MOMENTS OF INERTIA FOR RESONANT COLUMN TOP PLATEN

Calculation for the 70.50 mm top platen (I_{tp}). Units involved: grams force, centimeters, seconds, radians. **File:** c:\.papers\atenp\inertia.mcd. July,99, January 2001

$\phi_1 := 7.050$ $\quad \phi_2 := 3.833$

Top cap external and internal diameter,

$h_1 := 1.590 + 1.410$ $\quad h_2 := 1.590$

external and internal, heights and weights, and total weight of clamp

$w_1 := 137.47$ $\quad w_2 := 177.0$ $\quad w_{cl} := 43.715$

$d_t := 5.050$ $\quad \phi_t := 0.460$ $\quad h_t := 1.125$

Screw diametrical separation, diameter and height.

$w_t := 6.32$ $\quad g := 981$ $\quad \rho := 0.00274$

Screw weight, gravitational constant (g) and aluminum density [gr-s^2/cm].

Total volume of top cap = V_1(bottom) + V_2(anular top) - V_3(screw holes). Likewise total inertia I_{tp}= I_1 + I_2 - I_3 . I_1 and I_2 correspond to V_i and I_3 considers the inertia of screws and holes.
The computed density is higher than the density of aluminum because the valve fittings.

$$V_1 := \pi\cdot\left(\frac{\phi_1}{2}\right)^2\cdot(h_1 - h_2) \qquad V_2 := \pi\cdot\left[\left(\frac{\phi_1}{2}\right)^2 - \left(\frac{\phi_2}{2}\right)^2\right]\cdot h_2 \qquad V_3 := \pi\cdot\left(\frac{\phi_t}{2}\right)^2\cdot h_t$$

$$V_1 = 55.0411 \quad V_2 = 43.7207 \quad V_3 = 0.187 \quad V_T := V_1 + V_2 - 4\cdot V_3 \quad \rho_o := \frac{w_1 + w_2}{V_T\cdot g} \quad \rho_o = 3.2706\times 10^{-3}$$

$$I_1 := 0.5\cdot\frac{w_1}{g}\cdot\left(\frac{\phi_1}{2}\right)^2 \qquad I_2 := 0.5\cdot\frac{w_2}{g}\cdot\left[\left(\frac{\phi_1}{2}\right)^2 - \left(\frac{\phi_2}{2}\right)^2\right] \qquad I_3 := \left(\frac{w_t}{g}\right)\cdot\left(\frac{d_t}{2}\right)^2 - 4\cdot\rho\cdot V_3\cdot\left(\frac{d_t}{2}\right)^2$$

$$I_{tp} := I_1 + I_2 - I_3 \quad I_1 = 0.8706 \quad I_2 = 0.7896 \quad I_3 = 0.028 \quad I_{tp} = 1.6322 \quad \text{[gr-cm-s}^2\text{]}$$

L2 and L3, r2 and r3, are the lengths and radii of the clamp. r1i, r1o are the outside and inside radius of the rod.

$$L_2 := 0.5535 \quad L_3 := 2.512 \quad r_{1o} := \frac{0.635}{2} \quad r_{1i} := \frac{0.4480}{2} \quad r_2 := \frac{3.158}{2} \quad r_3 := \frac{2.522}{2}$$

The volume of the clamp outside the top bar is:

$$V_{cl} := \pi\cdot L_2\cdot\left(r_2^2 - r_{1i}^2\right) \qquad V_{cl} = 4.2482 \qquad \rho\cdot V_{cl}\cdot g = 11.4188$$

Inertia of the top-bar. r1, rt, and d1 are radii of rod and screws, and the distance between screws :

$$a := 7.100 \quad b := 3.704 \quad t := 2.533 \quad r_t := \frac{\phi_t}{2} \quad d_1 := \frac{d_t}{2} \quad I_{clamp} := 0.5\cdot\rho\cdot V_{cl}\cdot\left(r_2^2 - r_{1i}^2\right)$$

$$I_b := \left[\frac{1}{12}\cdot\rho\cdot a\cdot b\cdot t\cdot\left(a^2 + b^2\right) - \frac{\pi}{2}\cdot\rho\cdot t\cdot\left(r_{1i}^4 + 2\cdot r_t^4 + 2\cdot r_t^2\cdot d_1^2\right)\right] + I_{clamp} \qquad I_b = 0.9822 \qquad \text{[gr-cm-s}^2\text{]}$$

