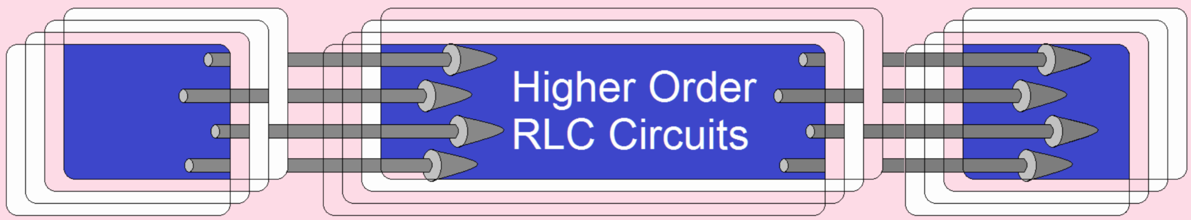
Part 3B:

1. Chapter 13 of Engineering Circuit Analysis, Hyat and Kemmerly. 4th edition.
2. Chapter 8 Schaums Outlines: Higher Order Circuits and Complex Frequency 6th Edition.

Level: Intermediate.

Circuiting PrerequisitesTo Laplace Transform Electric Circuits.



Part 3 - B

(Intermediate Level)

Apologies for any errors and omissions.

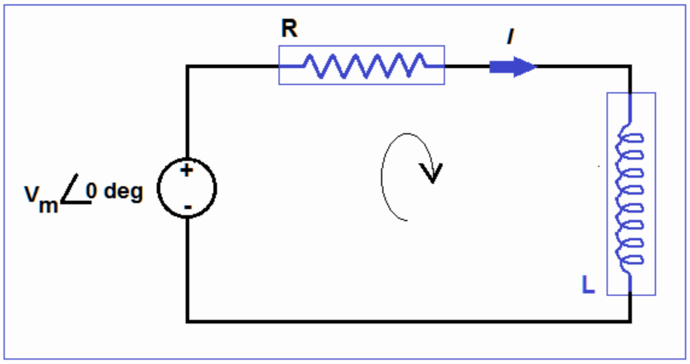
July 2020.

Engineering college year 2 course of 4 year program OR year 1 of 3 year program. Re-fresher OR Self Study. Graduate Study Review.
May be used in New Zealand, US, Malaysia, India, Pakistan, UK, and other Common Wealth Country engineering colleges.
Any errors and omissions apologies in advance.

Chapter 6 Part B.

13.5 Frequency Response As A Function of Time:

We end part A with a short introduction on section 13.5, we pick up from start of section 13.5, little refresher, and continue with section 13.5. We needed some intro level understanding of zeros and poles.



Voltage source: $V_m\angle 0$

Forced response exist because of the voltage source.

Frequency domain:

$$L = sL$$

$$I = \frac{V_m\angle 0}{R + sL}$$

$$s = \sigma + j\omega \qquad j\omega = 0$$

$$s = \sigma$$

$$v_s = V_m e^{(\sigma + j\omega)t} = V_m e^{(\sigma + 0)t}$$

$$v_s = V_m e^{\sigma t}$$

$$I = \frac{V_m\angle 0}{R + sL} = \frac{V_m\angle 0}{R + \sigma L}$$ Divide top and bottom of RHS by L

$$I = \frac{(V_m\angle 0)}{L}\left(\frac{1}{\frac{R}{L} + \frac{\sigma L}{L}}\right) = \frac{(V_m\angle 0)}{L}\left(\frac{1}{\frac{R}{L} + \sigma}\right)$$

$$I = \frac{V_m}{L}\left(\frac{1}{\sigma + \frac{R}{L}}\right)$$

Sigma is isolated (independent on its own).
Neglect 0 deg, rearrange bottom of 2nd term.
Vm is a phasor, dropped 0 deg because its understood for the first reference point in the circuit. We dont need to complicate the phase angle in the theory build-up.

$$I = \frac{V_m}{L\sigma + R}$$

$$I = \frac{V_m}{R + L\sigma}$$ <--- Frequency domain

Convert to time domain.

Previous notes from part A below:

2.---> $v(t) = V_0$ and when theta =0 deg, voltage is Vo for t=0 initial voltage.

But if we let sigma only equal 0 we get the general sinusoidal voltage, the one we usually see:

3.---> $v(t) = V_m \cdot cos(\omega t + \theta)$ $\cos(0\ deg) = 1$

And if we let omega only equal 0 we get the exponential voltage:

$$v(t) = V_m \cdot e^{\sigma t} \cdot cos(\theta)$$

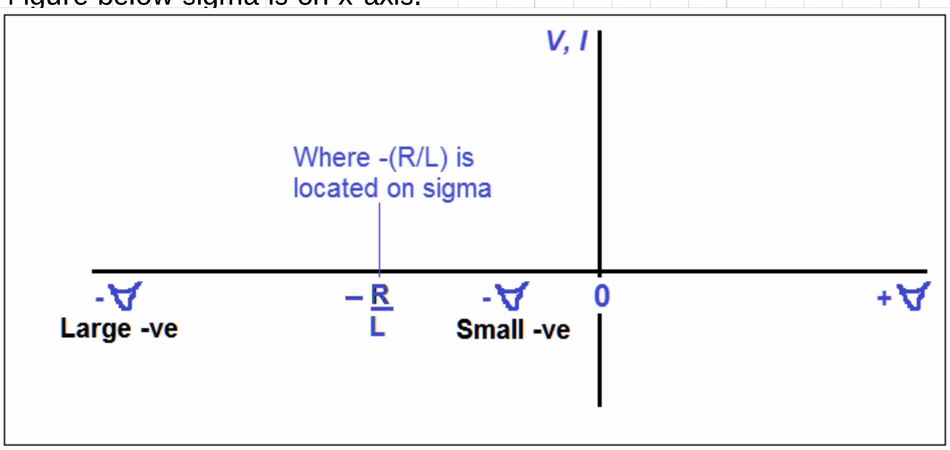
4.---> $v(t) = V_0 \cdot e^{\sigma t}$ ...when theta =0 deg, cos(0 deg) = 1, Vm cos (theta) = Vo..., Why Vo? Vm for sinusoidal maximum, so Vo for exponential's constant term.

We have number 1 the damped sinusoid and in it includes the special cases number 2 dc (constant), number 3 sinusoidal (general expression) and number 4 exponential function.

Instead of v(t) we have i(t)......points 3 and 4 above.

$$i(t) = \left(\frac{V_m}{R + L\sigma}\right) e^{\sigma t}$$ <--- Time domain

Next we look at some cases on the function's outcome relative to sigma.
Figure below sigma is on x-axis.



$$I = \frac{V_m}{L}\left(\frac{1}{\sigma + \frac{R}{L}}\right)$$ <----Sigma here is of interest to us relative to the outcome of I.

Next maybe I/You see the connection why we stopped and detoured to Zeros and Poles.
We did this before but did not include zeros and poles, I lacked that knowledge. Not You.

I. Case when sigma is large negative number: NEAR ZERO RESPONSE!

$e^{-(large)\,t}$ ---> exponentially decreasing rapidly
$V_m e^{-(large)\,t}$ ---> which makes the voltage decrease rapidly

$$I = \frac{V_m}{L}\left(\frac{1}{(-large)+\frac{R}{L}}\right) = \frac{V_m}{L}('-small\_value)$$

We see now the benefit of using this form, isolating sigma.

$I = small$ ---> our current is small in amplitude when sigma is a -ve large number.

II. Case when sigma is smaller negative closer to zero number: ZERO RESPONSE !

$e^{-(small)\,t}$ ---> exponentially decreasing slowly in comparison to case I.
$V_m e^{-(small)\,t}$ ---> which makes the voltage decrease slower in comparison.

$$I = \frac{V_m}{L}\left(\frac{1}{(-small)+\frac{R}{L}}\right) = \frac{V_m}{L}('-ve\_Or\_+ve\_larger\_in\_comparison\_to-caseI)$$

$I = larger\_than\_case\_I$ ---> current is larger in amplitude when sigma is a -ve small number.

III. Case when sigma close to -(R/L): POLE RESPONSE !

$e^{-\left(\frac{R}{L}\right)t}$ ---> dependent on values of R/L but its negative, decreasing exponentially.

$V_m e^{-\left(\frac{R}{L}\right)t}$ ---> voltage decrease.

$$I = \frac{V_m}{L}\left(\frac{1}{-\left(\frac{R}{L}\right)+\frac{R}{L}}\right) = \frac{V_m}{L}\left(\frac{1}{small}\right) = \frac{V_m}{L}('large)$$

<---(1/small) equal infinity we say arbitrarily large, in comparison to case I and II.

$I = Large$ ---> our current is largest in amplitude when sigma is -R/L.

VI. Case when sigma = 0: DC Case !

$$I = \frac{V_m}{L}\left(\frac{1}{0+\frac{R}{L}}\right) = \frac{V_m}{L}\left(\frac{L}{R}\right) = \frac{V_m}{R}$$

<---the forced response is Vm/R we have the dc case.

Next we example it for the above cases. This by giving values to the variables.
Example: All cases with jw =0.

$V_m := 1 \quad L := 1 \quad R := 1 \quad \frac{R}{L} = 1$

When $\sigma := 10$

$$I := \frac{V_m}{L} \cdot \left(\frac{1}{\sigma + \left(\frac{R}{L} \right)} \right) = 0.09$$

Case I...we have a small value

When $\sigma := 1000$

$$I := \frac{V_m}{L} \cdot \left(\frac{1}{\sigma + \left(\frac{R}{L} \right)} \right) = 0.000999$$

Case II...we have a ZERO

When $\frac{R}{L} = 1 \quad \sigma := -\left(\frac{R}{L} + 0.001 \right) = -1.001$ sigma close to R/L opposite in sign -ve.

$$I := \frac{V_m}{L} \cdot \left(\frac{1}{\sigma + \left(\frac{R}{L} \right)} \right) = -1000$$

Case III...we have a POLE
It can be -ve or +ve just so its arbitrarily high magnitude.

$\frac{R}{L} = 1 \quad \sigma := \left(\frac{R}{L} + 0.001 \right) = 1.001$

$$I := \frac{V_m}{L} \cdot \left(\frac{1}{\sigma + \left(\frac{R}{L} \right)} \right) = 0.49975$$

Here denominator expression gets greater than R/L, when sigma = 1.001, whereas when its -1.001 that added to R/L results in -0.001. We get a pole

When $\frac{R}{L} = 1$ and $\sigma := 0$

$$I := \frac{V_m}{L} \cdot \left(\frac{1}{0 + \left(\frac{R}{L} \right)} \right) = \frac{V_m}{L} \cdot \left(\frac{L}{R} \right) = \frac{V_m}{R}$$

Case VI...dc current, we have the dc case.

Positive values of sigma must provide positive amplitude responses, the larger amplitudes resulting when sigma is smaller. An infinite value of sigma provides a zero-amplitude response and sets a zero. Only critical frequencies are the pole at sigma = -R/L, and the zero at sigma = +/- infinity. We see the significance of plots at end of part 3A zeros and poles. Applicable here.

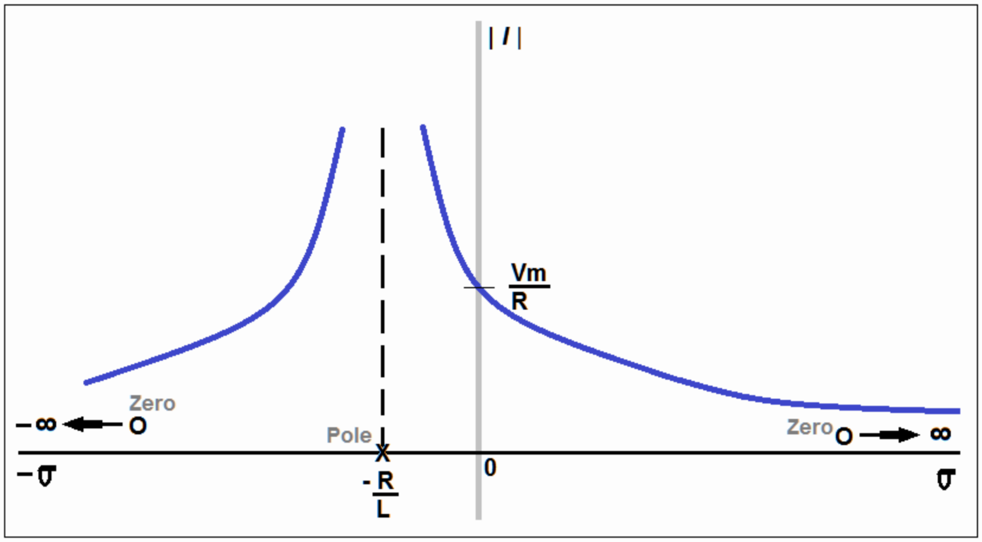


Figure above: Plot of the magnitude I versus sigma (neper frequency) for a series RL circuit excited by an exponential voltage source, Vme^(sigma_t).
Only pole located at sigma = -R/L, and zero is located at sigma = +/-infinity.
We seen this at end of part 3A.

May be you thought of this the practical value of so high a numerical value of the amplitude at the pole (sigma = near R/L)?
Well the engineers did. *Not a problem thats why they are engineers.*
The RL circuit we investigated had a forced response, because? we have a Vm.
How does the RL circuit behave if its to be a natural response?
This is the angle the engineers are going to take on next.

We have an RL circuit.
The appropriate form of natural response of the current:

$$i_n(t) = I_m e^{-\frac{Rt}{L}}$$

At t=0 we get the natural response of the amplitude Im:

$$i_n(0) = I_m e^{-\frac{R(0)}{L}}$$

$$i_n(0) = I_m$$

Now in this case we are not applying a forcing function, the only response we can get is the natural response, which of course in this condition is the complete response.

If we apply a forcing function of zero amplitude, same as no forcing function, our response is a finite amplitude. Finite, not infinite as in the forcing function case we seen on the previous page.

$$I := \frac{V_m}{L} \cdot \left(\frac{1}{\sigma + \left(\frac{R}{L}\right)} \right)$$

<--- We work this equation again but the forcing function Vm has to be dealt with because we are working in a natural response.

We have: $R := 1$ $L := 1$

For a pole we done studied-worked sigma has to be? near R/L with a negative sign so the resulting denominator is small. For now we make sigma = - (R/L) = -1 Np/s. Though thats infinite, its clearly a pole. We are looking for finite or course.

$$\sigma = -\left(\frac{R}{L}\right) = -1$$

$$I = \frac{V_m}{L} \cdot \left(\frac{1}{-\left(\frac{R}{L}\right) + \left(\frac{R}{L}\right)} \right)$$

<---We have a pole.

Question: With a forcing function $v_s = V_m e^{\sigma t}$

What amplitude Vm must have to cause a current I = 1A ?

In other words current something like: $i(t) = 1\ e^{\sigma t}$ at $s = \sigma + j\ \omega$

Lets evaluate what we can come up with so far.

$R := 1$ $L := 1$ $I := 1$

$$I = \frac{V_m}{L} \cdot \left(\frac{1}{\sigma + \left(\frac{R}{L}\right)} \right) = \frac{V_m}{1} \cdot \left(\frac{1}{\sigma + 1} \right) = V_m \left(\frac{1}{\sigma + 1} \right)$$

$$1 = V_m \left(\frac{1}{\sigma + 1} \right)$$

$V_m = \sigma + 1$ <--- We have Vm for voltage amplitude of forcing function.

Vm is the forcing function voltage amplitude to maintain a 1A current amplitude in this RL circuit.

Lets look at this one or two layers deeper on what we have equated above, and under the conditions we did. So it makes easier to understand what the engineers are about to say.

$$I = \frac{V}{R}$$

Lets say this is dc condition, I the current in the series RL circuit. Resistance will have to be near zero or small for a high current I value. In other-words, with R low, we have a high current, which if R is near 100% pure conductor, current I will be so high comparativley in the circuit its like saying I is a short circuit current. We have a pole.

Having said that, we know from our inductor definition, L, its a short circuit under dc conditions, the voltage across the inductor is zero. <--- That is the zero amplitude forcing function.

Next I try to understand what the engineers said:

'This is the amplitude of the forcing function that is required to maintain a current of 1A amplitude in the circuit. We note that a zero amplitude forcing function, or a short circuit, is is sufficient at the frequency of the pole.' - Hyat Kemerly page 346 4th edition.

$V_m = \sigma + 1$ <--- amplitude of forcing function.

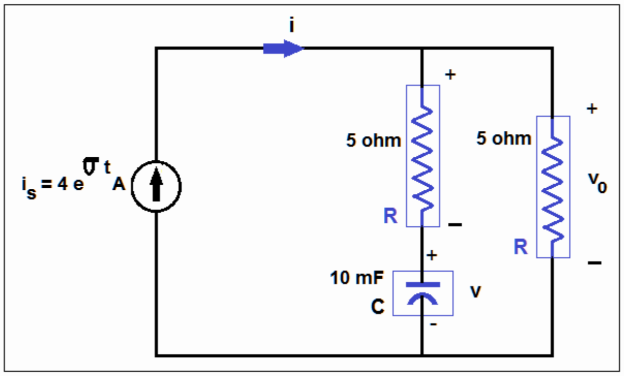
Vm above gives the natural response of 1A which is not a pole in the sense of infinity rather its finite. Maybe you are like me what did that accomplish. Well argument wise or discussion, we were led to belief a 1 A finite response could be accomplished. Under what conditions? Short circuit and zero amplitude forcing function, *thats what I am saying hope you do better.*

We seen the plots in end of part 3A and seen a figure above on the previous page. We can identify where the pole is at the neper frequency (sigma).

However, we have NOT yet sorted or defined the relationship between the frequency of the pole and the form of the natural response, AGREED, coming soon in examples the engineers said.

Next we work an RC circuit example, the author-engineers provided.

Example: RC Circuit.



The circuit's forcing function is the current source.

The forced response or desired response is the output voltage v0(t).

This voltage v0(t) is same across R, Series R and C, and across the current source.

Circuit impedance to the right of the current source:

$R1 := 5 \qquad R2 := 5 \qquad I_s = 4 e^{\sigma t}$

$$Z_{C\_calc} := \frac{1}{10 \cdot 10^{-3} \cdot s} = 100 \frac{1}{s}$$

Lets set Zc_calc to Zc:

$$Z_C := \frac{100.}{s}$$

$$Z_{left\_branch} = R1 + Z_C = 5 + \frac{100}{s}$$

Z_{input} : R2 parallel to Z_left_branch

$$Z_{input}(s) = \frac{(5) \cdot \left(5 + \frac{100}{s}\right)}{5 + \left(5 + \frac{100}{s}\right)} = \frac{25 + \frac{500}{s}}{10 + \frac{100}{s}}$$

In this circuit the current source: $4 e^{\sigma t}$
We do not have a j*w* in the exponent's power, nor phase angle given, which can be set to 0 should it arise because there is no voltage source to compare.

$s = \sigma + j\omega$
$s = \sigma + j0 = \sigma$

Now impedance as a function of sigma instead of s.

$$Z_{input}(\sigma) = \frac{25 + \frac{500}{\sigma}}{10 + \frac{100}{\sigma}}$$ Next simplify this expression

$$Z_{input}(\sigma) = \frac{25 + \frac{500}{\sigma}}{10 + \frac{100}{\sigma}}$$

Multiply by sigma top and bottom
Fix the bottom by pulling out 10/25 to 25/10
to the side, and multiplying 4 by (25/4)

$$= \frac{25\sigma + 500}{10\sigma + 100}$$

Divide by 25

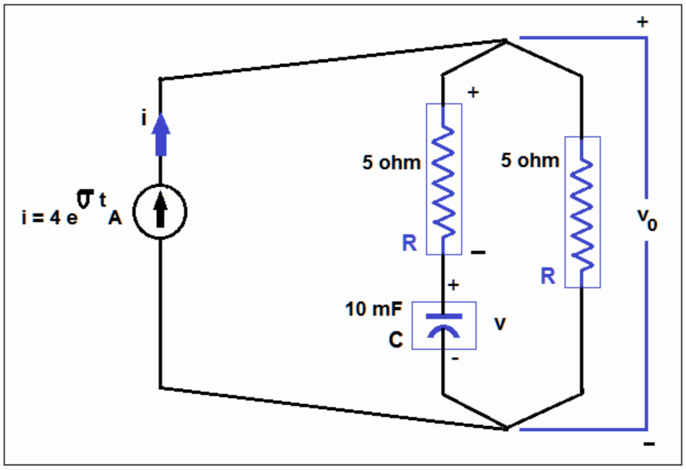
$$= \frac{1\sigma + 20}{\left(\frac{10}{25}\right)\sigma + 4}$$

Multiply by 25/10 to simplify bottom

$$= \frac{\left(\frac{25}{10}\right)(1\sigma + 20)}{\frac{25}{10}\left(\frac{10}{25}\sigma + 4\right)} = \frac{25}{10}\left(\frac{1\sigma + 20}{1\sigma + 10}\right)$$

$$Z_{input}(\sigma) = (2.5)\frac{1\sigma + 20}{1\sigma + 10}$$

This is what textbook has.



