Regression Kink Design

http://www.jasondebacker.com/classes/Lecture8_Notes_Kinks.pdf

The regression analysis is run first by retrieving β_1 in this equation:

$$E[Y|V=v] = \alpha_0 + \sum_{p=1}^{p}[\alpha_p(v-k)^p + \beta_p(v-k)^p * D], \quad (1)$$

- Where Y is the outcome variable of interest; k is the kink point; v is the running variable, and D allows the slopes to change at the kink.
- The β_1 retrieves the change in slope in outcomes at the kink point in the treatment variable.

Data

x	y
0.0923	$3.39 \cdot 10^{6}$
0.196	$3.05 \cdot 10^{6}$
0.350	$2.48 \cdot 10^{6}$
0.478	$1.97 \cdot 10^{6}$
0.584	$1.48 \cdot 10^{6}$
0.680	$1.09 \cdot 10^{6}$
0.770	$7.20 \cdot 10^{5}$
0.852	$3.60 \cdot 10^{5}$
0.890	$1.46 \cdot 10^{5}$
0.930	$1.5 \cdot 10^{3}$
0.952	$6.6 \cdot 10^{0}$
0.972	$3.5 \cdot 10^{-2}$
0.980	$3.5 \cdot 10^{-4}$
0.986	10^{-7}

Equations

$k := 0.92$ Kink point

Guesses

d	$a0$	$a1$	$a2$	$a3$	$a4$	$a5$	$a6$	$b1$	$b2$	$b3$	$b4$	$b5$	$b6$
1	1	1	1	1	1	1	1	1	1	1	1	1	1

Generate Fit

$$\begin{aligned} f(x, d, a0, a1, b1, a2, b2, a3, b3, a4, b4, a5, b5, a6, b6) := {} & a0 + (a1 \cdot (x-k) + b1 \cdot d \cdot (x-k)) \\ & + \left(a2 \cdot (x-k)^2 + b2 \cdot d \cdot (x-k)^2\right) \\ & + \left(a3 \cdot (x-k)^3 + b3 \cdot d \cdot (x-k)^3\right) \\ & + \left(a4 \cdot (x-k)^4 + b4 \cdot d \cdot (x-k)^4\right) \\ & + \left(a5 \cdot (x-k)^5 + b5 \cdot d \cdot (x-k)^5\right) \\ & + \left(a6 \cdot (x-k)^6 + b6 \cdot d \cdot (x-k)^6\right) \end{aligned}$$

$$\overrightarrow{f(x, d, a0, a1, b1, a2, b2, a3, b3, a4, b4, a5, b5, a6, b6)} - y = 0$$

$$\begin{bmatrix} d \\ a0 \\ a1 \\ b1 \\ a2 \\ b2 \\ a3 \\ b3 \\ a4 \\ b4 \\ a5 \\ b5 \\ a6 \\ b6 \end{bmatrix} := \mathbf{Minerr}(d, a0, a1, b1, a2, b2, a3, b3, a4, b4, a5, b5, a6, b6)$$

$$\begin{bmatrix} d \\ a0 \\ a1 \\ b1 \\ a2 \\ b2 \\ a3 \\ b3 \\ a4 \\ b4 \\ a5 \\ b5 \\ a6 \\ b6 \end{bmatrix} = \begin{bmatrix} -250.4441 \\ 5.7546 \cdot 10^{4} \\ 3.861 \cdot 10^{6} \\ 2.6998 \cdot 10^{4} \\ -1.0253 \cdot 10^{7} \\ -1.3132 \cdot 10^{5} \\ -3.3954 \cdot 10^{6} \\ -5.0396 \cdot 10^{5} \\ 5.2891 \cdot 10^{7} \\ -9.6861 \cdot 10^{5} \\ 9.3742 \cdot 10^{7} \\ -9.2406 \cdot 10^{5} \\ 4.8593 \cdot 10^{7} \\ -3.3637 \cdot 10^{5} \end{bmatrix}$$