Inertia of added mass r1: centre hole, r2: screw hole, d1: distance between screws:

$$a := 15.468 \quad b := 7.620 \quad t := 0.629 \quad r_1 := \frac{0.990}{2} \quad r_2 := \frac{0.487}{2} \quad d_1 := \frac{5.176}{2} \quad V_t := a\cdot b\cdot t - \pi\cdot t\cdot\left(r_1^2 + 2\cdot r_2^2\right) \quad \rho_s := \frac{572.18}{V_t\cdot g}$$

$$I_m := \left[\frac{1}{12}\cdot\rho_s\cdot a\cdot b\cdot t\cdot\left(a^2 + b^2\right) - \frac{\pi}{2}\cdot\rho_s\cdot t\cdot\left[r_1^{\,4} + 2\cdot r_2^{\,2}\cdot\left(r_2^{\,2} + d_1^{\,2}\right)\right]\right] + \frac{3.157}{g}\cdot d_1^{\,2} \qquad I_m = 14.6077$$ [gr-cm-s2]

DRIVEN PLATE INERTIA (I) AND PROBE'S STIFFNESS (K)

The inertia of driven plate includes two accelerometers and their connections

I_b= mass polar moment of inertia of top bar of calibration specimen. I_m= mass polar moment of inertia of added mass and screws. I_{tb}= mass polar moment of inertia of the top cap.
Mass density of aluminium 2.7 g/cm3 / 981 = 0.00274 g-s2/cm4

Calculation for the aluminum rod
ϕ=0.635

$\rho := 0.00274$

Probe length (L), mass polar moment of inertia (I_{sp}) and driven plate inertia I_d:

$\rho = 2.74 \times 10^{-3}$

$L := 16.104 \qquad I_{sp} := 0.5\cdot\rho\cdot L\cdot\pi\cdot\left[\left(\frac{0.635}{2}\right)^4 - \left(\frac{0.448}{2}\right)^4\right] \qquad I_{sp} = 5.2983 \times 10^{-4}$ [gr-cm-s2]

$\omega_1 := 8.22\cdot 2\cdot\pi \qquad \omega_2 := 7.50\cdot 2\cdot\pi$

Resonat frequencies without mass and with mass for the aluminum specimen (No.1)

Mass polar moment of inertia for the driving Plate: $I_{bs1} := I_b + \frac{1}{3}\cdot I_{sp}$

$$I_a := \frac{\left(I_{bs1} + I_m\right)\cdot\omega_2^{\,2} - I_{bs1}\cdot\omega_1^{\,2}}{\left(\omega_1^{\,2} - \omega_2^{\,2}\right)} \qquad I_a = 71.6146$$ [gr-cm-s2]

$I1 := I_a$

Torsional stiffness of the rod: $K_a := \frac{I_m\cdot\left(\omega_1\cdot\omega_2\right)^2}{\left(\omega_1^{\,2} - \omega_2^{\,2}\right)} \qquad K_a = 1.9365 \times 10^5$ [gr-cm]

Inertia for Driving Plate and the 70.5mm top cap: $I_6 := I_a + I_{tp} \qquad I_6 = 73.2468$ [gr-cm-s2]

Shear wave velocity for the aluminum specimen

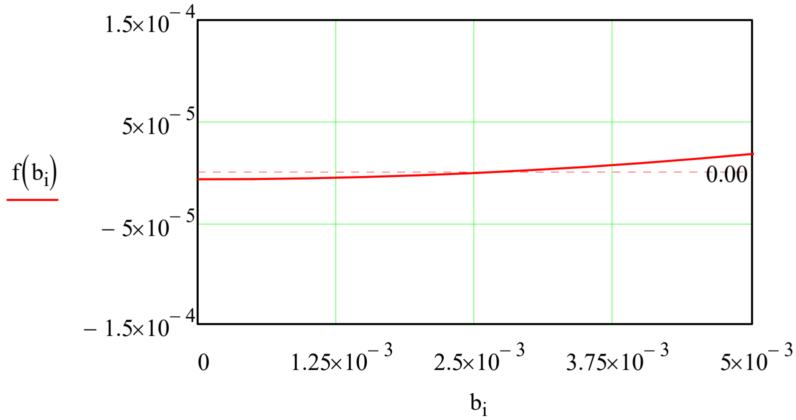
$I_d := I_a + I_b \qquad I_d = 72.5968$ [gr-cm-s2]

Iterative equation for shear wave velocity: $f(\beta) := \beta\cdot\tan(\beta) - \frac{I_{sp}}{I_d}$ (1)

Initial values to start iteration: $\beta := 0.01 \qquad b_i := 0.0, 0.0005 .. 0.05$

First root of equation (1): $a := \mathrm{root}(f(\beta), \beta) \qquad a = 2.70154 \times 10^{-3}$

Corresponding shear wave velocity: $v_s := \frac{\omega_1\cdot L}{a\cdot 100} \qquad v_s = 3.0788 \times 10^3$ [m/s]



Plot of equation (1) near the root "a"

Calculation for the aluminum rod ϕ=1.268 cm

$\rho := 0.00274$

L2 and L3, r2 and r3, are the lengths and radii of the clamp. r1i, r1o are the outside and inside radius of the rod.