The circuit impedance we calculated, and the current for the circuit provided, we can calculate V=I Z at the source branch.

V which is same across all branches in this circuit.

Vo = I Z_input

$v_0(\sigma) = I_s \cdot Z_{input}(\sigma)$ $\quad I_s = 4\,e^{\sigma t}$ at t=0 $\quad I_s = 4$ frequency domain

$$v_0(\sigma) = 4\cdot\left[(2.5)\frac{1\sigma + 20}{1\sigma + 10}\right] = 10\,\frac{1\sigma + 20}{1\sigma + 10} = \frac{10\,(1\sigma + 20)}{1\sigma + 10}$$

$$v_0(\sigma) := \frac{10\,(1\sigma + 20)}{1\sigma + 10}$$ <--- frequency domain expresion for v.

$$v_0(\sigma) = \frac{10\,(1\,\sigma + 20)}{1\,\sigma + 10}$$ <---Why the engineers wanted it in this form?

The form above for v0(sigma) allows us to evaluate for poles and zeros.
We see next. Yes, thats why they are Engineers.

Pole:

To evaluate for the pole we know the denominator has to be something close to zero, remember in RL circuit, we worked to make (sigma + R/L) close to zero as possible. Yes! That gets us the maximum value.

$$v_0(\sigma) = \frac{10\,(1\,\sigma + 20)}{1\,\sigma + 10}$$ <---1 sigma must equal something close to -10 sigma to get bottom close to zero.

$$v_0(\sigma) = \frac{10\,(1\,\sigma + 20)}{1\,(-10) + 10} = \frac{10\,(1\,\sigma + 20)}{-10 + 10} = \frac{10\,(1\,\sigma + 20)}{0}$$

We have a pole at : $\sigma = -10$

Zero:

To evaluate for the zero we know the numerator has to be something close to zero, so nothing divided by something is nothing, i.e. something of value close to zero. Yes! That gets us the minimum value.

$$v_0(\sigma) = \frac{10\,(1\,\sigma + 20)}{1\,\sigma + 10}$$ <---1 sigma must equal something close to -20 sigma to the top is close to zero.

$$v_0(\sigma) = \frac{10\,(1\,(-20) + 20)}{1\,\sigma + 10} = \frac{-200 + 200}{1\,\sigma + 10} = \frac{0}{1\,\sigma + 10}$$

We have a zero at : $\sigma = -20$

We need not worry about infinite frequency its not a critical frequency.

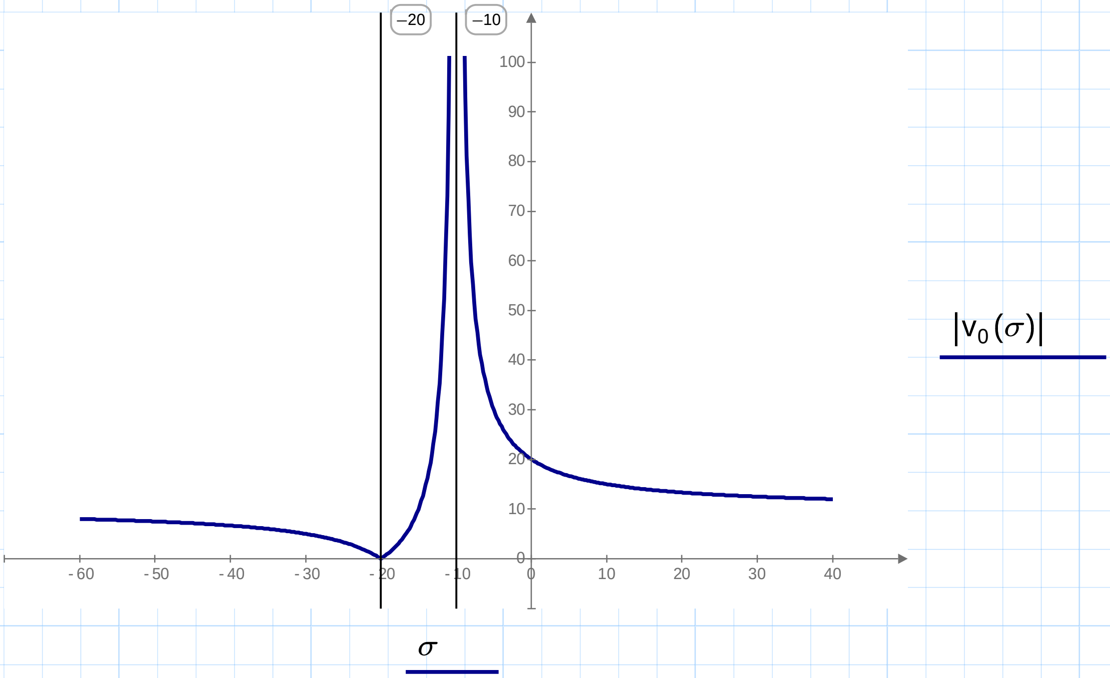
Time domain expression for vo(t) just place e^(sigma t) in the frequency domain expression.

$$v_0(t) = \frac{10\,(1\,\sigma + 20)}{1\,\sigma + 10}\,e^{\sigma t}$$ <--- time domain expresion for v.

clear (σ)

$$v_0(\sigma) := \frac{10\ (1\ \sigma + 20)}{1\ \sigma + 10}$$

The plot below the magnitude, which is indicated in the plot below as |v0(sigma)|.



Pole at -10 Np/s indicates a forcing function of 4A amplitude can produce a voltage of artibrarily large amplitude when the frequency (sigma) is brought close to - 10 Np/s.

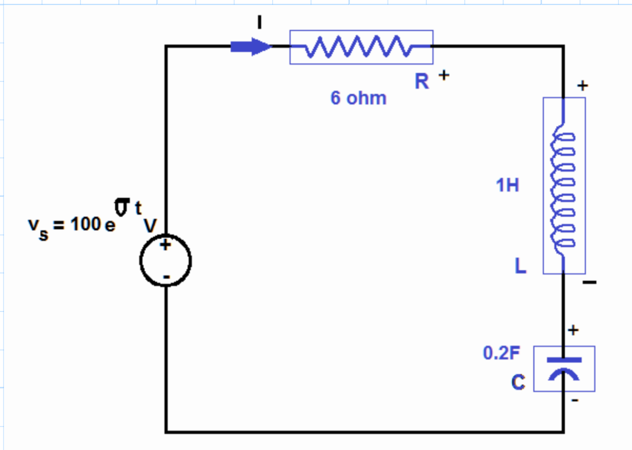
Zero at -20 Np/s indicates no finite amplitude from the source at this frequency is capable of generating any output voltage.

Now we have a come into a new method of circuit analysis where frequency, sigma - Np/s, provides for a forcing function specific frequencies where the output amplitude is significant (Pole) or insignficant (zero).

Next engineers show us a simple but complicated response from an RLC circuit.

Example: RLC Circuit.

Find the current I and plot the current magnitude vs sigma frequency?



$R := 6 \quad Z_R = 6$

$L := 1 \quad Z_L = s1$

$C := 0.2 \quad Z_C = \dfrac{1}{sC}$

Circuit impedance to the right of the current source:

$$Z_{input} = R + L + C$$

$$= 6 + s1 + \left(\frac{1}{s0.2}\right) = 6 + s1 + \left(\frac{1}{s\left(\frac{1}{5}\right)}\right)$$

$$Z_{input} = 6 + s + \frac{5}{s}$$

$$v(t) = 100\ e^{\sigma t}$$

$$V = 100$$

$$I = \frac{V}{Z_{input}} = \frac{100}{6 + s + \frac{5}{s}}$$

Factor the bottom expression, multiply top and bottom by 's'.

$$I = \frac{V}{Z_{input}} = \frac{100\ s}{6\ s + s^2 + 5}$$

$$6\ s + s^2 + 5 = s^2 + 6\ s + 5$$

$$(s+1)\ (s+5) = s^2 + s5 + s + 5 = s^2 + 6\ s + 5 \quad \text{factored.}$$

$$I = \frac{V}{Z_{input}} = \frac{100\ s}{(s+1)\ (s+5)}$$

$$I = 100 \cdot \frac{s}{(s+1)\,(s+5)}$$

$s = \sigma + j\ \omega$

$j\ \omega = 0$

$s = \sigma$ substitute sigma for s

$$I = \frac{100\ \sigma}{(\sigma+1)\,(\sigma+5)}$$

$$I(\sigma) := 100 \cdot \frac{\sigma}{(\sigma+1)\,(\sigma+5)}$$

Related to the solution later the differentiation of expression above on coming pages.

Pole:

To evaluate for the pole we know the denominator has to be something close to zero.

$$I(\sigma) = 100 \cdot \frac{\sigma}{(\sigma+1)\,(\sigma+5)}$$

<---1 sigma must equal something close to -1 sigma AND -5 sigma to get bottom close to zero.

$$I(\sigma) = \frac{100\ \sigma}{(-1+1)\,(-5+5)} = \frac{100\ \sigma}{(0)\,(0)} = 100 \cdot \frac{\sigma}{0}$$

Poles : $\sigma = -1$ and $\sigma = -5$

Zero:

To evaluate for the zero we know the numerator has to be something close to zero.

$$I(\sigma) = \frac{100\ \sigma}{(\sigma+1)\,(\sigma+5)}$$

<--- sigma in numerator must equal something close to 0 100 x 0 = 0.

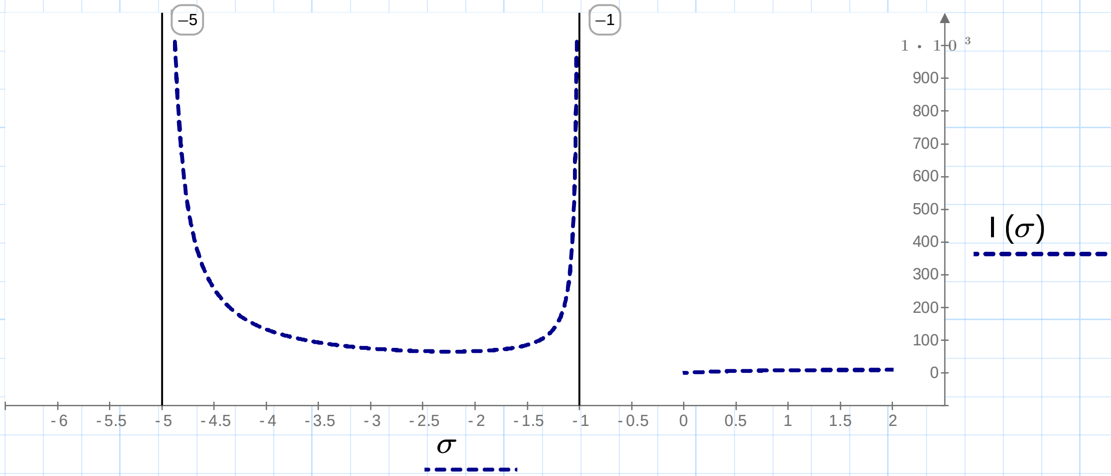
$$I(\sigma) = \frac{100 \cdot 0}{(\sigma+1)\,(\sigma+5)} = \frac{0}{(\sigma+1)\,(\sigma+5)}$$

Zero : $\sigma = 0$

Lets plot next page.

clear (σ, s) $\quad \sigma := -100, -99.99..100$

$$I(\sigma) := \frac{100 \cdot \sigma}{(\sigma+1)\ (\sigma+5)}$$



We have the plot sorted above, except for something, new, theory wise that needs looking into. Not just a passing thing but required to complete for advanced studies and work!

Discussion: We see the pole location at -1, and -5, and a zero at 0. The curve lines rising rise up higher and higher as the range is increased. But this plot does not have the two sides that make up the mountain-curve for a pole we see only one side for each pole. We do NOT need both sides leading up, the other sides do not exist for the sigma values. Otherwise we would see it. *If we extent the sigma-axis range we may see it but none were found for a realistic -sigma and + sigma.*

In a plot between 2 maximum we have a minimum? Yes. 2 mountain peaks a minimum in between peaks - valley, and between two minimums a maximum or peak-mountain in between.

A function may have a maximum and minium which may be found through its 1st derivative. Calculus stuff, max min. This is what the engineers said:

The response curve may be obtained by indicating the locations of all poles and zeros on the sigma axis, and placing vertical asymptotes at the poles. When this is done, we find that the **relative minimum** of the forced response must exist between two poles, and a **relative maximum** must be present at some frequency greater than zero.

Differentiation function: $\dfrac{100\ s}{(s+1)\ (s+5)}$

$u\ =\ 100\ s \qquad v\ =\ (s+1)\ (s+5)\ =\ \left(s^2+6\ s+5\right)$

$$\frac{d\left(\frac{u}{v}\right)}{ds} = \frac{v\left(\frac{du}{dx}\right)-u\left(\frac{dv}{dx}\right)}{v^2}$$

$$= \frac{\left(s^2+6\ s+5\right)\ (100)-(100\ s)\ (2\ s+6)}{((s+1)\ (s+5))^2}$$

$$= \frac{\left(s^2\ 100+600\ s+500\right)-\left(200\ s^2+600\ s\right)}{((s+1)\ (s+5))^2}$$

$$= \frac{-100\ s^2+500}{((s+1)\ (s+5))^2}$$

$$= \frac{-100\ s^2+500}{((s+1)\ (s+5))^2}$$

$$= \frac{-100\ \left(s^2-5\right)}{(s+1)\ (s+5)\ (s+1)\ (s+5)}$$

$$\frac{d\left(\frac{u}{v}\right)}{ds} = \frac{-100\cdot\left(s-\sqrt{5}\right)\ \left(s+\sqrt{5}\right)}{(s+1)^2\cdot(s+5)^2}$$ Next change s to sigma

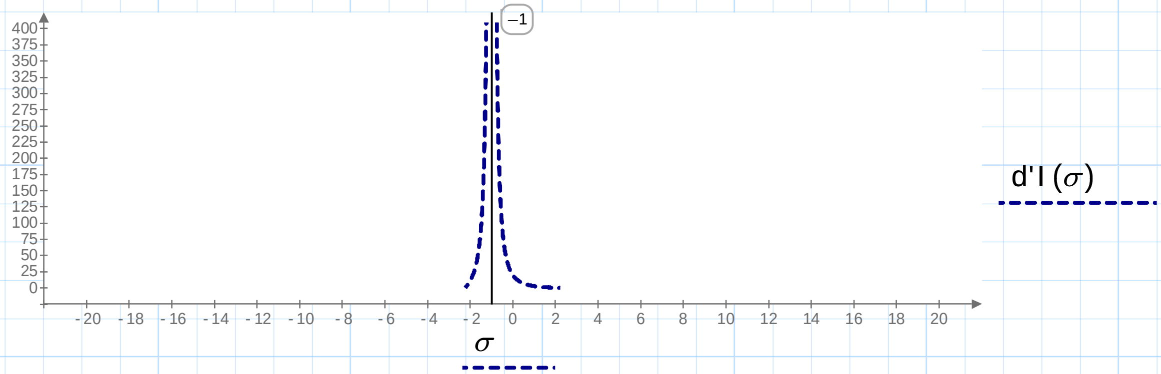
$$\frac{d\left(\frac{u}{v}\right)}{d\ \sigma} = \frac{-100\cdot\left(\sigma-\sqrt{5}\right)\ \left(\sigma+\sqrt{5}\right)}{(\sigma+1)^2\cdot(\sigma+5)^2}$$ <--- Zero
<--- Pole

The poles are identical at -1 and -5, but this time in 1st derivative.
A relative minimum must exist between the 2 poles; poles at -1 and -5?
The zeros, sigma = +sqrt(5) and sigma = -sqrt(5), a relative maximum must exist at some frequency greater than 0?

Poles 1st derivative: -1 and -5 <---maximum at these points
Zeros 1st derivative: $-\sqrt{5}$ and $\sqrt{5}$ <---minimum at these points

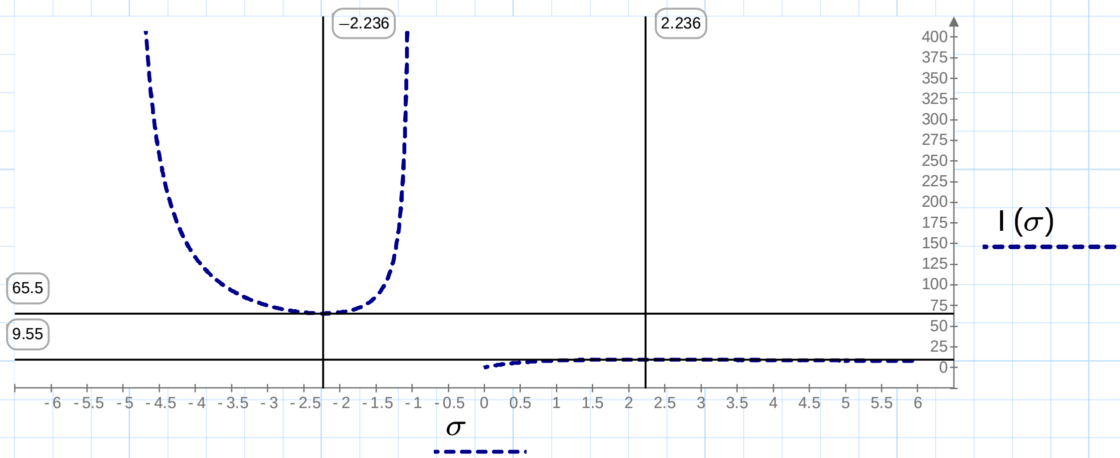
How do we solve this? Maybe do a plot of the derivative function?

$$\frac{d\left(\frac{u}{v}\right)}{d\,\sigma} = d'I(\sigma) := \frac{-100\cdot\left(\sigma-\sqrt{5}\right)\cdot\left(\sigma+\sqrt{5}\right)}{(\sigma+1)^2\cdot(\sigma+5)^2}$$



Plot above is the derivative of the function of I. We have a peak at -1 so its maximum and got noting at -5. Which -1 matches our pole in the I(sigma) plot. Otherwise, how do we interpret the derivative of function of current? Maximum and Minimum points maybe? The Engineers did not plot this function. We take the zeros of the derivative function, sqrt(5) and sqrt(-5), and search for their I magnitude values in the I(sigma) plot. Provided below, as the engineers performed.

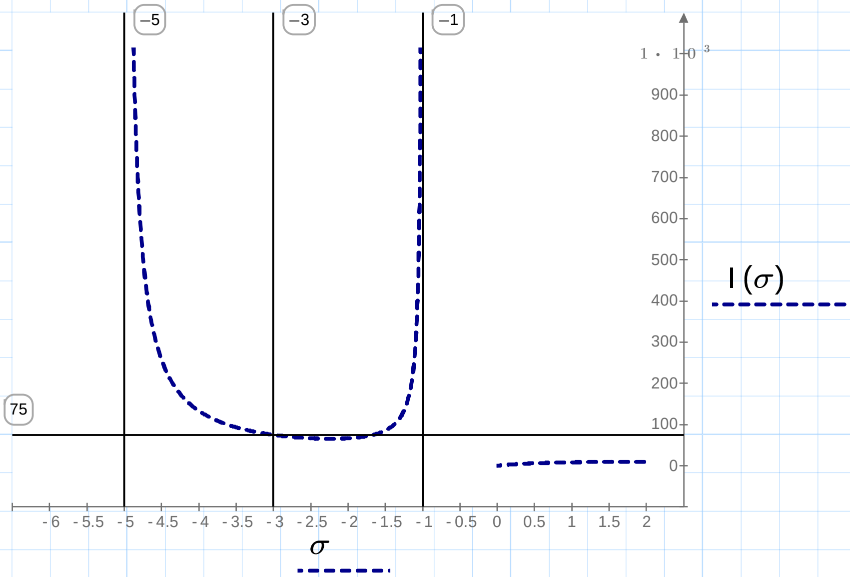
$\sqrt{5} = 2.236$ $\quad -\sqrt{5} = -2.236$



Zeros - 1st derivative: $-\sqrt{5}$ and $\sqrt{5}$
The curve to the left at -sqrt(5) the minimum value at: 65.5 Answer.
The curve to the right at sqrt(5) the maximum value at: 9.55 Answer.
As per textbook results.

