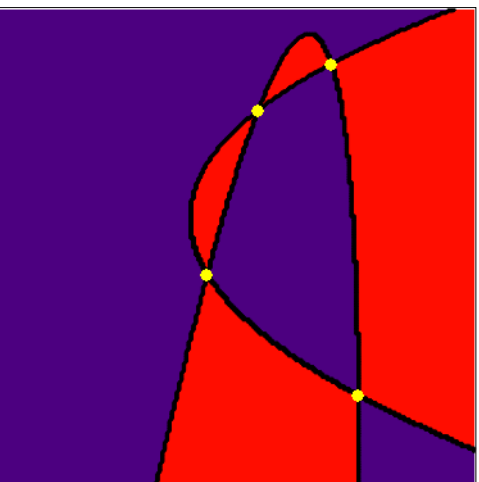
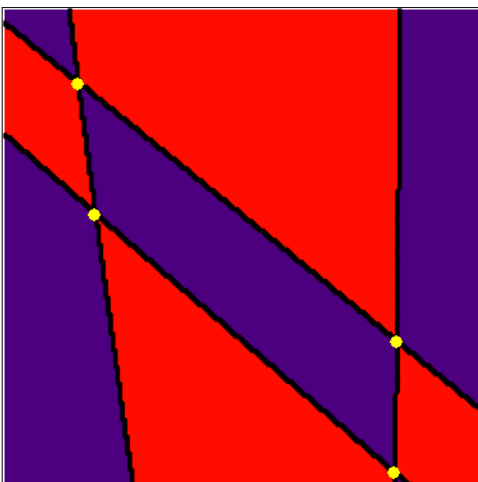
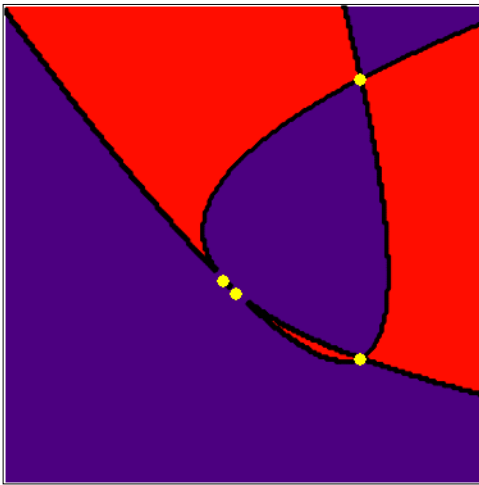
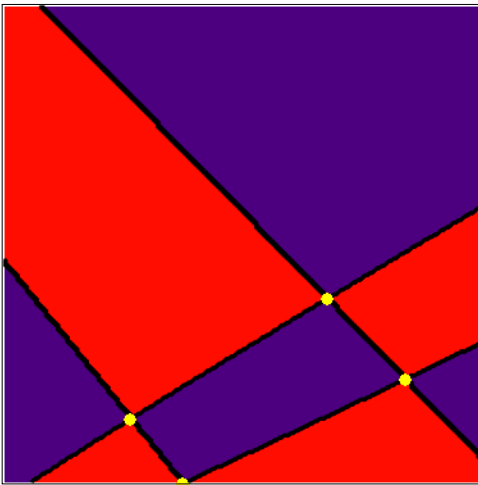
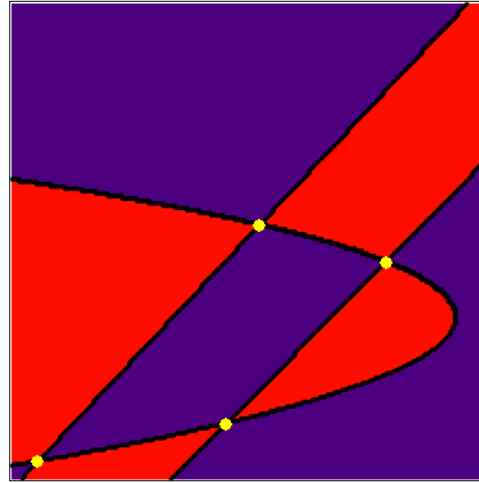
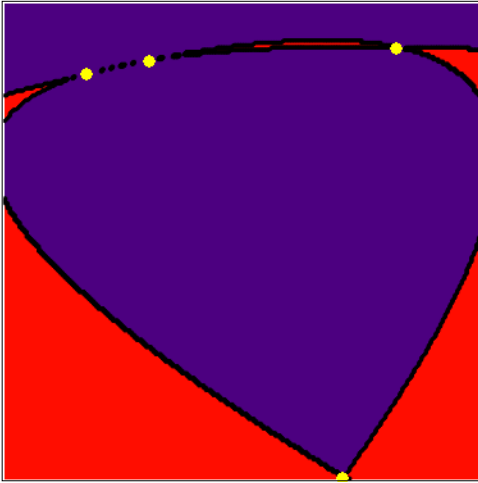


Red: the fifth point yields an ellipse

Green: the fifth point yields a hyperbola

Four points fixed, the fifth at random. A simulation using your code but with only a single free point. The boundary curves are the two parabolas that we can draw through these four points.

Here are some ways the regions of ellipse and hyperbola can look: purple is hyperbola, orange ellipse. These are made just by picking four random points and then contouring your discriminant D over the square for the remaining point.



The regions outside both parabolas and inside both parabolas give hyperbolas. The regions inside one parabola but not both give ellipses.

For four given points we could (laboriously) calculate the two parabolas, and (tediously) integrate to find the exact value of the hyperbola fraction. But then we have to somehow average this fraction over all possible choices of the four given points in a square. Note that for some choices of four points (when one of the four is contained in a triangle determined by the other three) the fifth point can **only** yield hyperbolas. About 30.6 percent of 4-point configurations in a square have a triangle containing the remaining point (so says my simulation). Another new constant? No, this problem has been solved and the true probability is $11/36$ (see E. Weisstein's article on Sylvester's Four-Point Problem at <http://mathworld.wolfram.com>).