$r_{1o} := \frac{1.271}{2}$ $\quad r_{1i} := \frac{1.076}{2}$ $\quad r_2 := \frac{3.158}{2}$

The volume of the clamp outside the top bar is:

$V_{cl} := \pi \cdot L_2 \cdot \left(r_2^{\,2} - r_{1i}^{\,2}\right)$ $\qquad V_{cl} = 3.8321$

Inertia of the top-bar. r1, rt, and d1 are radii of rod and screws, and the distance between screws :

$a := 7.100$ $\quad b := 3.704$ $\quad t := 2.533$ $\quad r_t := \frac{\phi_t}{2}$ $\quad d_1 := \frac{d_t}{2}$ $\quad I_{clamp} := 0.5 \cdot \rho \cdot V_{cl} \cdot \left(r_2^{\,2} - r_{1i}^{\,2}\right)$

$I_b := \left[\frac{1}{12} \cdot \rho \cdot a \cdot b \cdot t \cdot \left(a^2 + b^2\right) - \frac{\pi}{2} \cdot \rho \cdot t \cdot \left(r_{1i}^{\,4} + 2 \cdot r_t^{\,4} + 2 \cdot r_t^{\,2} \cdot d_1^{\,2}\right)\right] + I_{clamp}$ $\qquad I_b = 0.9787$ $\qquad$ [gr-cm-s^2]

$L := 16.108$ $\qquad I_{sp} := 0.5 \cdot \rho \cdot L \cdot \pi \cdot \left[\left(\frac{1.271}{2}\right)^4 - \left(\frac{1.076}{2}\right)^4\right]$ $\qquad I_{sp} = 5.4995 \times 10^{-3}$ $\qquad$ [gr-cm-s^2]

$\omega_1 := 22.75 \cdot 2 \cdot \pi$ $\qquad \omega_2 := 21.06 \cdot 2 \cdot \pi$

Resonat frequencies without mass and with mass for the aluminum specimen (No.1)

Mass polar moment of inertia for the Driven Plate: $\qquad I_{bs2} := I_b + \frac{1}{3} \cdot I_{sp}$

$$I_a := \frac{\left(I_{bs2} + I_m\right) \cdot \omega_2^{\,2} - I_{bs2} \cdot \omega_1^{\,2}}{\left(\omega_1^{\,2} - \omega_2^{\,2}\right)}$$

$I_a = 86.5255$ $\qquad$ [gr-cm-s^2]

$I2 := I_a$

Torsional stiffness of the rod: $\qquad K_a := \frac{I_m \cdot \left(\omega_1 \cdot \omega_2\right)^2}{\left(\omega_1^{\,2} - \omega_2^{\,2}\right)}$ $\qquad K_a = 1.788 \times 10^6$ $\qquad$ [gr-cm]

Inertia for Driven Plate and the 2.8" top cap: $\qquad I_{12} := I_a + I_{tp}$ $\qquad I_{12} = 88.1577$ $\qquad$ [gr-cm-s^2]

Shear wave velocity for the aluminum specimen

$I_d := I_a + I_b$ $\quad I_d = 87.5042$ [gr-cm-s2]

$$f(\beta) := \beta \cdot \tan(\beta) - \frac{I_{sp}}{I_d} \qquad (1)$$

Initial values to start iteration: $\beta := 0.001$ $\quad b_i := 0.0, 0.0005 .. 0.05$

First root of equation (1): $a := \text{root}(f(\beta), \beta)$ $\quad a = 7.9276 \times 10^{-3}$

Corresponding shear wave velocity: $v_s := \frac{\omega_1 \cdot L}{a \cdot 100}$ $\quad v_s = 2.9044 \times 10^{3}$ [m/s]

Calculation for the aluminum rod ϕ=1.930 cm $\quad \rho := 0.00274$

L2 and L3, r2 and r3, are the lengths and radii of the clamp. r1i, r1o are the outside and inside radius of the rod

$$r_{1o} := \frac{1.932}{2} \qquad r_{1i} := \frac{1.563}{2} \qquad r_2 := \frac{3.158}{2}$$

The volume of the clamp outside the top bar is: $V_{cl} := \pi \cdot L_2 \cdot \left(r_2^{\,2} - r_{1i}^{\,2}\right)$ $\quad V_{cl} = 3.2734$