Eventually we want a time domain response of the current.
So how do we get there.
And we see roots s1 and s2 from quadratic equation solution for series RLC circuit.

We got -5 to -1 the range in the plot below the average or mid-point will be at? -3.
We use that magnitude of I for forced response magnitude at sigma = -3 in e^-st.



At $\sigma = -3$ $\quad I(\sigma) = 75$

Under these conditions:

Forcing function: $v(t) = 100\, e^{-3t}$

Forced response: $I(\sigma) = 75\, e^{-3t}$

Engineers take it to the max. We are almost finished.
In part 3A we had the series and parallel RLC circuit.
Circuit here is series RLC. We had conditions based on solution to a quadratic equation.
We had complete response fc(t) = f_f(t) + f_n(t); sum of forced and natural response.
Should our circuit loose the voltage source vs(t), the components L and C will release energy into the circuit. C discharges current.
Lets say we did the t = <0 analysis when the voltage source was present.
Now we do the t=0 and t = >0 analysis without the source voltage.
We have poles and zero same as roots of the series RLC circuit.
Copy paste notes posted on next page from part 3A.

Review notes:

Next we see the solutions to a quadratic equation

$$s_1 = -\left(\frac{R}{2L}\right) + \sqrt{\left(\frac{R}{2L}\right)^2 - \left(\frac{1}{LC}\right)} = -\alpha + \beta$$

$$s_2 = -\left(\frac{R}{2L}\right) - \sqrt{\left(\frac{R}{2L}\right)^2 - \left(\frac{1}{LC}\right)} = -\alpha - \beta$$

$\alpha = \left(\frac{R}{2L}\right)$ <--- Exponential damping coefficient. $\omega_0 = \left(\frac{1}{\sqrt{LC}}\right)$

$\beta = \sqrt{\alpha^2 - (\omega_0)^2}$ Alpha and Beta are parts of root s1 and s2.

We have some serious conditions that make the solutions unique to each condition. These are shown and applied in the examples later. For example in our QUADRATIC EQUATIONS solution conditions:

$$Ax^2 + Bx + C = 0 \quad x1 = \frac{-B + \sqrt{B^2 - 4AC}}{2A} \quad x2 = \frac{-B - \sqrt{B^2 - 4AC}}{2A}$$

$B^2 - 4AC$
- > 0 Roots are real and unequal.
- $= 0$ Roots are real and equal.
- < 0 Roots are imaginary.

Roots of equation can be imaginary (squar-root of -ve number) - Underdamped case.

$$\sqrt{\alpha^2 - (\omega_0)^2} = \sqrt{-1}\,\sqrt{\omega_0^2 - \alpha^2} = j\sqrt{\omega_0^2 - \alpha^2} = \omega_d$$

<--- Natural resonant frequency

Sqrt(-1) ie j re-positions w0 and alpha above.

The conditions for series and parallel RLC circuits will be shown at end of parallel RLC circuit. General idea expressed below so you got the mystery out. *Check with your textbook.*
In RLC circuit the three cases are called
1. over damped: series (alpha > omega0) and parallel (alpha^2 > omega0^2).
2. critically damped: series and parallel (alpha = omega0).
3. underdampled: series (alpha < omega) & for parallel (alpha^2 < omega0^2).

Summary:

1). Decide if its a series or parallel RLC circuit.
2). This leads to selecting the correct alpha.
3). Next calculate omega0.
4). We have alpha and omega0.
Next meet one of the 3 conditions:
5). alpha > omega0 circuit is overdamped
solve for s1 and s2
natural response fn(t) = A1e^(s1t) + A2e^(s2t)
6). alpha = omega0 circuit is critically damped
solve for s1 and s2
natural response fn(t) = e^(-alpha)t (A1t + A2)
7). alpha < omega0 circuit is underdamped
solve for s1 and s2
natural response is:
fn(t) = e^(-alpha)t (A1(cos (wd)t + A2(sin (wd)t)
where (wd) = sqrt(w0^2 - alpha^2)
8). Last decision: If there are no independent sources acting in circuit after switching or discontinuity is completed, then the circuit is source-free and the natural response comprises the complete response. IF independent sources are still present then the circuit driven by a forced response must be determined. The complete response is then f(t) = f_f(t) +f_n(t).

We are looking at -5 and -1, these are real and unequal roots.

$s1 = -1 \qquad s2 = -5$

$\alpha = \left(\frac{R}{2 \cdot L}\right) = 3 \qquad \omega_0 = \frac{1}{\sqrt{(L \cdot C)}} = 2.236$

Alpha > Omega0: Over damped case.

Expression for current natural response: $i_n(t) = A_1 e^{-s1t} + A_2 e^{-s2t}$ <--over damped eq.

$$s1 = \left(\frac{-R}{2\ L}\right) + \sqrt{\left(\frac{R}{2\ L}\right)^2 - \left(\frac{1}{L \cdot C}\right)} = -1$$

Proven by evaluation --->

$$s2 = \left(\frac{-R}{2\ L}\right) - \sqrt{\left(\frac{R}{2\ L}\right)^2 - \left(\frac{1}{L \cdot C}\right)} = -5$$

This was surprising it tied here (same); complex frequency same as roots of quadratic equation.

Engineers conclusion:
As the frequency of a non-zero forcing function approaches either of these two frequencies, sigma = -1 and sigma = -5, an arbitrarily large forced response results.

Next the engineers said:
A zero amplitude forcing function may be associated with a finite amplitude forced response which can be intepreted as the natural response of the circuit.

My addition/suggestion:
Can we solve for A1 and A2 or have we already? We have s1 and s2 so if we got the magnitude of I at these points for A1 and A2 that may complete the function for natural response?

$$i_n(t) = A_1 e^{-1\ t} + A_2 e^{-5\ t}$$

From the plot, I(sigma) vs sigma, curve intersects -1, but not -5. So that makes me say A1 and A2 may not be both attained thru the plot. We did initial conditions and solved thru equations in the past and had 2 equations -this way. Sorry my suggestion was unsuccessful.
We come to this in section 13.7 Natural response and the s plane.

Completed section 13.5.
Next some worked examples from Schaums Section 8.7, 8.8, and 8.9. Then entering an important theory-study in the complex frequency plane in section 13.6. I work section 8.7-8.9 first to maybe make simpler/assist section 13.6.

8.7 Network Function and Pole-Zero Plots (Schaums).

I did poles and zeros from Hyat & Kemerly because it had a full explanation on it as a textbook would. Now in Schaums there is the same section, I will go thru this to reinforce or supplement recent efforts. Its the example problems in the Schaums section whose value I do not want to neglect. The notes in Schaums partially assumes I have studied poles and zeros in an EE textbook.

A network function H(s) is the
ratio of a circuit's
exponential form of output's, Y(s), complex amplitude
to
exponential form of input's, X(s), complex amplitude.

$$H(s) = \frac{Y(s)}{X(s)} = \frac{\text{Output(s)}}{\text{Input(s)}}$$ <--- industry standard format, H(s), Y(s), and X(s)

Unfortunately this is electrical engineering we will not be given some voltage magnitude across the desired output terminals in the circuit and the amplitude of the input voltage source maybe hopefully could be easy enough for us to determine, no measured quantitites will be provided, **INSTEAD** we have to apply equations for Y(s) and X(s), this too most likely in differential equation form to solve the circuit's required values. *Hope you liked that. Its true and may be a joke for some not all. Ok! Some engineers love differential equations.*

Do the units of output to input have to be the same V_out(s) / I_in(s)?
Should be otherwise V(s) / I(s) results in resistance Z(s) ohms,
whereas V(s) / V(s) is unitless.
Ratio typically is unitless.
BUT such is NOT the case here. We can have V(s) / I(s) resulting in resistance.

Electric circuits can have so many components and connections, the desired ratio Y(s)/X(s) dependent on what is of value-impact to the circuit.

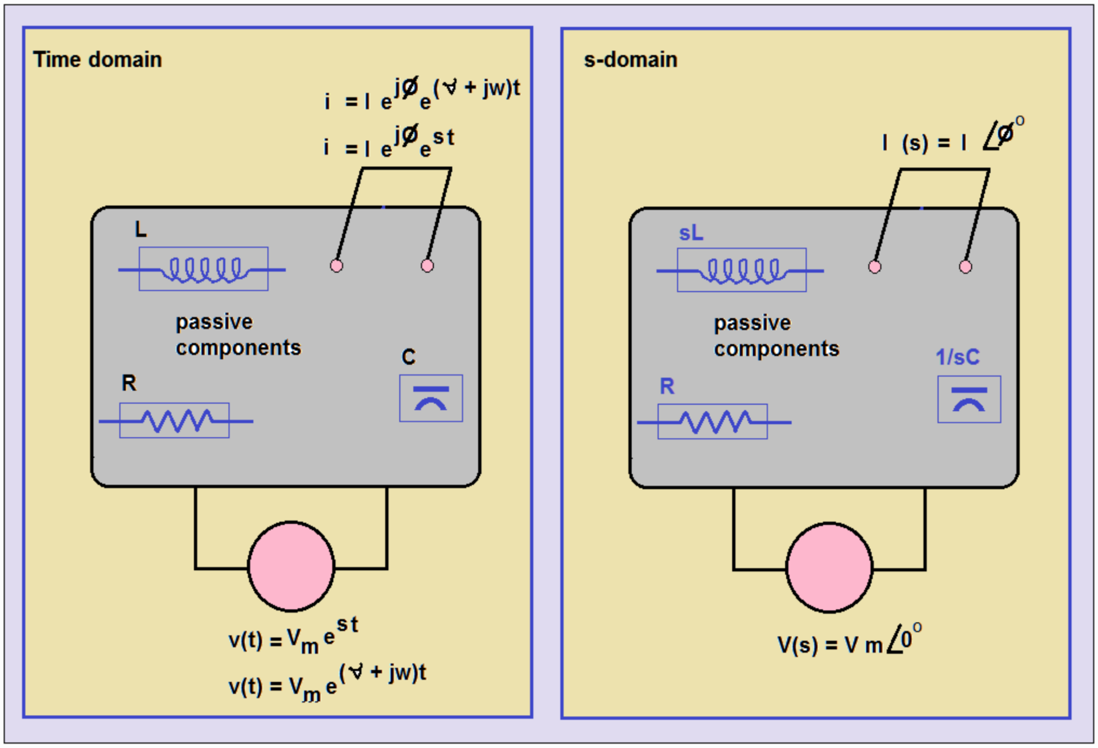
Lets get to the equations describing Y and X.
A 2nd order differential equation for y(t) looks like this: $a_2 \cdot \frac{d^2 y}{dt^2} + a_1 \cdot \frac{d^1 y}{dt} + a_0$

Typical form for function y(t) where a's are the coeficcients:

$$a_n \cdot \frac{d^n y}{dt^n} + a_{n\_minus\_1} \cdot \frac{d^{n-1} y}{dt^{n-1}} + a_{n\_minus\_2} \cdot \frac{d^{n-2} y}{dt^{n-2}} + \ldots + a_{n\_minus\_n} \cdot \frac{d^{n-n} y}{dt^{n-n}}$$

$$a_n \cdot \frac{d^n y}{dt^n} + a_{n\_minus\_1} \cdot \frac{d^{n-1} y}{dt^{n-1}} + a_{n\_minus\_2} \cdot \frac{d^{n-2} y}{dt^{n-2}} + \ldots + a_0$$ Correct? Verify.

We seen the above many times, just here for differential equation form.
I have to write the 'minus' in the subscript because of my text editor.



Before we continue with differential equations, the figure above is the essense of the new game new methods new……, to the left we have time-domain where s = sigma + jw, and to the right its corresponding s-domain. We have to work in this form, s-domain, not by choice rather by demand.

Time domain form:	**s-domain form:**
$x(t) = Xe^{st}$	$X(s)$
$y(t) = Ye^{st}$	$Y(s)$

To simplify expression: $a_n s^n + a_{n\_minus\_1} s^{n-1} + ... + a_{n\_minus\_n\_minus\_1} s^{n-(n-1)} + a_0$

Correct---> $a_n s^n + a_{n\_minus\_1} s^{n-1} + ... + a_1 s^1 + a_0$

If output (a) = input (b) in time domain e^(st) then we write as:

$$\left(a_n s^n + a_{n\_minus\_1} s^{n-1} + ... + a_1 s^1 + a_0 \right) e^{st} = \left(b_m s^n + b_{m\_minus\_1} s^{n-1} + ... + b_1 s^1 + b_0 \right) e^{st}$$

For the s-domain remove? e^(st):

$$\left(a_n s^n + a_{n\_minus\_1} s^{n-1} + ... + a_1 s^1 + a_0 \right) = \left(b_m s^n + b_{m\_minus\_1} s^{n-1} + ... + b_1 s^1 + b_0 \right)$$

Engineering college year 2 course of 4 year program OR year 1 of 3 year program. Re-fresher OR Self Study. Graduate Study Review.
May be used in New Zealand, US, Malaysia, India, Pakistan, UK, and other Common Wealth Country engineering colleges.
Any errors and omissions apologies in advance.

Now lets say maybe we got a little smart, we try to write it for the transfer function H(s), nothing more than expression of 'a' over that of 'b'.

$$H(s) = \frac{Y(s)}{X(s)} = \frac{\left(a_n s^n + a_{n\_minus\_1} s^{n-1} + \ldots + a_1 s^1 + a_0 \right)}{\left(b_m s^n + b_{m\_minus\_1} s^{n-1} + \ldots + b_1 s^1 + b_0 \right)}$$

I want to make it simple for myself, we done doing some factoring in the previous circuit solutions, we do the same here. *But should you choose to read what Schaums wrote: In linear lumped circuits made up of lumped elements the network function H(s) is a rational function of s and can be written in the following general form:*

$$= \frac{(s - z_1)(s - z_2) \ldots (s - z_u)}{(s - p_1)(s - p_2) \ldots (s - p_v)}$$

If we place a k infront that fixes it for any adjustments if not k=1, we usually place a variable or constant in front, same here.

$$H(s) = \frac{Y(s)}{X(s)} = k \frac{(s - z_1)(s - z_2) \ldots (s - z_u)}{(s - p_1)(s - p_2) \ldots (s - p_v)}$$

z_m: z1....z_u are the zeros of H(s)

p_n: p1...p_v are the poles of H(s)

k is some real number

Like we did in our previous circuit examples, here:

when s = z_m the response will be zero, regardless how great the excitation

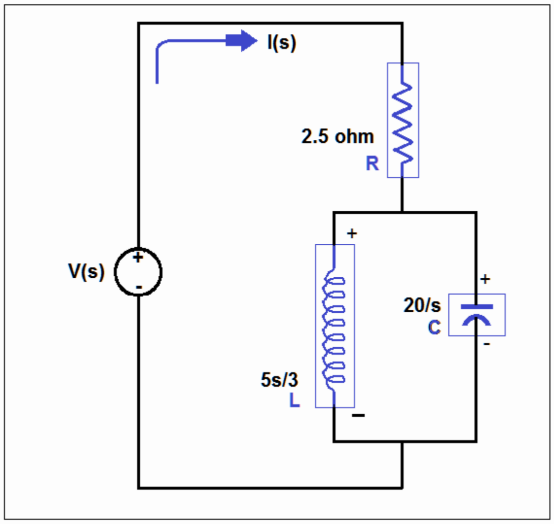
when s = p_n the response will be infinite (pole), regardless how small the excitation.

Exactly what we done in our zeros and poles studies.

The importance of all these relationships....come to work when:
H(s) is intepretated as the ratio of the response (in one part of the s-domain network) to the excitation (in another part of the network).

Next 1 example please!

Example 8.8:



A passive network in the s-domain is shown in the figure to the left.

Obtain the network function for the current I (s) due to an input voltage V(s).

Solution:

$$Z_R = 2.5 \qquad Z_C = \frac{20}{s} \qquad Z_L = \frac{5\ s}{3}$$

Our network function is H(s), output is I(s), and input is V(s).

$$H(s) = \frac{I(s)}{V(s)} = \frac{1}{Z(s)}$$

The understanding of 1/Z(s) is the circuit's total impedance.

L and C are in parallel:

$$\frac{(Z_L)\ (Z_C)}{Z_L + Z_C} = \frac{\left(\frac{5\ s}{3}\right)\left(\frac{20}{s}\right)}{\left(\frac{5\ s}{3}\right)+\left(\frac{20}{s}\right)} = \frac{\left(\frac{100\ s}{3\ s}\right)}{\left(\frac{5\ s}{3}\right)+\left(\frac{20}{s}\right)} = \frac{(3\ s)\cdot\left(\frac{100\ s}{3\ s}\right)}{(3\ s)\cdot\left(\frac{5\ s}{3}+\frac{20}{s}\right)}$$

$$= \frac{100\ s}{5\ s^2 + 60} = \frac{s(100)}{5\left(s^2+12\right)} = \frac{s(100)}{5\left(s^2+12\right)}$$

$$Z_{total} = Z_R + \left(\frac{(Z_L)\ (Z_C)}{Z_L + Z_C}\right) = 2.5 + \frac{s(100)}{5\left(s^2+12\right)}$$

$$H(s) = \frac{I(s)}{V(s)} = \frac{1}{2.5 + \dfrac{s(100)}{5\left(s^2+12\right)}}$$ multiply top and bottom by $5\left(s^2+12\right)$

$$= \frac{5\left(s^2+12\right)}{5\left(s^2+12\right)\cdot\left(2.5 + \dfrac{s(100)}{5\left(s^2+12\right)}\right)}$$

$$= \frac{5\left(s^2+12\right)}{12.5\left(s^2+12\right) + s(100)}$$

$$= \frac{5\left(s^2+12\right)}{12.5\,s^2 + 150 + 100\,s}$$ Divide the numerator and denominator by 12.5

$$= \frac{(0.4)\left(s^2+12\right)}{s^2 + 12 + 8\,s}$$

$$H(s) = (0.4)\,\frac{\left(s^2+12\right)}{s^2 + 8\,s + 12}$$ Numerator looks ok for next step. <--- Factor the denominator.

$$H(s) = (0.4)\cdot\frac{\left(s^2+12\right)}{(s+2)\,(s+6)}$$

We have H(s) in the form where we can identify the value of s for zero in numerator and pole for denominator. This is the difficult part to have it in the form that makes for working the next step which is setting the values of s for zero(s) and pole(s).

Numerator - Zero: $0.4\left(s^2+12\right)$
Condition for this term to equal 0: $\left(s^2+12\right)$

$s^2 = -12$ *So 0.4(-12 + 12) = 0.4(0) = 0.*

$s = \sqrt{-12}$ $\quad s = \sqrt{-1\cdot 12}$

$s = \sqrt{-1}\cdot\sqrt{12}$ $\quad s = j\sqrt{12}$

$s = +/-\ j\sqrt{12}$

Because its the root of j ie sqrt(-1), the final result of the root can be +ve or -ve.
Sqrt(25) = 5, 5 x 5 = 25,
5-5 = 5 x conj(5) = 5 x -5 = -25.*
Sqrt(-1 x 25) = +/- j5.

Zero = $+/-\ j\sqrt{12}$ Solved! Schaums answer.

Zero = $+\ j\sqrt{12}$ and $-\ j\sqrt{12}$

Next denominator.

Denominator - Pole:

Condition for these term to equal 0: $(s+2)\ (s+6)$

$s1 = -2 \qquad (-2+2) = 0$ Solved! Schaums answer.

$s2 = -6 \qquad (-6+6) = 0$ Solved! Schaums answer.

