations, diameters and areas:

$$No := [0\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ 13\ 14\ 15\ 16\ 17\ 18]^T$$

$$d_b := [0\ 0\ 0\ 0.375\ 0.5\ 0.625\ 0.75\ 0.875\ 1.00\ 1.128\ 1.27\ 1.41\ 0\ 0\ 1.693\ 0\ 0\ 0\ 2.257]^T \cdot in$$

$$A_b := [0\ 0\ 0\ 0.11\ 0.20\ 0.31\ 0.44\ 0.60\ 0.79\ 1.00\ 1.27\ 1.56\ 0\ 0\ 2.25\ 0\ 0\ 0\ 4.00]^T \cdot in^2$$

Bar numbers, diameters and areas are in the vector rows (or columns in the transposed vectors shown) corresponding to the bar numbers. Individual bar numbers, diameters, areas and development lengths and splices of a specific bar can be referred to by using the vector subscripts as shown in the example below.

Example: $No_5 = 5$ $d_{b_5} = 0.625\ in$ $A_{b_5} = 0.31\ in^2$

Calculations

The following values are computed from the entered material properties.

Nominal "one way" shear strength per unit area in concrete (ACI 318, 11.3.1.1, Eq. (11-3), 11.5.4.3):

$$v_c := k_v \cdot 2 \cdot \sqrt{\frac{f'_{c\_max}}{psi}} \cdot psi \qquad v_c = 126\ psi$$

Nominal "two way" concrete shear strength per unit area in slabs and footings, expressed as a function of β_c, d, α_s and b_o. β_c is the ratio of the long side to the short side of concentrated load or reaction area. d is the effective depth. α_s is equal to 40 for interior columns, 30 for edge columns and 20 for corner columns, and b_o is the critical shear perimeter for slabs and footings (ACI 318, 11.12.2.1, Eqs. (11-36), (11-37) and (11-38)):

$$v_{cp}\left(\beta_c, d, \alpha_s, b_o\right) := min\left(\begin{bmatrix} 2 + \frac{4}{\beta_c} \\ \frac{\alpha_s \cdot d}{b_o} + 2 \\ 4 \end{bmatrix}\right) \cdot k_v \cdot \sqrt{\frac{f'_{c\_max}}{psi}} \cdot psi$$

Modulus of elasticity of concrete for values of w_c between 90 pcf and 155 pcf (ACI 318, 8.5.1):

$$E_c := \left(\frac{w_c}{pcf}\right)^{1.5} \cdot 33 \cdot \sqrt{\frac{f'_c}{psi}} \cdot psi = 3644.147\ ksi$$

Strain in reinforcement at yield stress:

$$\varepsilon_y := \frac{f_y}{E_s} = 0.00207$$

Factor used to calculate depth of equivalent rectangular stress block (ACI 318, 10.2.7.3):

$$\beta_1 := \text{if}\left(\left(f'_c \geq 4 \cdot ksi\right) \cdot \left(f'_c \leq 8 \cdot ksi\right), 0.85 - 0.05 \cdot \frac{f'_c - 4 \cdot ksi}{ksi}, \text{if}\left(\left(f'_c \leq 4 \cdot ksi\right), 0.85, 0.65\right)\right)$$

$$\beta_1 = 0.85$$

Reinforcement ratio producing balanced strain conditions (ACI 318, 10.3.2):

$$\rho_b := \frac{\beta_1 \cdot 0.85 \cdot f'_c}{f_y} \cdot \frac{E_s \cdot \varepsilon_c}{E_s \cdot \varepsilon_c + f_y} = 2.851\%$$

Maximum reinforcement ratio (ACI 318, 10.3.3):

$$\rho_{max} := \frac{3}{4} \cdot \rho_b = 2.138\%$$

Minimum reinforcement ratio for beams (ACI 318, 10.5.1, Eq. (10-3)):

$$\rho_{min} := \frac{200}{f_y} \cdot \frac{\textbf{lbf}}{\textbf{in}^2} = 0.333\%$$