Inertia of the top-bar. r1, rt, and d1 are radii of rod and screws, and the distance between screws :

$$a := 7.100 \quad b := 3.704 \quad t := 2.533 \quad r_t := \frac{\phi_t}{2} \quad d_1 := \frac{d_t}{2} \quad I_{clamp} := 0.5 \cdot \rho \cdot V_{cl} \cdot \left(r_2^{\,2} - r_{1i}^{\,2}\right)$$

$$I_b := \left[\frac{1}{12} \cdot \rho \cdot a \cdot b \cdot t \cdot \left(a^2 + b^2\right) - \frac{\pi}{2} \cdot \rho \cdot t \cdot \left(r_{1i}^{\,4} + 2 \cdot r_t^{\,4} + 2 \cdot r_t^{\,2} \cdot d_1^{\,2}\right)\right] + I_{clamp} \qquad I_b = 0.9724$$ [gr-cm-s2]

$$L := 16.164 \qquad I_{sp} := 0.5 \cdot \rho \cdot L \cdot \pi \cdot \left[\left(\frac{1.932}{2}\right)^4 - \left(\frac{1.563}{2}\right)^4\right] \qquad I_{sp} = 0.0346$$ [gr-cm-s2]

$\omega_1 := 59.19 \cdot 2 \cdot \pi$ $\quad \omega_2 := 54.13 \cdot 2 \cdot \pi$ $\quad$ Resonat frequencies without mass and with mass for the aluminum specimen (No.1)

Mass polar moment of inertia for the Driven Plate: $I_{bs3} := I_b + \frac{1}{3} \cdot I_{sp}$

$$I_a := \frac{\left(I_{bs3} + I_m\right) \cdot \omega_2^{\,2} - I_{bs3} \cdot \omega_1^{\,2}}{\left(\omega_1^{\,2} - \omega_2^{\,2}\right)} \qquad I_a = 73.661$$ [gr-cm-s2]

$I3 := I_a$

Torsional stiffness of the rod: $K_a := \frac{I_m \cdot \left(\omega_1 \cdot \omega_2\right)^2}{\left(\omega_1^{\,2} - \omega_2^{\,2}\right)}$ $\quad K_a = 1.0324 \times 10^{7}$ [gr-cm]

Inertia for Driven Plate and the 2.8" top cap: $I_{19} := I_a + I_{tp}$ $\quad I_{19} = 75.2932$ [gr-cm-s2]

Shear wave velocity for the aluminum specimen

$I_d := I_a + I_b$ $\quad I_d = 74.6334$ [gr-cm-s2]

$$f(\beta) := \beta \cdot \tan(\beta) - \frac{I_{sp}}{I_d} \qquad (1)$$

Initial values to start iteration: $\beta := 0.005$ $\quad b_i := 0.0, 0.0005 .. 0.05$

First root of equation (1): $a := \mathrm{root}(f(\beta), \beta)$ $\quad a = 0.0215$

Corresponding shear wave velocity: $v_s := \frac{\omega_1 \cdot L}{a \cdot 100}$ $\quad v_s = 2.7909 \times 10^3$ [m/s]

Calculation for the aluminum rod ϕ=2.523 cm

$\rho := 0.00274$

L2 and L3, r2 and r3, are the lengths and radii of the clamp. r1i, r1o are the outside and inside radius of the rod

$$r_{1o} := \frac{2.523}{2} \qquad r_{1i} := \frac{1.926}{2} \qquad r_2 := \frac{3.158}{2}$$

The volume of the clamp outside the top bar is: $V_{cl} := \pi \cdot L_2 \cdot \left(r_2^{\,2} - r_{1i}^{\,2}\right)$ $\quad V_{cl} = 2.7229$

Inertia of the top-bar. r1, rt, and d1 are radii of rod and screws, and the distance between screws :

$$a := 7.100 \quad b := 3.704 \quad t := 2.533 \quad r_t := \frac{\phi_t}{2} \quad d_1 := \frac{d_t}{2} \quad I_{clamp} := 0.5 \cdot \rho \cdot V_{cl} \cdot \left(r_2^{\,2} - r_{1i}^{\,2}\right)$$

$$I_b := \left[\frac{1}{12} \cdot \rho \cdot a \cdot b \cdot t \cdot \left(a^2 + b^2\right) - \frac{\pi}{2} \cdot \rho \cdot t \cdot \left(r_{1i}^{\,4} + 2 \cdot r_t^{\,4} + 2 \cdot r_t^{\,2} \cdot d_1^{\,2}\right)\right] + I_{clamp} \qquad I_b = 0.9645$$ [gr-cm-s2]

$$L := 17.356 \qquad I_{sp} := 0.5 \cdot \rho \cdot L \cdot \pi \cdot \left[\left(\frac{2.523}{2}\right)^4 - \left(\frac{1.926}{2}\right)^4\right] \qquad I_{sp} = 0.1249$$ [gr-cm-s2]