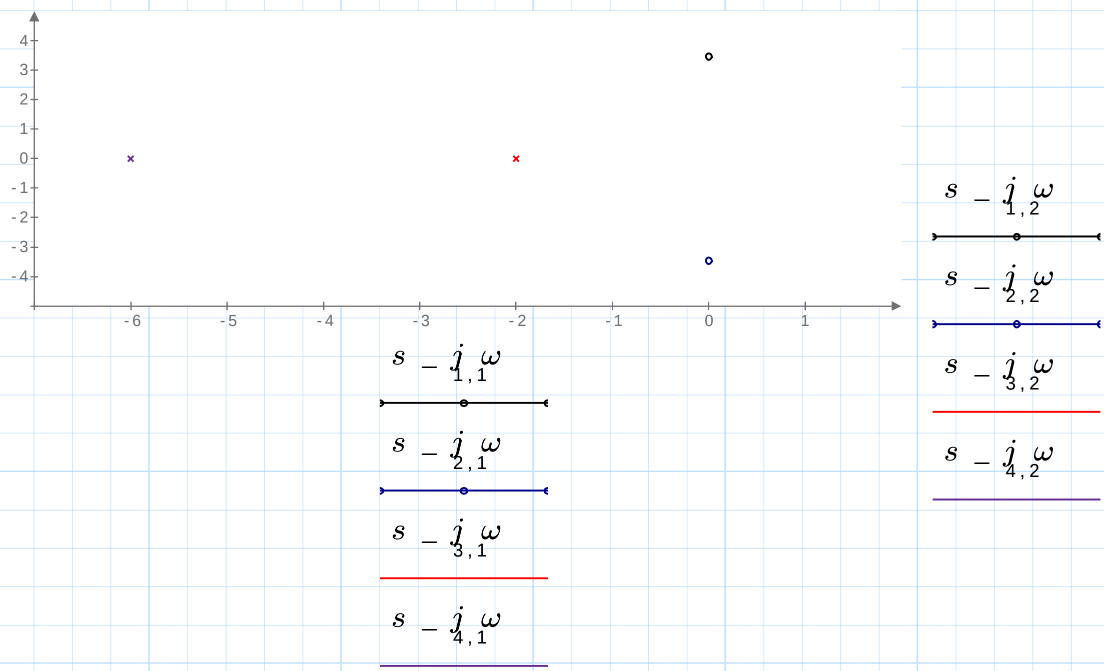
Next 'poles and zeros' plot(s):

$\text{clear}(s, j\ \omega)$ clear variables register
$\text{ORIGIN}(1,1)$ set matrix origin at 1,1 instead of default 0,0.

$$s\_j\omega := \begin{bmatrix} 0 & \sqrt{12} \\ 0 & -\sqrt{12} \\ -2 & 0 \\ -6 & 0 \end{bmatrix}$$

Matrix s_jw identifies the 4 pointsl zeros and poles.

Plot shows the zeros 'O' and poles 'X'. As shown in Schaums. Answer.
Plot of each (Re, Im) part in combination, like (x,y) coordinates.
See the subscripts they are elements of the matrix, real and imaginary parts separated.



Engineer conclusions next page. Far from finished.

Conclusion:

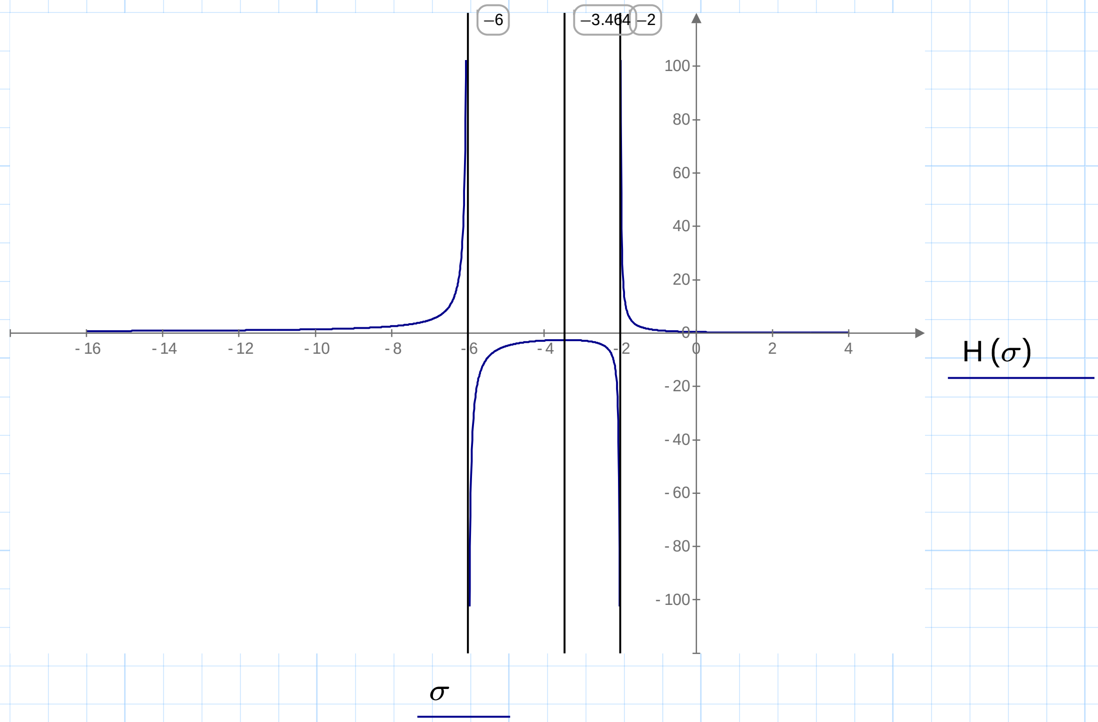
- The numerator of H(s) is zero when s = +/- j Sqrt(12).
- A voltage function v(t) for corresponding V(s) results in zero current.
- The zeros and poles can be plotted in a complex s plane (as we did Re and Im parts using matrix).
- In this circuit the zeros occured in complex conjugate pairs **j Sqrt(12)** and **- j Sqrt(12).**

My Addition/Suggestion:

Can we plot the transfer function H(s) and maybe identify some of the zeros and poles? Give it a try.

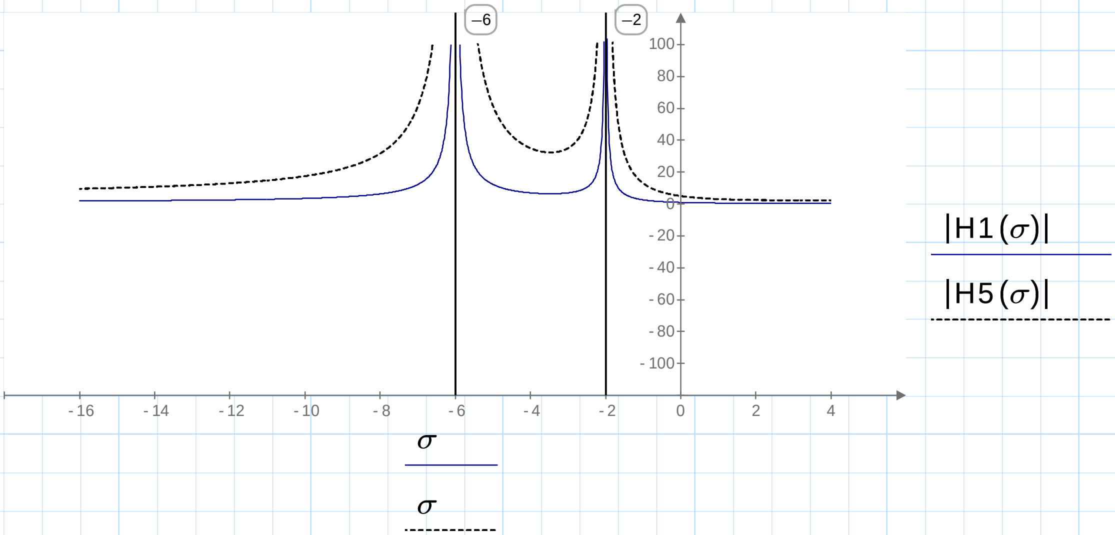
clear (s)

$$H(\sigma) := (0.4) \cdot \frac{(\sigma^2 + 12)}{(\sigma + 2)\ (\sigma + 6)}$$



Discussion: We see success for poles at -2 and -6 but this we knew by inspection of the transfer function H(s). The factor k is 0.4 , *this was verified/compred against by a multiplier of 5* in next plot, went further away from poles, when removed (ie 0.4 and 5) resulted in a better plot we see next, and maybe more matching analysis method. You may verify. The almost 0 at Sqrt(-12) = -3.464, *ignoring -ve square root just placed the -ve sign(- sqrt(12))*, did almost occur thats the peak of the bottom curve seen near -4.

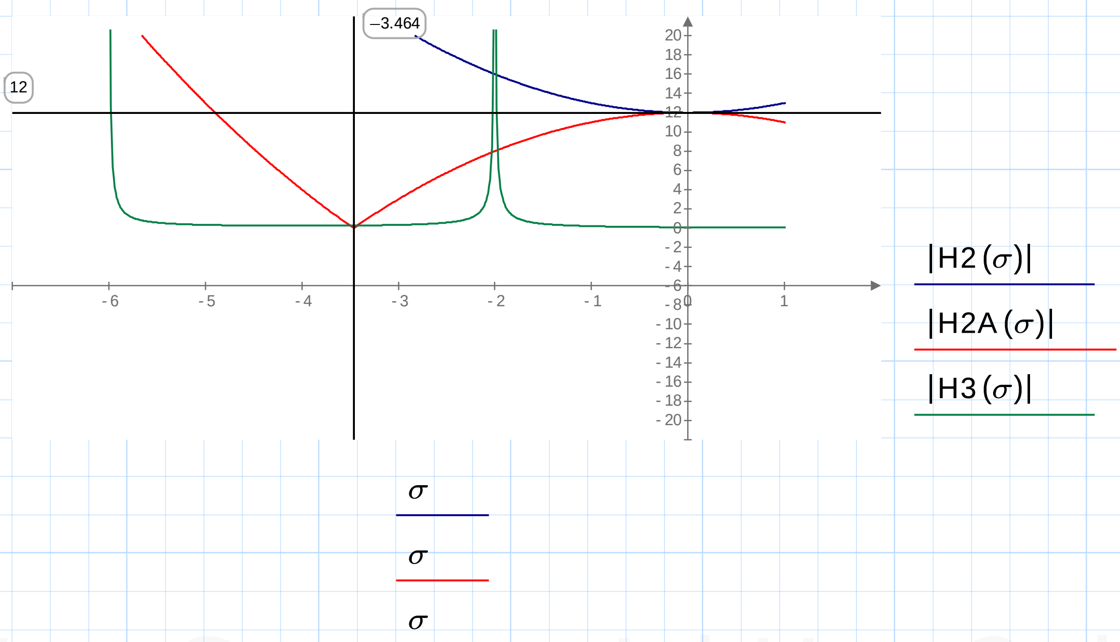
clear (s) $\quad H1(\sigma) := \dfrac{(\sigma^2 + 12)}{(\sigma + 2)\ (\sigma + 6)} \quad H5(\sigma) := \dfrac{5 \cdot (\sigma^2 + 12)}{(\sigma + 2)\ (\sigma + 6)}$ <---Multiplier 5



I got near 0 at -3.464. How to get -12/12? What if I split the expression into 2? H2 and H3 Maybe. H2(0) = s^2+12; Got +12 at sigma=0. What if H2A(0) = - s^2 +12? Got 12 at 0, and 0 at -3.464, but the curve is above x-axis. My exercise been on how plots may help. Not direct answers, since here we could inspect, it was just my suggestion on use of plots.

$H2(\sigma) := (\sigma^2 + 12)$ $\quad H2A(\sigma) := (-\sigma^2 + 12)$ $\quad H3(\sigma) := \dfrac{1}{(\sigma + 2)\ (\sigma + 6)}$

s=0 at 12 $\quad$ s=0 at 12 and s=-3.464 at 0



8.8 Forced Response:

We seen forced response in RLC circuits, but here we are doing it differently and using the s-plane (frequency domain) method. We have a section on Natural Response coming next, which we will see again in the Hyat Kemerly textbook. We cover both.

We got these equations from section 8.7, provided below. We continue with the coversion to phasor form.

$$H(s) = \frac{Y(s)}{X(s)} = \frac{\left(a_n s^n + a_{n\_minus\_1} s^{n-1} + ... + a_1 s^1 + a_0\right)}{\left(b_m s^n + b_{m\_minus\_1} s^{n-1} + ... + b_1 s^1 + b_0\right)}$$

$$= \frac{(s - z_1)\,(s - z_2).....\,(s - z_u)}{(s - p_1)\,(s - p_2)....\,(s - p_v)}$$

$$H(s) = \frac{Y(s)}{X(s)} = k\,\frac{(s - z_1)\,(s - z_2).....\,(s - z_u)}{(s - p_1)\,(s - p_2)....\,(s - p_v)}$$

z_m: z1....z_u are the zeros of H(s)
p_n: p1...p_v are the poles of H(s)
k is some real number

We got this far prior.

H(s) known as transfer function and super important in electrical engineering, we come across sorts of transfer functions in EE, it is merely a ratio of some specific output divided by some specific form of input. Ratio of V(s)/I(s), Vo(s)/Vi(s), I2(s)/I1(s),.....we got the idea. It can be I(s)/V(s) where I(s) is the current source, and V(s) the voltage across the output terminals.

Then we said
Like we did in our previous circuit examples, here:
when s = z_m the response will be zero, regardless how great the excitation
when s = p_n the response will be infinite (pole), regardless how small the excitation.

Continuing *with my wording (Needs Correction) provided next page :*

s - z_m equal what ? $s = z_m$ Response zero

$(s - z_m) =$ Response equal something, surely not zero....

$(s - z_m) = N_m \angle \alpha_m$ m=1,2,3,4........

This is phasor for all m_th =1,2,3,4.....u

similarly s - p_n equal what ?

$(s - p_n) =$ Response equal something, surely not zero....

$(s - p_n) = D_m \angle \beta_n$ n=1,2,3,4........

This is phasor for all n_th =1,2,3,4....v.

The last sentences on the previous page may not be clear.

s = z_m is for the mth term of the zero in numerator.

When we re-arrange it, s - z_m same as
(sigma + jw) - (z_sigma + z_jw) = N_m Angle (alpha_m).
An example coming helps clear this.

s is complex frequency, and the root (pole/zero) is also complex frequency.
I use the word 'root' because the roots of the quadratic equation of a RLC circuit were exactly the same as complex frequency in one worked example.

Lets write as engineers did, seems to be 'intuitive' to our math experience.

Now setting s - z_m = $N_m \angle \alpha_m$ m = 1,2,3.......u

s - p_n = $D_n \angle \beta_n$ n = 1,2,3.......v

Now our H(s) in polar form: $H(s) = k \cdot \left(\frac{N_m \angle \alpha_m}{D_n \angle \beta_n} \right)$

$$H(s) = k \cdot \left(\frac{N_m \angle \alpha_m}{D_n \angle \beta_n} \right) = \frac{(N_1 \angle \alpha_1)\ (N_2 \angle \alpha_2) \ldots (N_u \angle \alpha_u)}{(D_1 \angle \alpha_1)\ (D_2 \angle \alpha_2) \ldots (D_v \angle \alpha_v)}$$

We needed to get here --->

$$H(s) = k. \frac{N_1 N_2 \ldots N_u}{D_1 D_2 \ldots D_v} \angle (\alpha_1 + \alpha_2 \ldots \alpha_u) - (\beta_1 + \beta_2 \ldots \beta_v)$$

Discussion: The engineers used the words 'Now setting (s - z_m).... so on the RHS eventually we have a phasor expression N_m Angle(alpha_m).

The RHS is the result of LHS after working it thru an equation, simplifying it, resulting in a transfer function. The numerator and denominator of that original function represented ? voltage or current or impedance, it may been in ? exponential form to begin then in phasor form, then it got reduced through factorisation for zeros and poles, so NOW we say yes the LHS can be something phasor resulting with RHS something phasor.

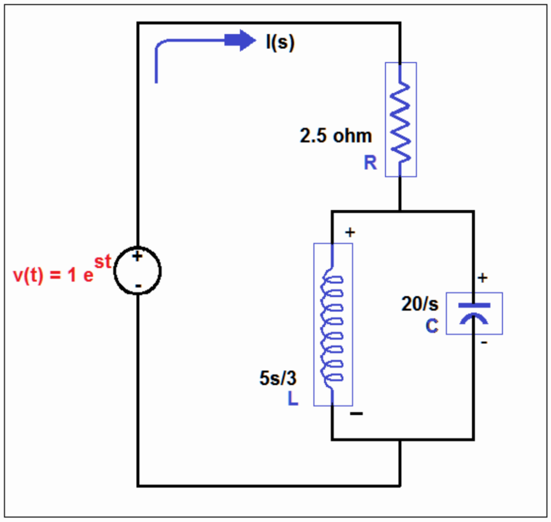
The coming example work thru all this so this mystery is solved.

We have a network in other words a circuit, it maybe small and simple or large and complex, our forcing function on this network is **s** = sigma + jw. The reponse of our network to that forcing function can be determined by measuring the lengths of the vectors from the zeros and poles to **s**, and same for the angles these vectors make with the positive sigma axis in the pole-zero plot.

Seems agreeable we seen this in maths course many times.

One example from Schaums should convince all.
Next example 8.9 on this picks up with example 8.8.

Example 8.9 RLC circuit:



Test the response of the network of example 8.8 to an exponential voltage excitation v = 1e^(st), where s = 1 Np/s.

Solution:

$s = \sigma + jw$

$\sigma = 1$

$\omega = 0$

$s = 1 + j0$

Origin(1,1)

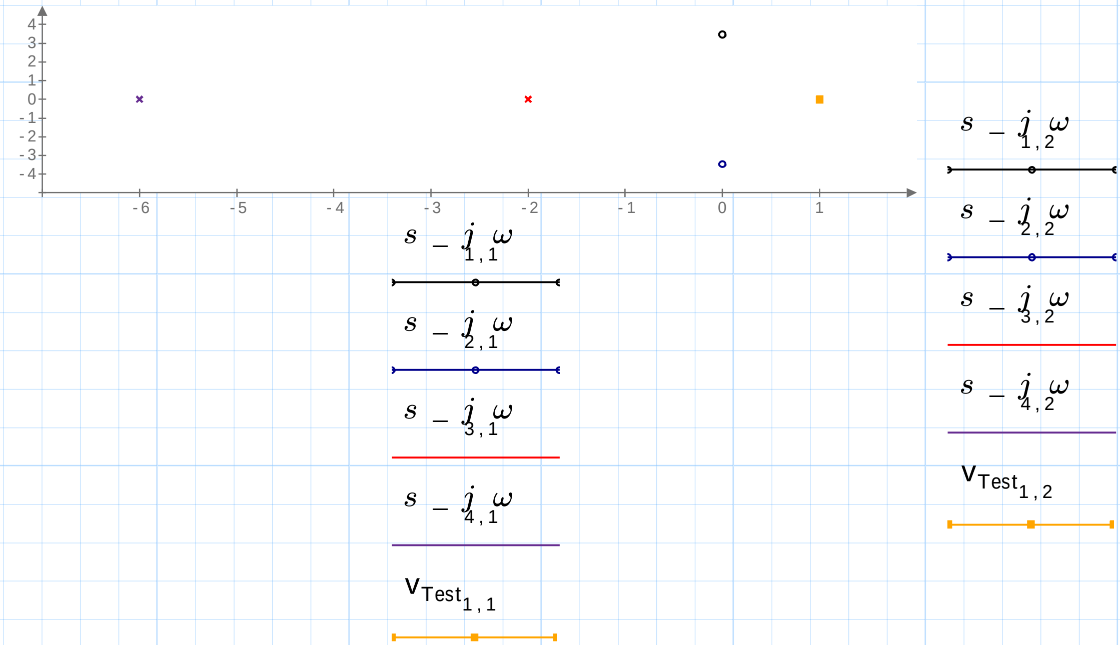
$v_{Test} := [1 \;\; 0]$ Matrix for plot.

Paragraph at top of page gives us some instructions on how to solve this.
Using vector lenghts and angles.

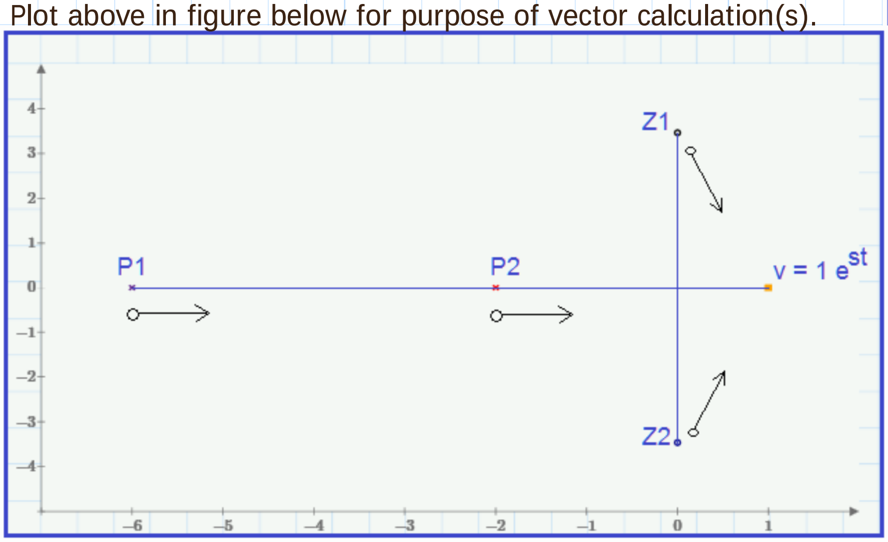
To do that we place the s = 1 + j0 on the pole zero plot.