Shrinkage and temperature reinforcement ratio (ACI 318, 7.12.2.1):

$$\rho_{temp} := \begin{cases} \text{if } f_y \le 50\ \textbf{ksi} \\ \quad 0.002 \\ \text{else if } f_y \le 60\ \textbf{ksi} \\ \quad 0.002 - \dfrac{f_y}{60\ \textbf{ksi}} \cdot 0.0002 \\ \text{else if } \dfrac{0.0018 \cdot (60\ \textbf{ksi})}{f_y} \ge 0.0014 \\ \quad \dfrac{0.0018 \cdot (60\ \textbf{ksi})}{f_y} \\ \text{else} \\ \quad 0.0014 \end{cases} = 0.18\%$$

Preferred reinforcement ratio:

$$\rho := \frac{1}{2} \cdot \rho_{max} = 1.069\%$$

Flexural coefficient K, for rectangular beams or slabs, as a function of ρ (ACI 318, 10.2):
(Moment capacity $\phi M_n = K(\rho)F$, where $F = bd^2$)

$$K(\rho) := \phi_f \cdot \rho \cdot \left(1 - \frac{\rho \cdot f_y}{2 \cdot 0.85 \cdot f'_c}\right) \cdot f_y$$

Factors for adjusting minimum beam and slab thickness h_{min} for use of lightweight concrete and yield strengths other than 60 ksi (ACI 318, 9.5.2.1, see footnotes to Table 9.5 (a)):

Adjustment factor for minimum thickness for concrete weights between 90 and 120 pcf:

$$q_1 := \text{if}\left(w_c \le 112 \cdot pcf, 1.65 - 0.005 \cdot \frac{w_c}{pcf}, \text{if}\left(w_c \le 120 \cdot pcf, 1.09, 1\right)\right) = 1$$

Adjustment factor for minimum thickness for yield strengths other than 60 ksi:

$$q_2 := 0.4 + \frac{f_y}{100 \cdot ksi} = 1$$

Adjustment factor for minimum thickness combining factors for concrete weight and for yield strengths other than 60 ksi:

$$Q := q_1 \cdot q_2 = 1$$

Development and splice lengths of reinforcing bars

Basic tension development length l_{dbt} (ACI 318, 12.2.2 and 12.2.3.6):

No. 3 through No. 11 bars: $n := 3..11$

$$X1_n := 0.04 \cdot A_{b_n} \cdot \frac{f_y}{\sqrt{f'_{c\_max} \cdot lbf}} \qquad X2_n := 0.03 \cdot d_{b_n} \cdot \frac{f_y}{\sqrt{\frac{f'_{c\_max}}{psi}} \cdot psi}$$

$$l_{dbt_n} := \text{if}\left(X1_n > X2_n, X1_n, X2_n\right)$$

$$l_{dbt}^{T} = [0\ \ 0\ \ 0\ \ 10.7\ \ 14.2\ \ 17.8\ \ 21.3\ \ 24.9\ \ 30\ \ 37.9\ \ 48.2\ \ 59.2]\ in$$

No. 14 bars: $l_{dbt_{14}} := 0.085 \cdot \frac{f_y \cdot in^2}{\sqrt{f'_{c\_max} \cdot lbf}} = 80.638\ in$

No. 18 bars: $l_{dbt_{18}} := 0.125 \cdot \frac{f_y \cdot in^2}{\sqrt{f'_{c\_max} \cdot lbf}} = 118.585\ in$

Tension development length (ACI 318, 12.2.1):

No. 3 through No. 11 bars:

$$l_{dt_n} := \text{if}\left(k_w \cdot l_{dbt_n} \ge 12 \cdot in, k_w \cdot l_{dbt_n}, \text{if}\left(k_w \cdot l_{dbt_n} > 0 \cdot in, 12 \cdot in, 0 \cdot in\right)\right)$$

$$l_{dt}^{T} = [0\ \ 0\ \ 0\ \ 12\ \ 14.2\ \ 17.8\ \ 21.3\ \ 24.9\ \ 30\ \ 37.9\ \ 48.2\ \ 59.2]\ in$$