$\omega_1 := 105.13 \cdot 2 \cdot \pi$ $\quad \omega_2 := 96.38 \cdot 2 \cdot \pi$

Resonat frequencies without mass and with mass for the aluminum specimen (No.1)

Mass polar moment of inertia for the Driven Plate: $I_{bs4} := I_b + \frac{1}{3} \cdot I_{sp}$

$$I_a := \frac{\left(I_{bs4} + I_m\right) \cdot \omega_2^{\,2} - I_{bs4} \cdot \omega_1^{\,2}}{\left(\omega_1^{\,2} - \omega_2^{\,2}\right)} \qquad I_a = 75.9513$$ [gr-cm-s2]

$I4 := I_a$

Torsional stiffness of the rod: $K_a := \frac{I_m \cdot \left(\omega_1 \cdot \omega_2\right)^2}{\left(\omega_1^{\,2} - \omega_2^{\,2}\right)}$ $\quad K_a = 3.3579 \times 10^7$ [gr-cm]

Inertia for Driven Plate and the 2.8" top cap: $I_{25} := I_a + I_{tp}$ $\quad I_{25} = 77.5835$ [gr-cm-s2]

Shear wave velocity for the aluminum specimen

$I_d := I_a + I_b$ $I_d = 76.9157$ [gr-cm-s^2]

$f(\beta) := \beta \cdot \tan(\beta) - \frac{I_{sp}}{I_d}$ (1)

Initial values to start iteration: $\beta := 0.005$ $b_i := 0.0, 0.0005 .. 0.05$

First root of equation (1): $a := root(f(\beta), \beta)$ $a = 0.0403$

Corresponding shear wave velocity: $v_s := \frac{\omega_1 \cdot L}{a \cdot 100}$ $v_s = 2.8454 \times 10^3$ [m/s]

$I_{avg} := \frac{1}{3} \cdot (I_6 + I_{12} + I_{19})$ $I_{a\_avg} := \frac{1}{4} \cdot (I1 + I2 + I3 + I4)$ $I_{a\_avg} = 76.9381$

Average driving plate inertia + top cap inertia= $I_{avg} = 78.8993$ $I_{bs4} = 1.0061$

$$\sigma := \frac{\sqrt{(I_6 - I_{avg})^2 + (I_{12} - I_{avg})^2 + (I_{19} - I_{avg})^2 + (I_{25} - I_{avg})^2}}{3}$$ $\sigma = 3.8356$

$I1 + I_{bs1} = 72.597$ $I2 + I_{bs2} = 87.506$ $I3 + I_{bs3} = 74.6449$ $I4 + I_{bs4} = 76.9574$

EVALUATION OF ROOT "a" TO COMPUTE WAVE VELOCITY

I = mass polar moment of inertia of the driven plate

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Inertia of driving plate + top platten : $I_d := I_{avg}$ [gr-cm-s^2] $g := 981$ [cm/s^2]

Iterative equation for shear wave velocity: $f(\beta, I_{sp}, I_d) := \beta \cdot \tan(\beta) - \frac{I_{sp}}{I_d}$ (1)

Root "a" for Barco sand: $I_d = 78.8993$

Sample 1, e=0.614: $W_t := 935.67$ $\phi := 7.0563$ $H := 14.573$ $I_{sp} := \frac{1}{2} \cdot \left(\frac{W_t}{g}\right) \cdot \left(\frac{\phi}{2}\right)^2$

Initial values to start iteration: $\beta := 0.40$ $I_{sp} = 5.9363$

First root of equation (1): $a := root(f(\beta, I_{sp}, I_d), \beta)$ $a = 0.27091$

Total Inertia of sand sample: $I_t := I_{avg} + I_{sp}$ $I_t \cdot 9.81 \cdot 10^{-5} = 8.3224 \times 10^{-3}$

$I_{sa} := \frac{1}{2} \cdot (W_t) \cdot \left(\frac{\phi}{2}\right)^2$ $I_{sa} = 5.8235 \times 10^3$

$I_{dn} := 981 \cdot I_d$ $I_{dn} = 7.74 \times 10^4$ $a := root(f(\beta, I_{sa}, I_{dn}), \beta)$ $a = 0.27091$