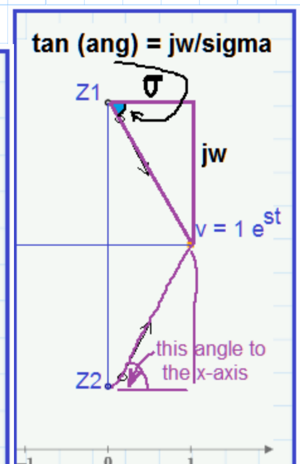
Next we take each point's (poles & zeros) distance and angle to this point (1, j0).

We have the voltage excitation 1e^(st) in the plot - vTest.



Plot above in figure below for purpose of vector calculation(s).





Absolute value of vector magnitude is taken. Vector is magnitude and angle.

Magnitude P1 to v: $|-6+(-1)| = 7$ Angle to +ve x-axis: 0 deg

Magnitude P2 to v: $|-2+(-1)| = 3$ Angle to +ve x-axis: 0 deg

Magnitude Z1 to v: $z1_{squared} := \sqrt{12^2 + 1^1} = z1 := \sqrt{12+1} = \sqrt{13}$

Angle located in 1st quadrant.

Z1 angle to v ie +ve x-axis: $\operatorname{atan}\left(\sqrt{\frac{12}{1}}\right) = 73.898$ deg

You as Engineer say angles need be set from 0 deg, +ve x-axis, anti-clockwise, then do it yourself.

Magnitude Z2 to v: $z2_{squared} := \sqrt{-12^2 + 1^1} = z1 := \sqrt{12+1} = \sqrt{13}$

Z1 angle to v ie +ve x-axis: $\operatorname{atan}\left(\frac{-\sqrt{12}}{1}\right) = -73.898 \text{ deg}$

Angle located in 4th quadrant

You as Engineer say angles need be set from 0 deg, +ve x-axis, anti-clockwise, then do it yourself.

We got something vectors, ok, this is same saying we got phasors, ok.
What do we do with them now?

$$H(s) = k.\frac{N_1 N_2 .. N_u}{D_1 D_2 .. D_v} \angle(\alpha_1 + \alpha_2 .. \alpha_u) - (\beta_1 + \beta_2 .. \beta_v)$$

<----Remember this? Plug into the original H(s) transfer function.

N1 N2.....Nu represent zeros Z1 Z2.....Zu
and similalry for D1 D2...Dv represent poles P1 P2....Pv.

$$H(\sigma) := (0.4)\frac{(\sigma^2 + 12)}{(\sigma+2)(\sigma+6)}$$

<--- That transfer function.

$Z_1 := \sqrt{13}$ $\quad Z_2 := \sqrt{13}$ $\quad P_1 := 7$ $\quad P_2 := 3$

$angZ_1 := 0$ $\quad angZ_2 := 0$ $\quad angP_1 := 73.898$ $\quad angP_1 := -73.898$

$$H(response\_to\_v) = (0.4)\cdot\frac{(Z_1 \cdot Z_2)}{(P_1 \cdot P_2)} = 0.248$$

$$\frac{angZ1 + angZ_2}{angP_1 + angP_2} = \frac{(0+0)}{73.9 + (-73.9)} = 0 \text{ deg}$$

$H(s\_response\_to\_v) := 0.248\angle 0$ deg Answer.

You as Engineer say angles need be set from 0 deg, +ve x-axis, anti-clockwise, then do it yourself, the final answer should be as above.

In time domain: $i(t) = 0.248 \cdot 1\, e^{st}$ $\quad$ v(t) = 1e^(st)

$= 0.248 \cdot e^{st}$

$i(t) = 0.248 \cdot v(t)$ Answer. Good exercise.

Engineer conclusion:
Result 'implies' in the time domain i(t)=0.248 v(t), since v(t) = 1e^st, where s = 1 + j0 = 1, such that the exponent becomes e^1t, this exponent creates an infinite upward curve. Hence, current and voltage become infinite. Therefore, for most practical purpose sigma be either negative or zero.

The above geometrical method does not seem to require knowledge of the analytic expression for H(s) as a rational function. From H(s) known zeros and poles in the pole-zero plot, the expression can be written to within the constant factor k.
- Like we did in our example k was from our previous transfer function calculation.

8.9 Natural Response

Will go over this section in Schaums now because its example fall in the method of analysis. However, Natural Response will be visited in a continuing section in Hyat and Kemerly where there is deeper explaining on with a more complicated circuit in comparison.

We seen the forced OR steady state response,
and it is in getting that response
that the complex frequency method is most useful.

So what about the natural response,
the case when there is no voltage or current source in the circuit?

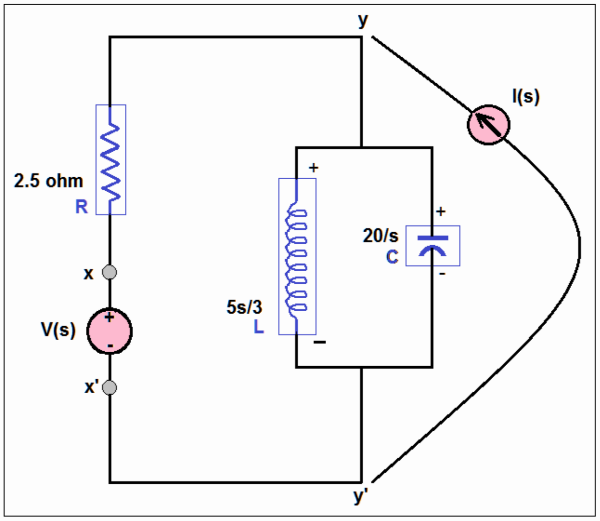
Just so happens the poles (denominator of H(s)) are the natural frequencies which define the transient response. We know how to get them.
Without going into much detail as Schaums is intended for a supplement text, we know the poles in the network function represent the natural frequencies of the? natrual response. Natural response correesponds to natural frequencies - poles.

Why poles represent transient response? Because of the rise in the function near the pole its steep and thats a transient. So thats why poles are the natural frequencies that define the transient pole.

Example 8.10 RLC circuit

Same circuit as in example 8.8 now updated with voltage source V(s) is placed between xx'.
Obtain the natural response?

Solution:



Same circuit as far as the transfer function H(s) is concerned.

$$H(s) = (0.4) \cdot \frac{(s^2 + 12)}{(s+2)\ (s+6)}$$

We know the poles are -2 and -6 from previous exercise.

We have 2 poles, what does that say about the natural or transient current response i(t) in time domain equation for this circuit?
Specifically the form of equation.

I am saying why can't I do this.......units of components ?
Our circuit is parallel LC then series to R.
Fortunately, for both series and parallel LC omega0 is the same - 1/Sqrt(LC)
The units are Ohm H and F.

$$\omega_0 := \frac{1}{\sqrt{L \cdot C}}$$

We have: $Z_L = Ls = \frac{5}{3}s$

$L = \frac{5}{3}$

$Z_C = \frac{1}{Cs} = \frac{20}{s}$

$\frac{1}{C} = 20$

$C = \frac{1}{20}$

At end of example 8.8 information provided where:
R = 2.5 Ohm
L = (5/3)H.
C = (1/20)F.
w = Sqrt(12)
So we don't have to guess on the units of the components.

$$\omega_0 := \frac{1}{\sqrt{\left(\frac{5}{3}\right)\cdot\left(\frac{1}{20}\right)}}$$

$$= \frac{1}{\sqrt{\frac{5}{60}}} = \sqrt{12} = 3.464$$

Alpha Series:

$$\alpha = 2\cdot\left(\frac{R}{L}\right) = \frac{2\cdot 2.5}{\left(\frac{5}{3}\right)} = 3$$

Alpha Parallel:

$$\alpha = \frac{1}{2\cdot R\cdot C} = \frac{1}{2\cdot 2.5\cdot\left(\frac{1}{20}\right)} = 4$$

If the circuit is series as we have alpha < omega0 ie 3 < 3.464
We typically use a sinusoidal form for oscillations - under damped.
If the circuit is parallel we have alpha > omega0 ie 4 > 3.464
We typically use an exponential expression - over damped.
Our circuit is parallel as we started in example 8.8 (alpha > omega) over damped case.

From previous pages 'Example RLC circuit' we have expression for i_n(t).......

current natural response: $i_n(t) = A_1 e^{-s1t} + A_2 e^{-s2t}$ <--over damped eq.

Already -ve exponentials, e^-s1t and e^-s2t, for decreasing response in the?
natural response condition - of course there is no external source.

We have for poles -2 and -6 so we just plug in 2 and 6.

$$i_n(t) = A_1 e^{-2t} + A_2 e^{-6t}$$ Answer.

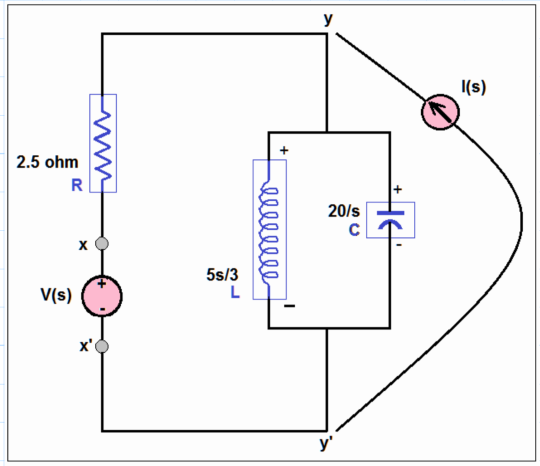
To compute A1 and A2 we rely on initial conditions method of calculations. Not required for this question just the time domain natural response. Seems some things are easier here. *For me there was just so much information that to look back and make certain which technique to apply seems over whelming....resulting in the regular looking up in the many chapters of the textbook. Same for most except there are exceptional graduates and engineers. No denying that.*

Example 8.11 RLC circuit:

Same circuit as example 8.10.

The circuit or network is now driven by current I(s) across terminals yy'.

Find the network transfer function?

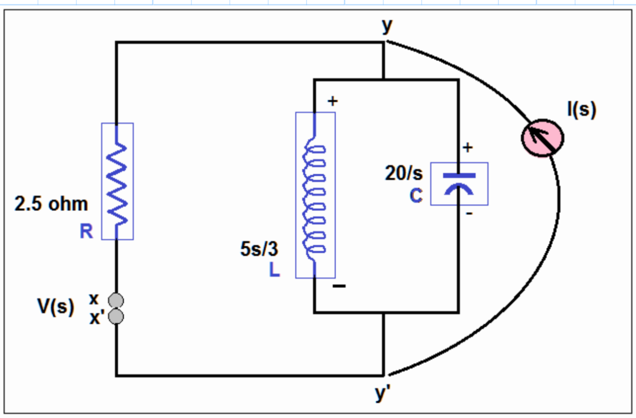


Solution:

Do we disregard the voltage source V(s)?

No, we were not told too. Voltage V(s) is in series to resistor R.
So how do we do this? The passive components are all in parallel.
LC in parallel and then in parallel to R and V(s) which are in series.

V(s) is all voltage it is not a source of resistance, it can be seen as an ? Open Circuit? or Short Circuit?
We have voltage across its terminals, no resistance across its terminals, so each end of the terminal is connected to the other, because there is no resistance across it.
So for the circuit analysis its a short circuit. NOW R is parallel to LC. With I(s) driving the circuit or I(s) supplying current to the circuit as the main supply source.



The updated figure looks better for the passive component parallel connection.

First we do the LC parallel connectionwhich we done in the past, then second we parallel R to it.

$$Z_{LC} = \frac{(Z_L)\,(Z_C)}{Z_L + Z_C} = \frac{s\,(100)}{5\,(s^2 + 12)}$$

$$Z_{Total} = \frac{Z_R \cdot (Z_{LC})}{Z_R + Z_{LC}}$$

$$Z_{Total} = \frac{2.5 \cdot \left(\frac{s(100)}{5\,(s^2+12)}\right)}{2.5 + \left(\frac{s(100)}{5\,(s^2+12)}\right)} = \frac{\left(\frac{1}{2}\right)\left(\frac{s(100)}{(s^2+12)}\right)}{2.5 + \left(\frac{s(100)}{5\,(s^2+12)}\right)}$$

Multiply top and bottom by $\frac{(s^2+12)}{s(100)}$

$$Z_{Total} = \frac{\left(\frac{1}{2}\right)}{2.5\left(\frac{(s^2+12)}{s(100)}\right) + \left(\frac{1}{5}\right)} = \frac{1}{\frac{5 \cdot (s^2+12)}{s(100)} + \left(\frac{2}{5}\right)}$$

Multiply top and bottom by $5\,s(100)$

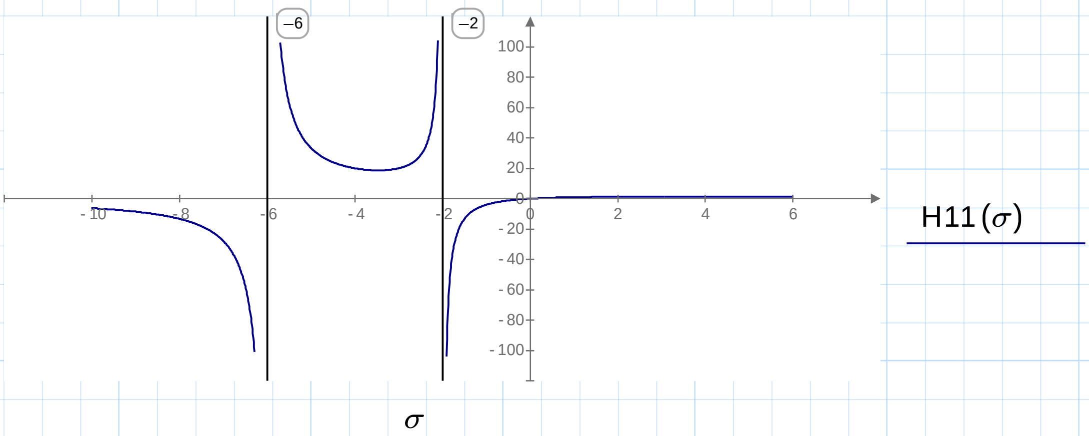
$$Z_{Total} = \frac{5\,s(100)}{25 \cdot (s^2+12) + 2\,s(100)} = \frac{500\,s}{25\,s^2 + 200 \cdot s + 300} = \frac{20\,s}{s^2 + 8 \cdot s + 12}$$

$$Z_{Total} = \frac{20\,s}{(s+2)\,(s+6)}$$ Answer.

Comments:
The poles are, same as the example we started 8.8, s1 = -2 and s2 = -6.
The zero?

Numerator we have 20s, the only way that can be 0 is s = 0.
Zero is 0.

$$H11(\sigma) := \frac{20\,\sigma}{\sigma^2 + 8 \cdot \sigma + 12}$$ Visibly the zero is at 0 and poles -2 and -6.
Lots better plot when the 20 is made 1 in numerator.



Break/Recess Time:

Remaining sections in Schaums are 8.10 Magnitude and Frequency Scaling, and 8.11 Higher Order Active circuits. Contents of 8.10 can be explained later thru Hyat & Kemerly. 8.11 is using Op-Amp for the circuit component and thats not critical here.

Schaums Chapter 16 Laplace does not use a single Op-Amp in any example, solved and unsolved problem. Op-Amp is a very useful device but here for me to pick up on it requires I pick up a chapter on Op-Amp. Then work the example/problems but for my objective its not critical here.

I want to get to starting solved problems and unsolved problems in Schaums Chapter 8, since we covered adequate theory and examples. So I will finish Chapter 13 of Hyat & Kemerly and section 8.10 of Schaums, then start Part C on solving partial completed may be some unsolved problems.

Then I said I could do Op-Amps at end of Part B. Do only the op-amp circuits relevant and referenced in the chapters we took-up, thats more than adequate. We dont look at it as a semi-conductor device, just the application in electric circuits. Short on theory more on how to apply its characteristics in electric circuits. Adequate. FORGET IT.

Update: Finally at end of this part B I said plan to do brief Op-Amp in part D. Part C working solved problems, related problems in Schaums that are related to material worked thus far, and maybe some related problemsfrom Controls on circuit transfer functions.

Next into section 13.6 complex frequency plane. Here some basics on this topic, so I understand it from a working angle to solve electric circuit problems, not a perspective something rather more solid than that.

Section 13.6 Complex Frequency Plane.

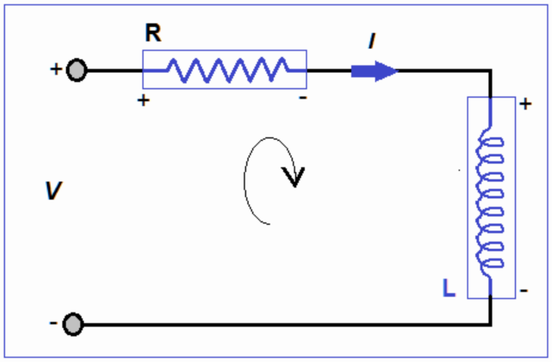
We done: s = $\sigma + j\ \omega$

When σ = 0
s = $0 + j\ \omega$ <---Done this

When ω = 0
s = σ + j0 <---Done this

Now we want to work with both σ and $j\ \omega$ <---Now both together this is the s-plane.

We have a circuit, RL, R = 3 ohm and L = 4H.



Z_R = 3 Z_L = s4
Z_{Total} = 3 + s4
s : s4 <---Got it!
$4\ (\sigma + j\ \omega)$
σ ---> $\sigma\ 4$ <---Got it!
$j\ \omega$ ---> $j\ 4\ \omega$ <---Got it!
$Z(\sigma) = 3 + \sigma\ 4$ <---Got it!

$Z(j\ \omega) = 3 + j\ 4\ \omega$ <---Got it!

s = $\sigma + j\ \omega$
When s = σ + j0 <---We seek a graphical intepretation of impedance
$Z(\sigma)$ = $3 + (\sigma\ 4 + j0)$ <---Variation with sigma, Z(sigma), so let w = 0
$Z(\sigma)$ = $3 + \sigma\ 4$ From inspection we see: Zero: -3/4 *3+(-3/4)4 =0*
Pole: Infinite.

We do not have a denominator. Some numerical value divided by something not there ie ~0, results with infinity. 4/0.0000000001~0 = LARGE --> Infinite.

Also when sigma = 0:
$Z(\sigma)$ = $3 + \sigma\ 4$
When σ = 0
$Z(0)$ = $3 + 0 \cdot 4$ = 3 <--- Non-critical frequency.

Next s = $\sigma + j\ \omega$ <---We seek a graphical intepretation of impedance
s = $0 + j\ \omega$ <---Variation with jw, Z(jw), so let sigma = 0
$Z(j\ \omega)$ = $3 + (0 + j\ 4)\omega$
$Z(j\ \omega)$ = $3 + j\ 4\ \omega$ <---Leave the w (omega) in, get the function Z(jw).

Magnitude: $|Z(j\omega)| = \sqrt{3^2 + 4^2\,\omega^2}$

$= \sqrt{9 + 16\,\omega^2}$

Angle: $ang\_Z(j\omega) = \tan^{-1}\left(\frac{4\,\omega}{3}\right)$

We had 1 pole at infinity, when sigma = (-3/4), but now, when we study the magnitude expression of Z(jw) its possible when w =0 we get another pole a minimum pole at w = 0.

$$|Z(j\omega)| = \sqrt{9 + 16\,\omega^2}$$

$$|Z(j0)| = \sqrt{9 + 16\,(0)}$$

$$|Z(j0)| = \sqrt{9} = 3$$

At *w=0* Z(j0) =3.