No. 14 bars: $l_{dt_{14}} := k_w \cdot l_{dbt_{14}} = 80.638\ in$

No. 18 bars: $l_{dt_{18}} := k_w \cdot l_{dbt_{18}} = 118.585\ in$

"Top Reinforcement" (ACI 318, 12.2.4.1):

$$l_{dt\_top_n} := \mathbf{if}\left(1.3 \cdot k_w \cdot l_{dbt_n} \ge 12 \cdot in, 1.3 \cdot k_w \cdot l_{dbt_n}, \mathbf{if}\left(1.3\ k_w \cdot l_{dbt_n} > 0\ in, 12\ in, 0\ in\right)\right)$$

$$l_{dt\_top}^{T} = [0\ \ 0\ \ 0\ \ 13.9\ \ 18.5\ \ 23.1\ \ 27.7\ \ 32.4\ \ 39\ \ 49.3\ \ 62.7\ \ 77]\ in$$

$$l_{dt\_top_{14}} := 1.3 \cdot k_w \cdot l_{dbt_{14}}$$

$$l_{dt\_top_{14}} = 104.8\ in$$

$$l_{dt\_top_{18}} := 1.3 \cdot k_w \cdot l_{dbt_{18}}$$

$$l_{dt\_top_{18}} = 154.2\ in$$

Tension lap splice lengths, (ACI 318, 12.15.1):

No. 3 through No. 11 bars:

$$A\_splice_n := \mathbf{if}\left(l_{dt_n} > 12 \cdot in, l_{dt_n}, 12 \cdot in\right)$$

$$A\_splice^{T} = [0\ \ 0\ \ 0\ \ 12\ \ 14.2\ \ 17.8\ \ 21.3\ \ 24.9\ \ 30\ \ 37.9\ \ 48.2\ \ 59.2]\ in$$

$$B\_splice_n := \mathbf{if}\left(1.3 \cdot l_{dt_n} > 12 \cdot in, 1.3 \cdot l_{dt_n}, 12 \cdot in\right)$$

$$B\_splice^{T} = [0\ \ 0\ \ 0\ \ 15.6\ \ 18.5\ \ 23.1\ \ 27.7\ \ 32.4\ \ 39\ \ 49.3\ \ 62.7\ \ 77]\ in$$

"Top Reinforcement" (ACI 318, 12.2.4.1):

$$A\_top_n := \mathbf{if}\left(l_{dt\_top_n} > 12 \cdot in, l_{dt\_top_n}, 12 \cdot in\right)$$

$$A\_top^{T} = [0\ \ 0\ \ 0\ \ 13.9\ \ 18.5\ \ 23.1\ \ 27.7\ \ 32.4\ \ 39\ \ 49.3\ \ 62.7\ \ 77]\ in$$

$$B\_top_n := \mathbf{if}\left(1.3 \cdot l_{dt\_top_n} > 12 \cdot in, 1.3 \cdot l_{dt\_top_n}, 12 \cdot in\right)$$

$$B\_top^{T} = [0\ \ 0\ \ 0\ \ 18\ \ 24\ \ 30.1\ \ 36.1\ \ 42.1\ \ 50.7\ \ 64.1\ \ 81.4\ \ 100]\ in$$

Lap splices of No. 14 and No. 18 bars are not permitted except as provided in ACI 318, 12.16.2 and 15.8.2.3.

Basic compression development length (ACI 318, 12.3.2):

No. 3 through No. 11 bars:

$$X3_n := 0.02 \cdot d_{b_n} \cdot \frac{f_y \cdot in}{\sqrt{f'_c \cdot lbf}} \qquad X4_n := 0.0003 \cdot \frac{in^2}{lbf} \cdot d_{b_n} \cdot f_y$$

$$l_{dbc_n} := \text{if}\left(X3_n \geq X4_n, X3_n, X4_n\right)$$

$$l_{dbc}^{T} = [0\ \ 0\ \ 0\ \ 7.1\ \ 9.5\ \ 11.9\ \ 14.2\ \ 16.6\ \ 19\ \ 21.4\ \ 24.1\ \ 26.8]\ in$$