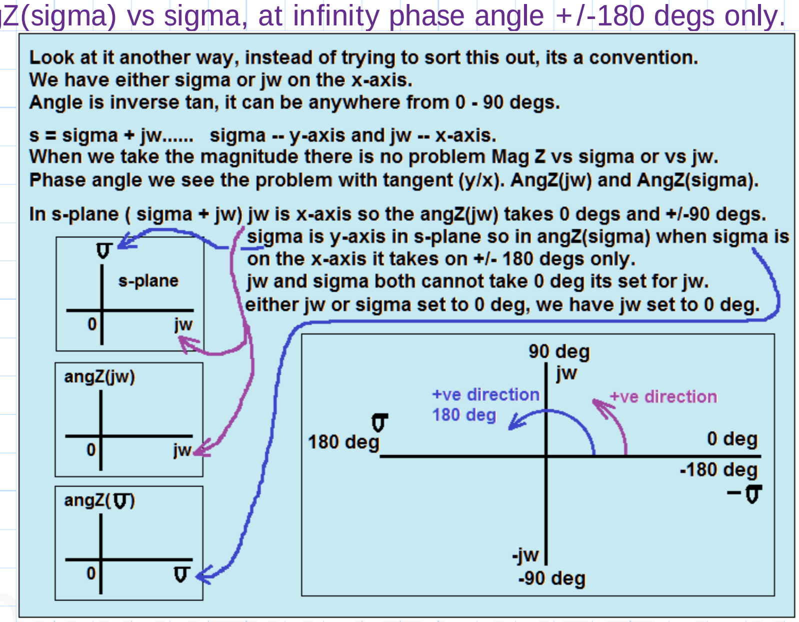
Some agrement there applying both sigma and jw in the pole location. Each separately.

Phage angle is an inverse tanget term. Tanget unlike the sin and cos which can go in circles, the tangent is restricted in some way. For example a vector at 90 how that got represented tan^1(y/x)..... y/x = y/(x=0) = y/0 = ? We have some value y axis and x-axis we have 0. y=4 x =0, tan^-1(4/0) = ? Obviously no result, check your calculator. Check tan(90)? No result.

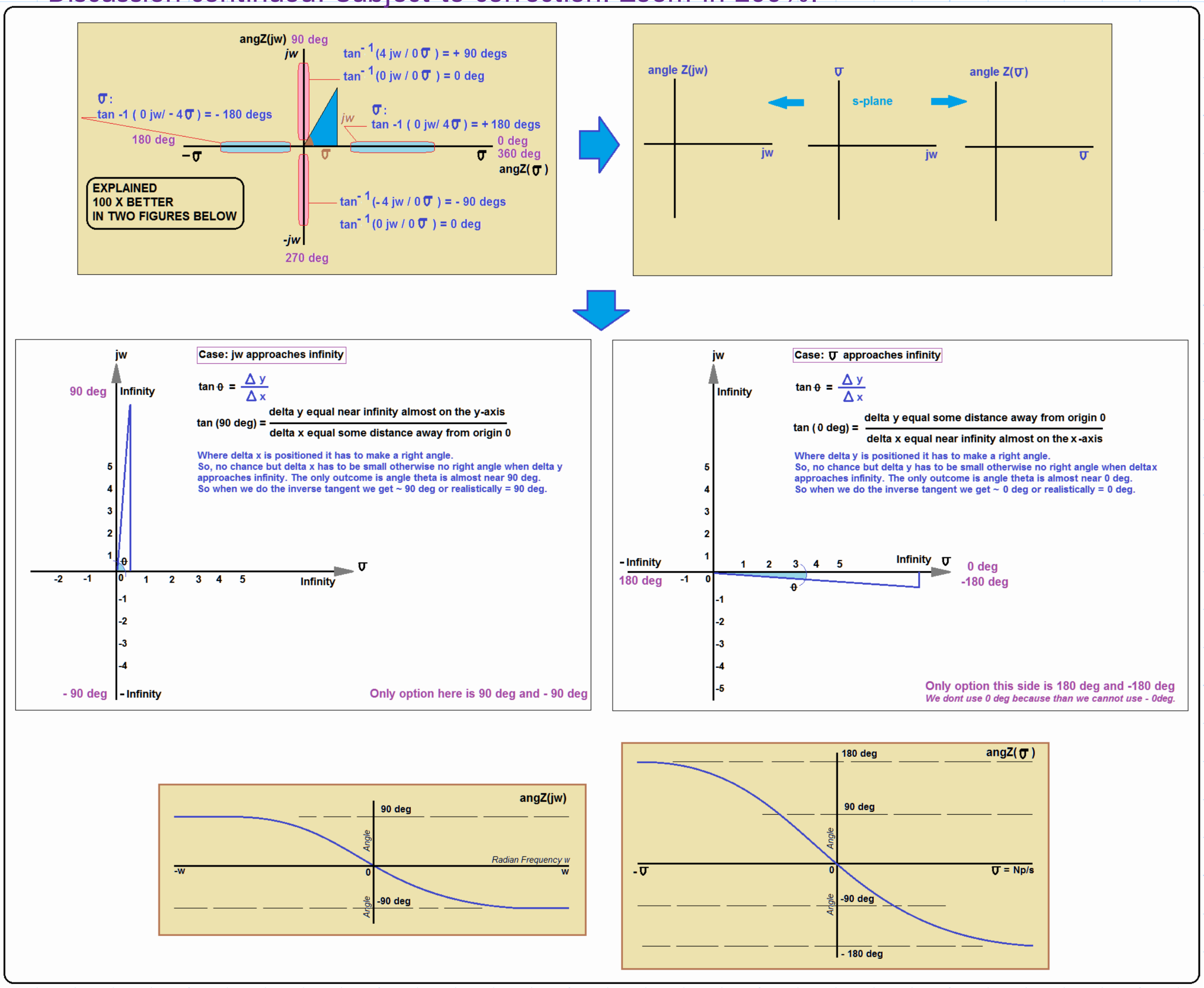
Phase angle, angZ(jw) is an inverse tangent function, 0 deg at w=0, and +/- 90 deg at w = +/- infinity. *See figure next page.* Phase angle plotted as ang Z(*jw*) versus *w*. Similar for sigma, angZ(sigma) vs sigma, at infinity phase angle +/-180 degs only.

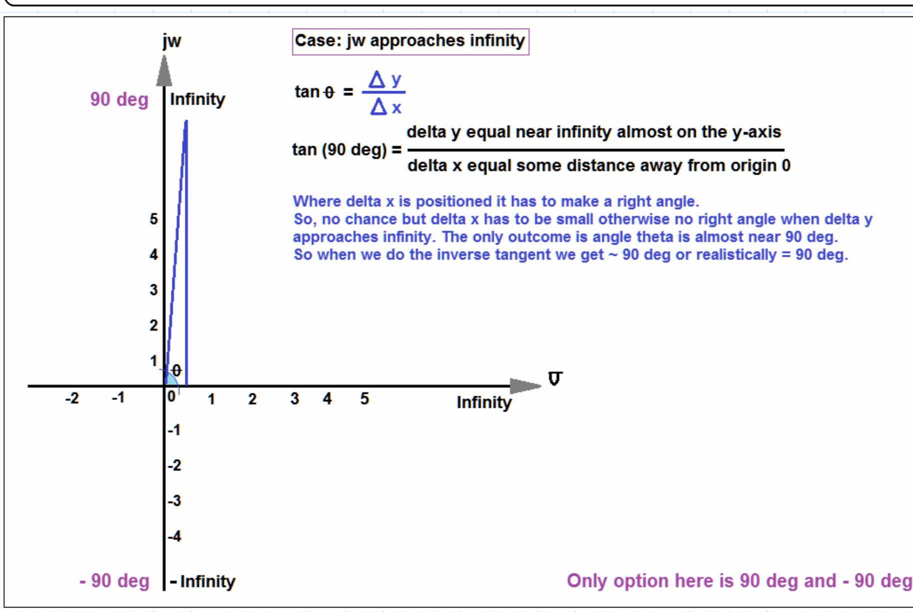
Discussion: ----->
If there is a mistake in my discussion you correct it maybe little off *(because I was trying to fit in too much in too little-and not willing to take it off)* so see next page better.

MAY BE A REASON FOR THE MADNESS. Solution later.



Discussion continued: Subject to correction. Zoom-in 200%.





<--- jw approaching infinity case shown here in larger scale. Same as in figure above. Well maybe me more than you that I get the specifics on this.

Each case 2 plots are required:

sigma x-axis: $|Z(\sigma)|$ and $\text{angZ}(\sigma)$

jw x-axis: $|Z(j\omega)|$ and $\text{angZ}(\omega)$

From our functions experience with regards to exponential and sinusoidal, we find sigma in exponential term and *w* in sinusoidal term.

$i(t) = 56\,e^{-st}$ <---- Exponential

Independent variable **s**

$v(t) = 124 \cdot \cos(\omega t + \theta)$ <---- Sinusoidal

Independent variable **t**

Note: There is only one independent variable sigma in the case of exponential and w in the case of sinusoidal.

Personally I see limitations when it comes to comparing here, you may know better:

between $|Z(\sigma)|$ and $|Z(j\omega)|$

between $\text{angZ}(\sigma)$ and $\text{angZ}(\omega)$

Of course it cannot be done.

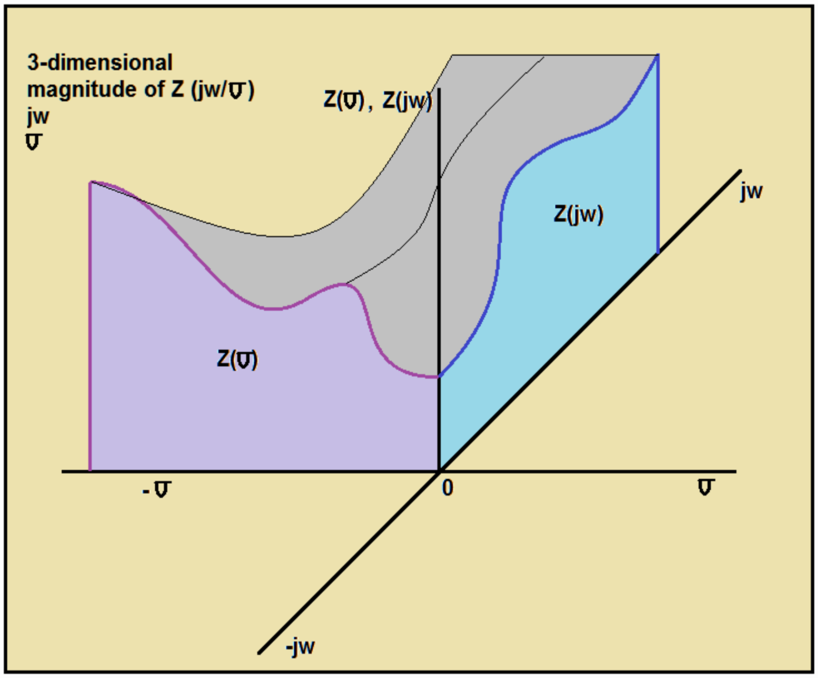
Engineers came out with a 3-dimensional one ------->

I work it as I progress with the engineers.

3D? Not easy for me.

Hope you got it down good.

MAY BE A REASON FOR THE MADNESS. Solution ----->



Added a 3rd axis perpendicular to sigma and jw axis, and passing thru the origin. We understand the math here, the surface is the function f(sigma, jw).

A better method of representing the magnitude of some complex response graphically involves using a three dimensional model - Hyat & Kemerly.

Before moving into 3D s-plane, lets study the 2D s-plane.
The s-plane below has the time domian functions at strategic locations.
Gives us a general idea on how time domain function behave corresponding to those sigma-jw locations.

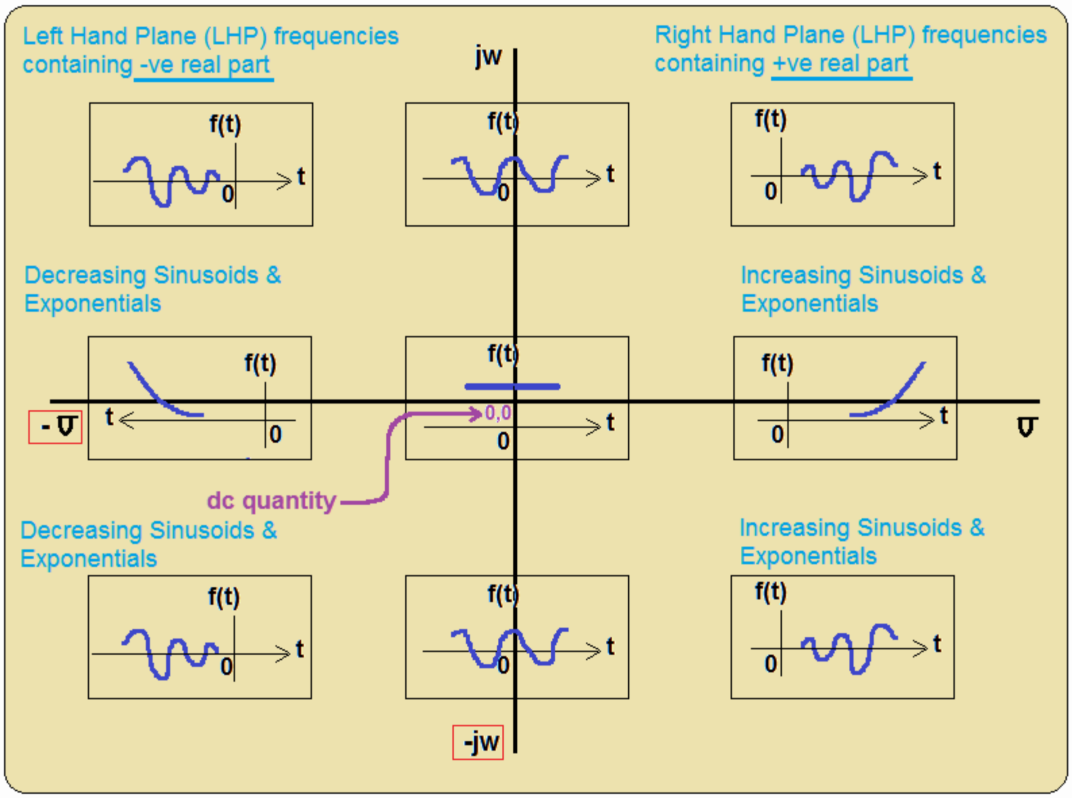
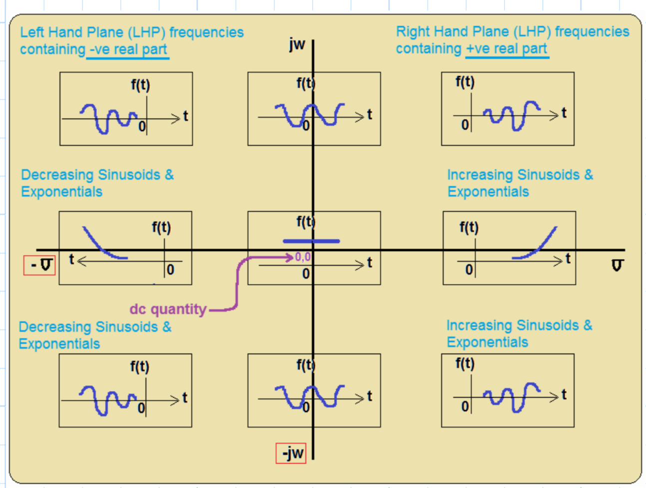


Figure above summarises relationship between time domain and various regions of s-plane.

The response magnitude is determined for every value of s (sigma + jw), the resultant plot is a surface (grey) lying above (or just touching) the s-plane.

Figure to the right has above, not one that is touching the bottom. See next figure for touching.

Magnitude can be that of anything... |Z|, |Y|,.... Here let magnitude of Z equal |Z| = 4 at s = -6*sigma* +j10w.



$$|Z| = 4 = -6\sigma + j10\omega$$

Lets work a simple exercise to get the basic idea.
RL series circuit.
R= 3 ohm Z_R + 3 in frequency domain
L = 1H in Frequency domain: s1.

$Z(s) = 3 + s \cdot 1 = 3 + s$ where $s = \sigma + j\,\omega$

$Z(s) = 3 + s$ $\quad Z(s) = 3 + (\sigma + j\,\omega)$

$Y(s) = \dfrac{1}{Z(s)} = \dfrac{1}{3+s}$ where $s = \sigma + j\,\omega$

$Y(s) = \dfrac{1}{Z(s)} = \dfrac{1}{3 + (\sigma + j\,\omega)}$

$|Y(s)| = \dfrac{1}{\sqrt{3^2 + (\sigma^2 + j^2\,\omega^2)}}$ next, sigma is the real part so its factored with 3.

$|Y(s)| = \dfrac{1}{\sqrt{(3+\sigma)^2 + \omega^2}}$ $\quad j := \sqrt{-1} \quad j^2 = -1 \quad |j^2| = 1$ <---
<--- magnitude in terms of sigma and jw

$|Y(s)| = \dfrac{1}{\sqrt{(3+\sigma)^2 + \omega^2}}$ <--- we have a sigma in the denominator which is for the pole.

Seems like so far only option we have is for the pole analysis at the denominator.

$Y(s) = \dfrac{1}{3+s}$

What value must s be for the denominator to be 0?

$s = -3 + j0 \qquad \omega = 0$

$Y(s) = \dfrac{1}{3 + (-3 + j0)} = \dfrac{1}{0 + j0}$

$Y(s) = \dfrac{1}{3 + (-3 + j0)} = \dfrac{1}{0 + j0}$ This means for us---> $\dfrac{1}{\sim 0}$ = Infinite.

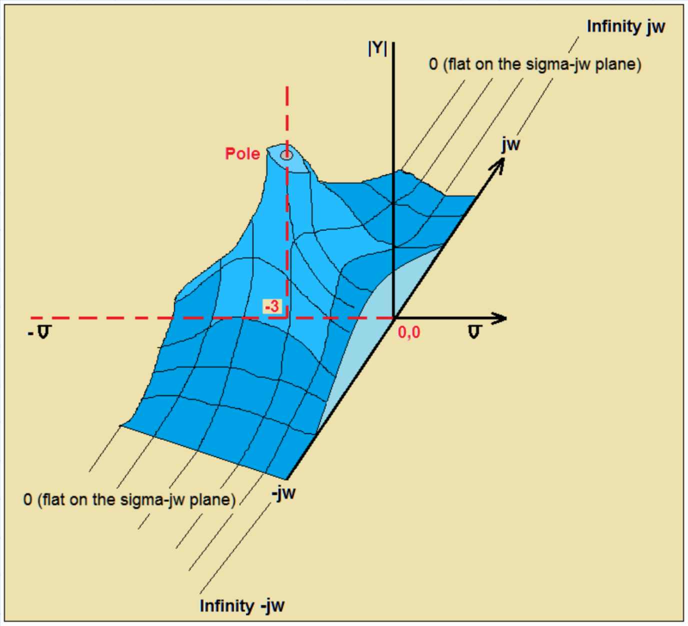
We have a pole at: $-3 + j\,0\,\omega$ <---Infinite value here.

And what happens when s is infinite in Y(s):

$$Y(s) = \frac{1}{3 + (\text{Infinite})} \quad \frac{1}{\text{Infinite}} = \sim 0$$

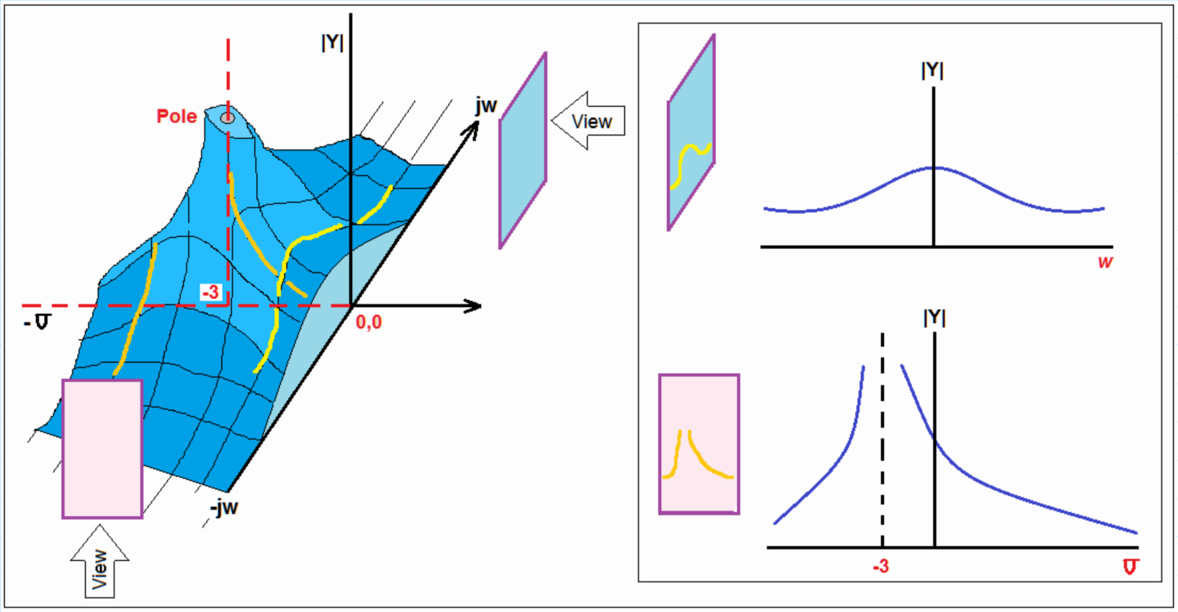
We have Y(s) = 0 when s is infinite.

Conclusion:
Our model RL circuit is infinitely high over the point s = -3 + j0 and it is zero height at all points infinitely far away from the origin (0,0). See figure below.



<---A cutaway view of the model.

Below section views,
Y(s) as function of w, and
Y(s) as function of sigma.

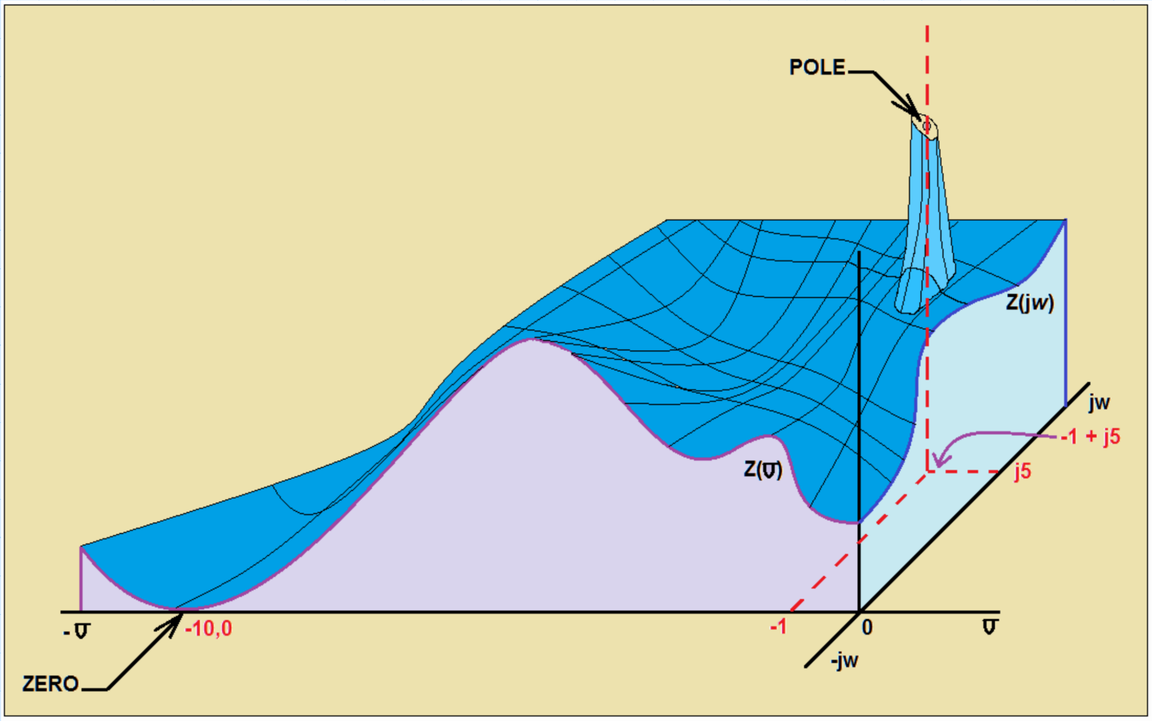


What we are calling the model is the surface and the coordinates are the plane sigma-jw. So when we look from the |Y|-sigma vertical plane can analyse |Y| with respect to sigma. The other vertical plane |Y|-jw we can analyse |Y| with respect to jw. Now from the top view sigma-jw we can see poles and in the coming figure we can see zeros too. The surface is able to capture information relative to both sigma and jw.

We have in the bottom of figure on previous page with 2 side views; magnitude of Y vs sigma and magnitude of Y vs jw.

We can also make it angle Z, angle V, angle I, magnitude of V2/V1,.......
.....Cleaver Engineers.

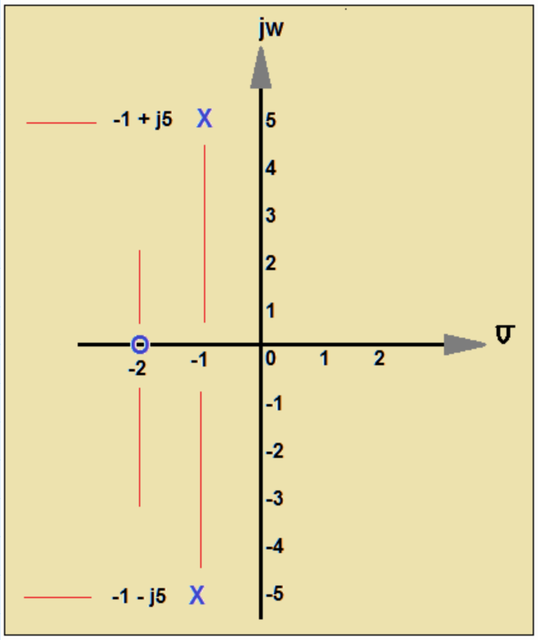
Figure below like wise for side views and top view.
We have a zero at (-10,j0) and pole at (-1, j5)



Short Talk: I say the above surface does not represent a real circuit....too rough......too many slopes. Its my sketch just to capture the technique. May make a golf course, next sketch closer to circuits. It will surprise me if there are such surfaces for circuits. It could be if you can come up with the? transfer function. Okay got a joke in. There maybe sofware for this graphic requirement but basically its solving the transfer function. Is the surface change or contour helpful for the circuit performance? I dont know, check with local enigneer. Yes in filter design.
There is so much software there than there is need - Karl Bogha.

Configuration of poles and zeros, sigma vs jw, is called pole-zero constellation.

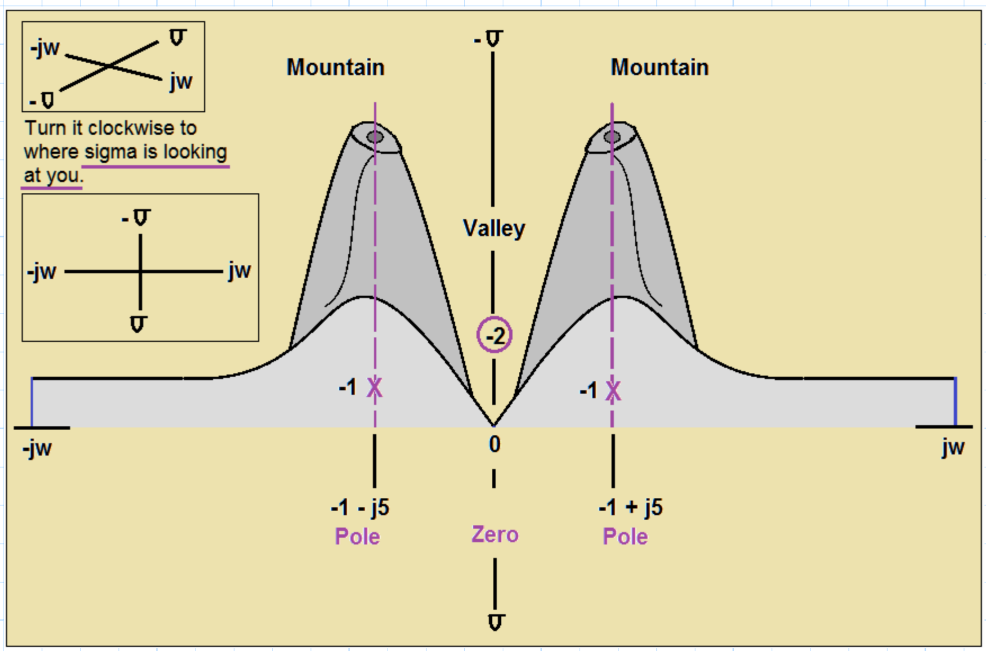
Here is located all the critical frequencies of some frequency-domain quantity.
We seen for example those quanitites for impedance Z(s) in the frequency domain.



<---This is a pole-zero constellation.
Its like looking at the night sky and seeing stars, X, and planets/moon O.

<---We have two poles at -1 + 5j and the other at -1 - 5j.
We have a zero at -2 +0j

Rough sketch of a section view or cut-away view on the longitudinal (y-axis ie jw) we have the 3D perspective of it below. The upper Left Hand Plane (LHP) part of model provided on next page - Engineer style.



Lets say I got the basic understanding on how to read the coordinates, sigma + j-omega, and next have the 3D surface perspective or view constructed.

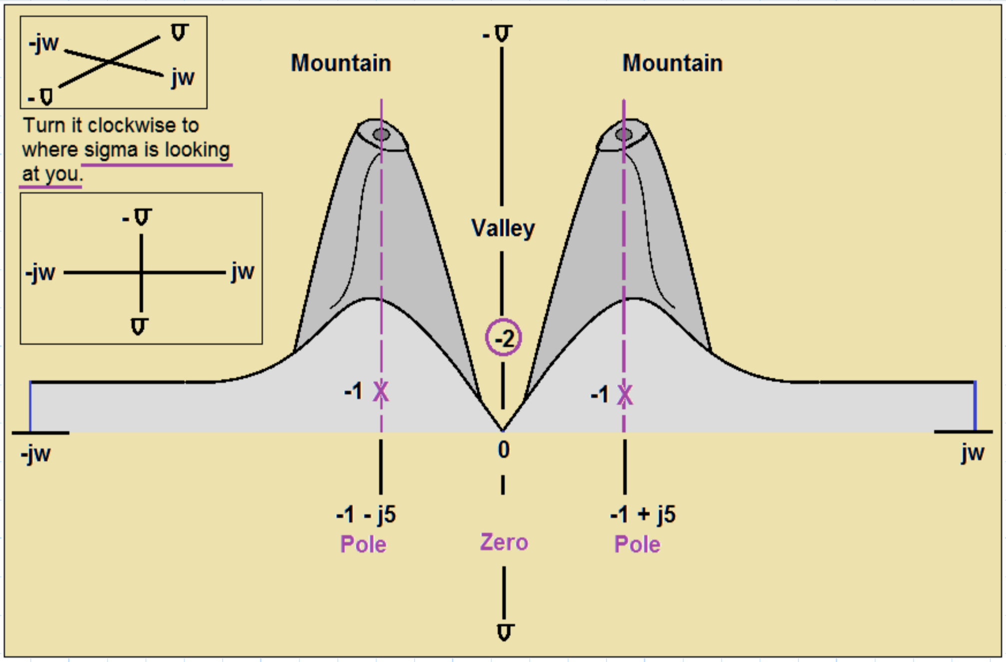


Figure above shows the upper portion, LHP, of the model.
We use the above pole-zero configuration, upper LHP, for the coming theory-example.
Next, working in reverse in comparison to past exercises.

We got poles and a zero, we need to construct Z(s) the original expression.

s:	$\sigma + j\ \omega$	The opposite or other	$s + ('-(\sigma + j\ \omega))$
Pole 1:	$-1 + j5$	way to where we got	$s + (1 - j5)$
Pole 2:	$-1 - j5$	the poles/zeros from,	$s + (1 + j5)$
Zero 1:	$-2 + j\ 0\ \omega$	those expression looks like this ---->	$s + (2 + j0)$

So now our Z(s) looks like this with the factor k:

$$Z(s) = k\left(\frac{s + (2 + j0)}{(s + (1 - j5))\ (s + (1 + j5))}\right)$$

$$Z(s) = k\left(\frac{s + 2 + j0}{(s + 1 - j5)\ (s + 1 + j5)}\right)$$

$$Z(s) = k\left(\frac{s + 2}{(s + 1 - j5)\ (s + 1 + j5)}\right)$$

$$(s + 1 - j5)\ (s + 1 + j5) = s^2 + s + s5j + s + 1 + j5 + -sj5 - j5 - j^2\ 25$$
$$= s^2 + 2\ s + 1 + 25$$
$$= s^2 + 2\ s + 26$$

$$Z(s) = k\left(\frac{s + (2 + j0)}{(s + (1 - j5))\ (s + (1 + j5))}\right)$$

$$Z(s) = k\left(\frac{s + 2}{s^2 + 2\ s + 26}\right)$$

To continue we need some other information on the circuit.
Engineers said lets say at s=0, Z(s) = 1. So we have a value for the function at s=0.

What do we do with it? Huh?

Substitute s =0 and make Z(s) = 1 or the Z(s) expression, see what result we get for k!

$$Z(s) = k\left(\frac{s + 2}{s^2 + 2\ s + 26}\right)$$

$$1 = k\left(\frac{0 + 2}{0^2 + 2\ 0 + 26}\right)$$

$$1 = k\left(\frac{2}{26}\right)$$

$$k = 13$$

Now plug k=13 into the Z(s) expression, and we have the full expression for Z(s).

$$Z(s) = 13\left(\frac{s + 2}{s^2 + 2\ s + 26}\right)$$ <---Satisfied.

If we were given Z(s) above and told to find the poles and zeros, could we do it?
Zeros: $s + 2$ ---> For it to equal 0 we need s = -2; sigma = -2 , jw = 0
Poles: $s^2 + 2\ s + 26$ ---> We need to factor this expression.

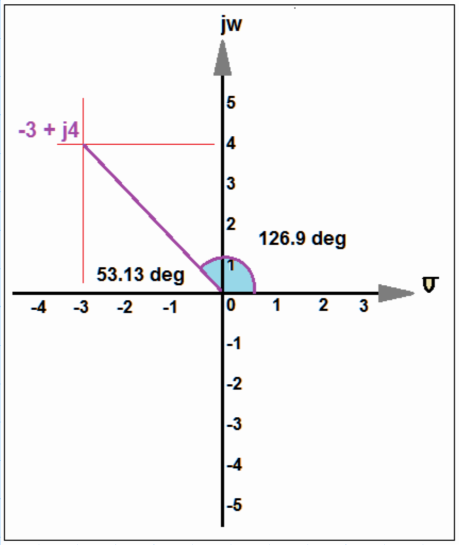
$(s - 5)\ (s - 5) = s^2 + s5 + s5 + 25$ <--- Not there.
$(s - 5)\ (s + 5) = s^2 + s5 - s5 + 25 = s^2 + 25$ <--- Not there.
Obviously now I have to resort to some other method and that requires some Math application, and obviously the factor exists since we started with it.

$(s + 1 - j5)\ (s + 1 + j5)$ <--- Here you knew that.

$(s + 1 - j5)\ (s + 1 + j5)$ ---> For it to equal 0 we need s = ?
s = -1 + 5j; sigma = -1, jw = 5j (first term)
s = -1 - 5j; sigma = -1, jw = -5j (second term)

Agreeablee. *My experience was 'going in circles' hope yours was better.*

Now, said the engineers we are going to look at a new powerful tool. Hyat & Kemerly said we have seen the qualitative, verbal stuff or word associative like good bad high low.... zero-pole, side of the pole-zero constellation. Engineers said there is a quantitative side to this, which I take for evaluating something of numerical value, thats what they meant. So, lets get to this new powerful method. Do I belief it so? Yes. Eventually we get to filter design, Laplace for circuits, signal processing,.......



s : Complex frequency

<---- $s1 = -3 + j4$

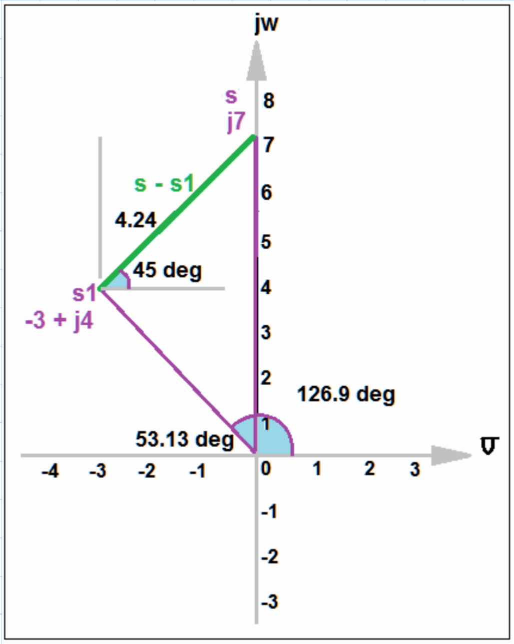
$$mag\_s1 = \sqrt{(-3)^2 + (4)^2} = 5$$

Phase angle in the 2nd quadrant: $\operatorname{atan}\left(\frac{4}{-3}\right) = -53.13 \text{ deg}$

-53.13 its in the 2nd quadrant and it would be negative, but we measure it from the positive real sigma side.

$$angle\_s1 = (180 - 53.13) \text{ deg} = 126.87 \text{ deg}$$

Next add a vector s= j7 to the vector s1 we already have in the figure.



$s1 = -3 + j4$
$s = j7$

Difference between s and s1?
The difference is the vector line drawn from end point of s1= -3+j4 to end point s = j7.

$$s - s1 = j7 - (-3 + j4)$$
$$= j7 + 3 - j4$$
$$s - s1 = 3 + j3$$
$$mag(s - s1) = \sqrt{(3)^2 + (3)^2} = 4.243$$

Phase angle in the 2nd quadrant: $= \operatorname{atan}\left(\frac{3}{3}\right) = 45 \text{ deg}$

$$s - s1 = 4.24\angle 45 \text{ deg}$$

Also from figure above: $s1 + (s - s1) = (-3 + j4) + (3 + j3) = 7j = s$ Correct.

Ok. I seen this in my engineering math course or calculus course. Its used in every engineering discipline, civil, mechanical,......electrical. Basic vectors. Sorry, hold-on we getting there.

We seen some graphical representation and intepretation of vectors.
So next how about an application here.

I try to start now:
I have an admittance Y(s):

$$Y(s) = s + 2$$
$$= (\sigma + jw) + 2$$

Our Y(s) is not a fraction term, its all numerator and here there is the 'zero'.
What value must s take on to make Y(s) = 0?

$$0 = (\sigma + jw) + 2$$
$$0 = (-2 + j0w) + 2$$
$$0 = (-2 + j0) + 2 = 0$$

So now we make s2 = -2+j0

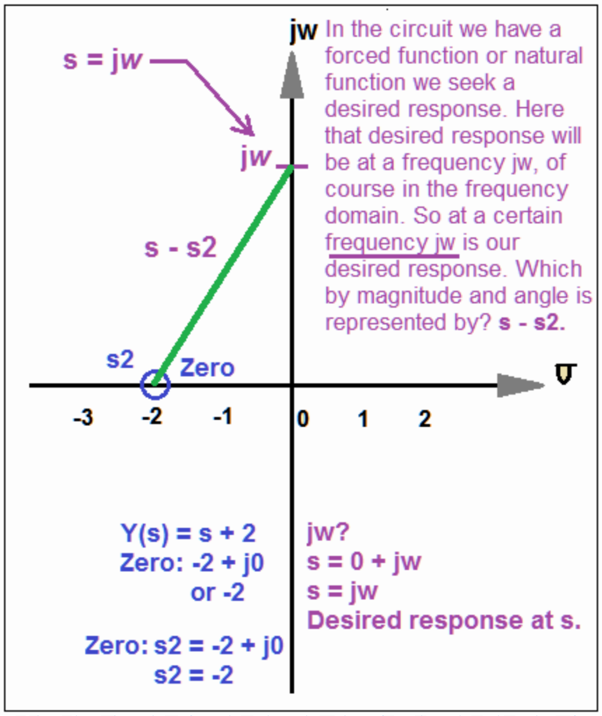
$$s2 = -2 + j0$$

The factor s+2, which came from Y(s) = s + 2, can be written as (s - s2) :

$$s - s2 = s - (-2 + j0) = s + 2 + j0 = s + 2 \text{ <--- } Y(s)$$

$$s - s2 = s + 2 \text{ <--- } Y(s)$$ <--- Difference vector (s - s2) represents? Y(s)

In our last figure we had (s - s1), this was with s = 7jw and s1 = -3 + j4.
Need (s - s2) be something same as (s - s1). Got it. NOW see figure below with s - s1.



Study the figure to the left, read the notes. Important stuff in there.

We can slide 'jw' up and down the y-axis. The GREEN line can travel on the jw (y-axis) while one end held at sigma = -2 the zero location.

The magnitude of s+2 may now be visualised as *w* varies from 0 to infinity. *w* the frequency can be at: infinity jw, 0, and infinity -jw.