No. 14 bars:

$$X3_{14} := 0.02 \cdot d_{b_{14}} \cdot \frac{f_y \cdot in}{\sqrt{f'_c \cdot lbf}} \qquad X4_{14} := 0.0003 \cdot \frac{in^2}{lbf} \cdot d_{b_{14}} \cdot f_y$$

$$l_{dbc_{14}} := \text{if}\left(X3_{14} \geq X4_{14}, X3_{14}, X4_{14}\right) \qquad l_{dbc_{14}} = 32.1\ in$$

No. 18 bars:

$$X3_{18} := 0.02 \cdot d_{b_{18}} \cdot \frac{f_y \cdot in}{\sqrt{f'_c \cdot lbf}} \qquad X4_{18} := 0.0003 \cdot \frac{in^2}{lbf} \cdot d_{b_{18}} \cdot f_y$$

$$l_{dbc_{18}} := \text{if}\left(X3_{18} \geq X4_{18}, X3_{18}, X4_{18}\right) \qquad l_{dbc_{18}} = 42.8\ in$$

Compression development length (ACI 318, 12.3.1 and 12.3.2):

$$l_{dc_n} := \text{if}\left(l_{dbc_n} > 8 \cdot in, l_{dbc_n}, \text{if}\left(l_{dbc_n} > 0 \cdot in, 8 \cdot in, 0 \cdot in\right)\right)$$

$$l_{dc}^{T} = [0\ \ 0\ \ 0\ \ 8\ \ 9.5\ \ 11.9\ \ 14.2\ \ 16.6\ \ 19\ \ 21.4\ \ 24.1\ \ 26.8]\ in$$

$$l_{dc_{14}} := l_{dbc_{14}} \qquad l_{dc_{14}} = 32.1\ in$$

$l_{dc_{18}} := l_{dbc_{18}}$ $\qquad l_{dc_{18}} = 42.8\ in$

Compression lap splice lengths (ACI 318 12.16.1):

No. 3 through No. 11 bars:

$$X5_n := \mathbf{if}\left(f_y > 60 \cdot \boldsymbol{ksi}, \left(0.0009 \cdot \frac{f_y}{\boldsymbol{psi}} - 24\right) \cdot d_{b_n}, 0.0005 \cdot \frac{f_y}{\boldsymbol{psi}} \cdot d_{b_n}\right)$$

$$X6 := \mathbf{if}\left(f'_c < 3 \cdot \boldsymbol{ksi}, \frac{4}{3}, 1\right)$$

$$CompSpl_n := \mathbf{if}\left(l_{dc_n} \geq X5_n \cdot X6, l_{dc_n}, X5_n \cdot X6\right)$$

$$CompSpl^{\mathrm{T}} = [0\ \ 0\ \ 0\ \ 11.3\ \ 15\ \ 18.8\ \ 22.5\ \ 26.3\ \ 30\ \ 33.8\ \ 38.1\ \ 42.3]\ in$$

Lap splices of No. 14 and No. 18 bars are not permitted except as provided in ACI 318, 12.16.2 and 15.8.2.3.

Summary

Specified compressive strength of concrete: $f'_c = 4\ ksi$

Specified yield strength of reinforcement: $f_y = 60\ ksi$

Unit weight of concrete: $w_c = 145\ pcf$

Modulus of elasticity of reinforcement (ACI 318, 8.5.2): $E_s := 29000 \cdot ksi$

Modulus of elasticity of concrete (ACI 318, 8.5.1): $E_c = 3644\ ksi$

Strain in concrete at compression failure (ACI 318, 10.3.2): $\varepsilon_c := 0.003$

Strain in reinforcement at yield stress: $\varepsilon_y = 0.002069$

Strength reduction factor for light weight concrete (ACI 318, 11.2.1.1.): $k_v = 1$

Factor for increasing development and splice lengths for lightweight aggregate concrete (ACI 318, 12.2.4.2): $k_w = 1$

Strength reduction factor for flexure (ACI 318, 9.3.2.1): $\phi_f := 0.9$

Strength reduction factor for shear (ACI 318, 9.3.2.3): $\phi_v := 0.85$

Factor used to calculate depth of equivalent rectangular stress block (ACI 318, 10.2.7.3): $\beta_1 = 0.85$

Reinforcing bar numbers, diameters, and areas:

$No_{14} = 14$ $\quad d_{b_{14}} = 1.693\ in$ $\quad A_{b_{14}} = 2.25\ in^2$

$No_{18} = 18$ $\quad d_{b_{18}} = 2.257\ in$ $\quad A_{b_{18}} = 4\ in^2$

$$\begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} \quad \begin{bmatrix} 0.375 \\ 0.5 \\ 0.625 \end{bmatrix} \quad \begin{bmatrix} 0.11 \\ 0.2 \\ 0.31 \end{bmatrix}$$

$$No_n = \begin{bmatrix} 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \end{bmatrix} \qquad d_{b_n} = \begin{bmatrix} 0.625 \\ 0.75 \\ 0.875 \\ 1 \\ 1.128 \\ 1.27 \\ 1.41 \end{bmatrix} in \qquad A_{b_n} = \begin{bmatrix} 0.31 \\ 0.44 \\ 0.6 \\ 0.79 \\ 1 \\ 1.27 \\ 1.56 \end{bmatrix} in^2$$

Bar sizes No. 3 through No. 11:

Tension Development Length and Splices

$$No_n = \begin{bmatrix} 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \end{bmatrix} \qquad l_{dt_n} = \begin{bmatrix} 12 \\ 14.23 \\ 17.788 \\ 21.345 \\ 24.903 \\ 29.978 \\ 37.947 \\ 48.193 \\ 59.198 \end{bmatrix} in \qquad A\_splice_n = \begin{bmatrix} 12 \\ 14.23 \\ 17.788 \\ 21.345 \\ 24.903 \\ 29.978 \\ 37.947 \\ 48.193 \\ 59.198 \end{bmatrix} in \qquad B\_splice_n = \begin{bmatrix} 15.6 \\ 18.499 \\ 23.124 \\ 27.749 \\ 32.374 \\ 38.972 \\ 49.332 \\ 62.651 \\ 76.957 \end{bmatrix} in$$

"Top Bars" Compression Development Length and Splices

$$No_n = \begin{bmatrix} 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \end{bmatrix} \qquad A\_top_n = \begin{bmatrix} 13.874 \\ 18.499 \\ 23.124 \\ 27.749 \\ 32.374 \\ 38.972 \\ 49.332 \\ 62.651 \\ 76.957 \end{bmatrix} in \qquad B\_top_n = \begin{bmatrix} 18.037 \\ 24.049 \\ 30.061 \\ 36.074 \\ 42.086 \\ 50.663 \\ 64.131 \\ 81.446 \\ 100.044 \end{bmatrix} in$$

$$l_{dc_n} = \begin{bmatrix} 8 \\ 9.487 \\ 11.859 \\ 14.23 \\ 16.602 \\ 18.974 \\ 21.402 \\ 24.097 \\ 26.753 \end{bmatrix} in \qquad CompSpl_n = \begin{bmatrix} 11.25 \\ 15 \\ 18.75 \\ 22.5 \\ 26.25 \\ 30 \\ 33.84 \\ 38.1 \\ 42.3 \end{bmatrix} in$$

Bar size No. 14:

$l_{dt_{14}} = 80.6\ in$ $l_{dt\_top_{14}} = 104.8\ in$ $l_{dc_{14}} = 32.1\ in$

Bar size No. 18:

$l_{dt_{18}} = 118.6\ in$ $l_{dt\_top_{18}} = 154.2\ in$ $l_{dc_{18}} = 42.8\ in$

Notes

1) This application is limited to bars which meet the requirements of ACI Section 12.2.3.1 for no increase in basic development length to account for bar cover, bar spacing and enclosing transverse reinforcement.
2) The factors of ACI 318 Sections 12.2.3.4 and 12.2.3.5 for reducing basic development length are not applied.