When s=0, Y(0)=s+2 =0+2 =2<--Magnitude
Magnitude 2 and angle of 0 deg.
0 deg because s = 0 = 0 + j0.
As w increases from j0 magnitude increases slowly at first and then almost linearly with w. Phase angle increases almost linearly at first and then gradually approaches 90 deg when w becomes infinite. *Hyat & Kemerly.*

Step by step separating sigma and jw for plotting.
Plot of magnitude/angle versus frequency w.

$Y(s) = s+2$ (Remember because 2 is a real part it will be on sigma axis).

$s = 0 = 0\sigma + j0\omega$

$Y(j0\omega) = j0\omega + 2 = 2$ *On jw axis, when jw=0 what is Y(j0w)? 0+2=2*

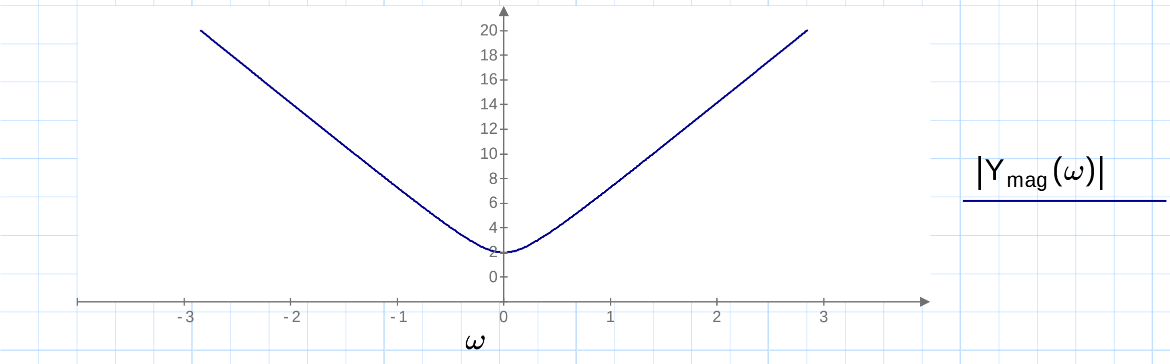
$Y(0\sigma) = 0\sigma + 2 = 2$ *On sig axis, when sig=0 what is Y(0sig)? 0+2=2*

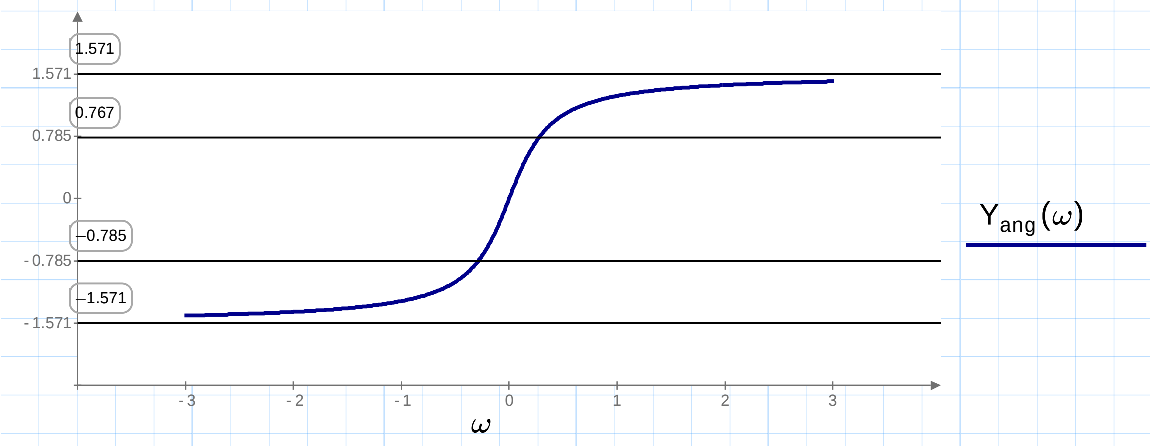
$Y(s) = s+2$

$s_{set\_freq} = j7\omega$ <--- jw = j7w set frequency clear(ω) $\omega := -10, -9.99..10$

$Y(\omega) = Y(j0\omega) + s_{set\_freq}(j0\omega) = 2 + (7\omega)$

$Y(\omega) := 2 + 7\cdot\omega$ $Y_{mag}(\omega) := \sqrt{(2^2 + 7^2\cdot\omega^2)}$ $Y_{ang}(\omega) := \left(\mathrm{atan}\left(\frac{7\cdot\omega}{2}\right)\right)$





In the angle plot above, y-axis degree values for 90 and 45 deg given below.

90 deg = 1.571

45 deg = 0.785

What we accomplished from the two plots on the previous page:

Plots of |Y(jw)| and ang Y(jw) as they might be obtained from the vector as s moves up or down the jw axis from the origin.

Comment: It may be simple for some to do those plots but to split them up into sigma and jw was an extra effort on my part. So though it may look simple or basic but we got a tool or technique for the desired response by moving up and down the vertical axis ie (+jw 0 -jw) axis. At each point on the jw axis, vector (s - s2), will provide a magnitude and phase angle. Let's not forget we started with Y(s) = s +2, and that could be L =1, R = 2, so thats (s*1 + 2) equal s + 2. Y(s) is admittance and the inverse of it (1/(s +2)) equal impedance Z(s).

Next thru the learning/theory side of this section in Hyat & Kemerly, we have another example. A frequency domain function given by a quotient, more realistic.

$$V(s) = \frac{s+2}{s+3}$$

We make V(s) into 2 functions V1 and V2:

$V1(s) := s + 2$

$V2(s) := s + 3$

Here we have two functions, zero and pole, and we work like previous exericse, we need to move up and down jw axis. Difference is there may be a relationship between the numerator and denominator functions to take into consideration.
If V(s) = V2(s)/V1(s) we may have relationship between the voltage vectors.

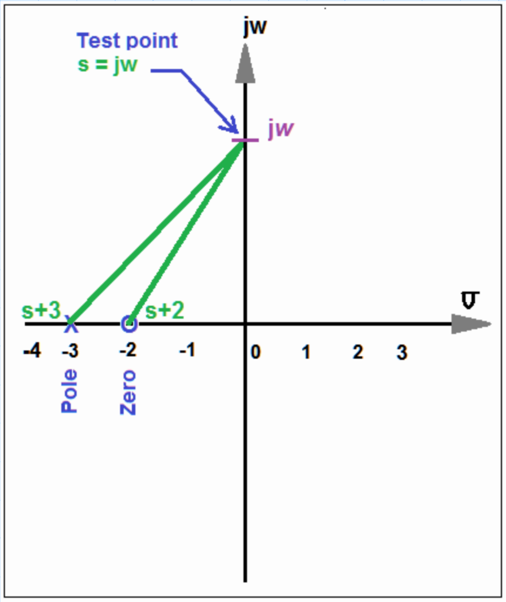
Bullet point notes below from Hyat & Kemmerly:

1. Draw the pole-zero constellation of the frequency-domain function under consideration in the s plane, and locate a test point corresponding to the frequency at which the function is to be evaluated.

$s + 2$	$s + 3$
$(\sigma + j\,\omega) + 2$	$(\sigma + j\,\omega) + 3$
$(-2 + j\,0)\omega + 2$	$(-3 + j\,0)\omega + 3$
$(-2 + j0) + 2$	$(-3 + j0) + 3$
0	0
Zero at: $-2 + j0$	Pole at: $-3 + j0$

<---Another way of writing take the w out comes with more experience/exercises.

2. Draw an arrow from 'each pole and each zero' to the test point.



<---Test point *jw* moving up and down the y-axis.

Angle formed by (s+2) with sigma is greater than the angle formed by (s+3). Because the (s+2) slope is steeper.

3. Determine the length (magnitude) of each pole arrow and zero arrow, and the value of each pole arrow angle and zero-arrow angle.

For our example we set the test point at 4jw.
s = j4w.

Zero: (-2,0)

$$Z_{mag} := \sqrt{(-2)^2 + 4^2}$$

$$Z_{mag} = 4.472$$

$$Z_{ang} := \operatorname{atan}\left(\frac{4}{-2}\right) = -63.435 \text{ deg}$$

Pole: (-3,0)

$$Z_{pole} := \sqrt{(-3)^2 + 4^2}$$

$$Z_{pole} = 5$$

$$Z_{pole} := \operatorname{atan}\left(\frac{4}{-3}\right) = -53.13 \text{ deg}$$

$$V(s) = \frac{4.472\angle -63.44 \text{ deg}}{5\angle -53.13 \text{ deg}}$$

$$= \frac{4.472}{5} = 0.894 \qquad = -63.435 - (-53.13) = -10.305$$

$$V(s) = 0.894\angle -10.3 \text{ deg}$$ Answer.

4. Divide the product of the zero-arrow lengths by the product of the pole arrow lengths.This quotient is the magnitude of the frequency domain function for the assumed frequency of the test point [within a multiplying constant, since F(s) and **k**F(s) have the same pole zero constellations].

5. Subtract the sum of the pole-arrow angles from the sum of the zero arrow angles. The resultant difference is the angle of the frequency domain function, evaluated at the frequency of the test point. The angle does not depend upon the value of the real multiplying constant k. *End of points 1-5.*

Plots:

Step by step separating sigma and jw for plotting.
Plot of magnitude/angle versus frequency jw.

$V1(s) = s + 2$

$V1(s) = s(\sigma + j\omega) + 2$

$s = 0 = 0\sigma + j0\omega$

$V1(j0\omega) = j0\omega + 2 = 2$ drop 0-sigma in this expression hold 2.

$V1(0\sigma) = 0\sigma + 2 = 2$ drop 0-jw in this expression hold 2.

$s_{set\_freq}(j\omega) = j4\omega$ <--- jw = j4w set 'test-point' frequency

$V1(\omega) = V1(j0\omega) + s_{set\_freq}(j\omega) = 2 + (4\omega)$ clear (ω) $\omega := -10, -9.99..10$

$V1(\omega) := 2 + 4\cdot\omega$ $V1_{mag}(\omega) := \sqrt{(2^2 + 4^2\cdot\omega^2)}$ $V1_{ang}(\omega) := \left(\mathrm{atan}\left(\frac{4\cdot\omega}{2}\right)\right)$

$V2(s) = s + 3$

$V2(s) = s(\sigma + j\omega) + 3$

$s = 0 = 0\sigma + j0\omega$

$V2(j0\omega) = j0\omega + 3 = 3$ drop 0-sigma in this expression hold 3.

$V2(0\sigma) = 0\sigma + 3 = 3$ drop 0-jw in this expression hold 3.

$s_{set\_freq}(j\omega) = j4\omega$ <--- jw = j4w set 'test-point' frequency

$V2(\omega) = V2(j0\omega) + s_{set\_freq}(j\omega) = 3 + (4\omega)$ clear (ω) $\omega := -10, -9.99..10$

$V2(\omega) := 3 + 4\cdot\omega$ $V2_{mag}(\omega) := \sqrt{(3^2 + 4^2\cdot\omega^2)}$ $V2_{ang}(\omega) := \left(\mathrm{atan}\left(\frac{4\cdot\omega}{3}\right)\right)$

$V(s) = \frac{s+2}{s+3}$ $V1(s) := s + 2$

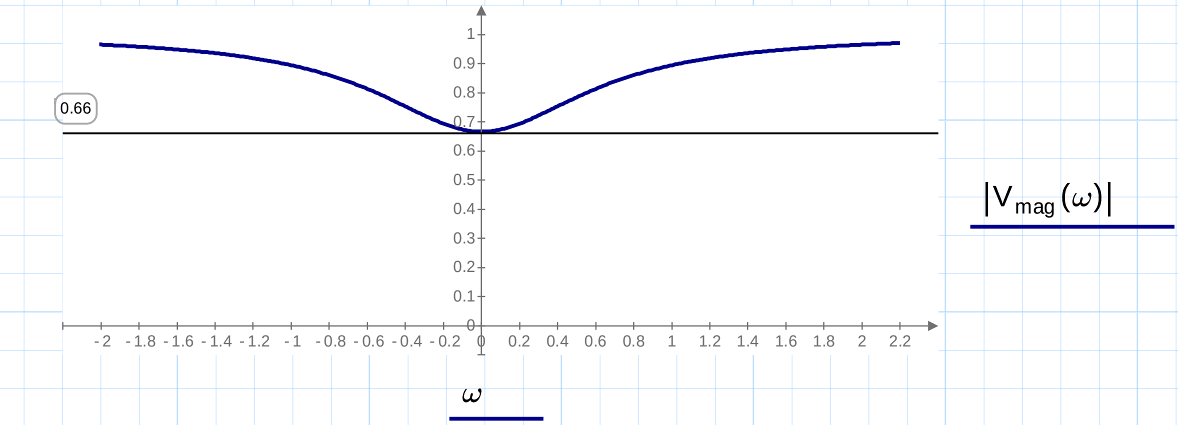
$V2(s) := s + 3$ clear (ω) $\omega := -10, -9.99..10$

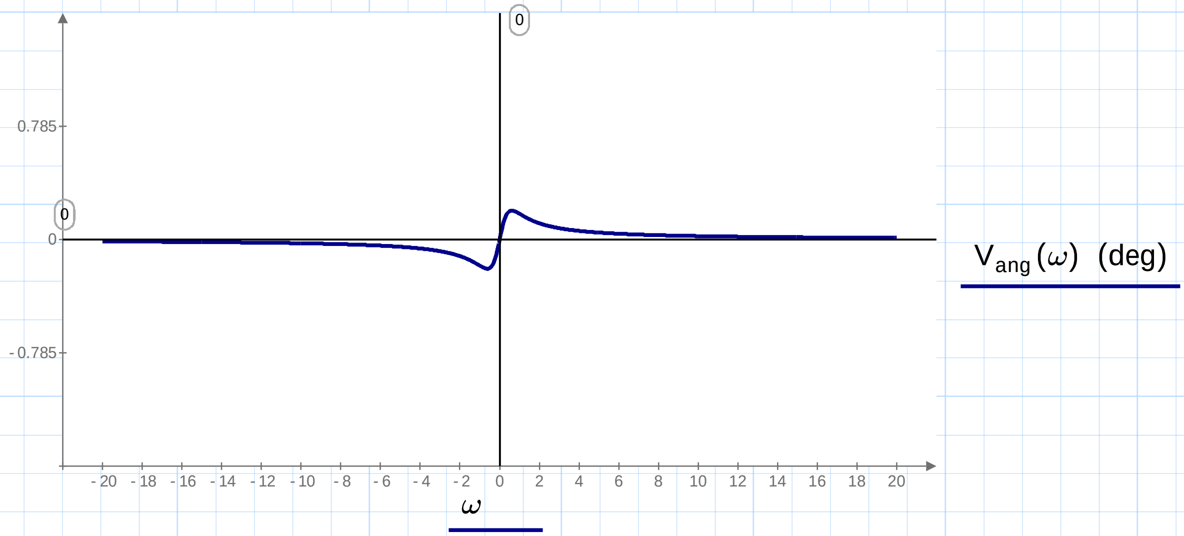
$V_{mag}(\omega) := \frac{\sqrt{(4 + 16\cdot\omega^2)}}{\sqrt{(9 + 16\cdot\omega^2)}}$ $V_{mag}(\omega) = k\cdot\left(\frac{\sqrt{(4 + 16\cdot\omega^2)}}{\sqrt{(9 + 16\cdot\omega^2)}}\right)$ *The form we seek and not have to solve for k here, we did that in previous exercise.*

clear (ω) $\omega := -20, -19.99..20$

$V_{ang}(\omega) := \left(\mathrm{atan}\left(\frac{4\cdot\omega}{2}\right) - \mathrm{atan}\left(\frac{4\cdot\omega}{3}\right)\right)$ deg

Next the magnitude and angle plots.





In the angle plot above, y-axis degree values for 45 deg given below.
45 deg = 0.785

Engineer conclusion: An investigation of the variation of the magnitude of V(s) versus w is made by allowing s to move from the origin up the jw axis and considering the ratio of the distance from zero to s = jw and the distance from the pole to the same point on the jw axis. Magnitude: The ratio seen is at 2/3 (0.66) at w = 0, and approaches unity (1) as w becomes infinite. Angle: Consideration of the difference of the 2 phase angles shows that angV(jw) is 0 deg at w=0, increases at first as w increases since the angle of the vector s+2 is greater than that of s+3, and then decreases with a further increase in w, finally approaching 0 deg at infinite frequency, where both vectors (V1(w) & V2(w)) posses 90 deg angles (90 deg - 90 deg = 0 deg).
The results can be seen in the plots above. Same as textbook plots. - *Hyat & Kemerly.*

Design Suggestion:

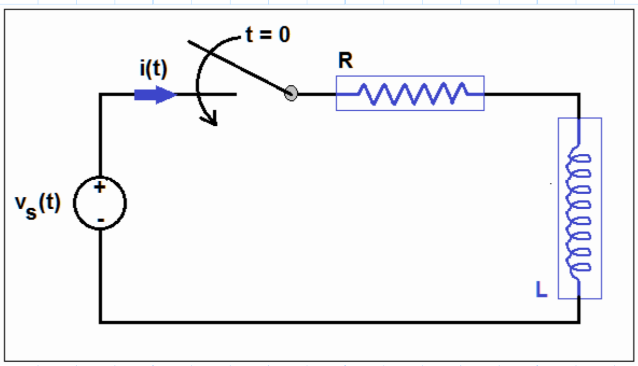
If it were necessary to increase the hump in the phase (angle) response (shown in angle plot), we can see we must provide a greater difference in the angles of the two vectors (V1 and V2). This may be achieved by either moving the zero closer to the origin OR by locating the pole farther from the origin, or both. - *Hyat & Kemerly.*

Give it a rest here, do examples/exercises in the worked example part.

13.7 Natural Response and the s-plane

Hyat & Kemerly: There is tremendous amount of information contained in the pole-zero plot of some forced response in the s-plane. We shall find out how a complete current response, natural plus forced, produced by an arbitrary forcing function can be quickly written from the pole-zero configuration of the forced current response and from the initial conditions; the method is similarly effective in finding the complete voltage response produced by an arbitrary source.

Coming theory-example may look similar to some recent ones before section 13.6, and here we connect it to pole-zero. There is more in-sight here which was not looked into. Reason why The Mahatma Engineers say this is a powerful method for circuit analysis!



Close switch at t=0.
Voltage source v_s(t) comes on,
current i(t) flows in the circuit.

i(t) = i_n(t) + i_f(t)
= natural and forced response

Our voltage source is in the form of v_s(t) is in time domain.
We need this time domain voltage source easily transformed into frequency domain.
That of course is helpful and then we can find the forced response. (Remember from previous exercises).

The engineers suggested v_s(t) be equal to 1/(1 + t^2): $v_s(t) = \dfrac{1}{(1+t^2)}$

$$I_f(s) = \frac{V_s}{R + sL}$$ current in frequency domain

$$I_f(s) = \frac{\frac{V_s}{L}}{\frac{R}{L} + \frac{sL}{L}} = \frac{1}{L}\left(\frac{V_s}{\frac{R}{L} + s}\right) = \frac{1}{L}\left(\frac{V_s}{s + \frac{R}{L}}\right)$$

$$I_f(s) = \frac{\frac{V_s}{L}}{\frac{R}{L} + \frac{sL}{L}} = \frac{1}{L}\left(\frac{V_s}{\frac{R}{L} + s}\right) = \frac{1}{L}\left(\frac{V_s}{s + \frac{R}{L}}\right)$$

Notes: Form of the natural response is independent of the forcing function (FF). The FF contributes with other initial conditions ONLY to the magnitude of the natural response. To find the proper form replace all independent sources by their internal impedances; here v_s(t) is replaced by a short circuit.

$$I_f(s) = \frac{1}{L}\left(\frac{V_s}{s + \frac{R}{L}}\right)$$

<--- To get this into time domain i_f(t) replace s, R, and L by their values, then reinsert e^st, and take the real part.

Note: I_f(s) can also be got by substituting w, sigma, R and L.
Since s = sigma + j*w*.

We form a transfer function first to explain better next.

$$H(s) = \frac{I_f(s)}{V_s(s)}$$

Current the output terminal called the response, divided by the input the voltage source.

$$H(s) = \frac{I_f(s)}{V_s(s)} = \frac{\left(\frac{V_s}{L\cdot\left(s + \frac{R}{L}\right)}\right)}{V_s} = \left(\frac{V_s}{L\cdot\left(s + \frac{R}{L}\right)}\right)\cdot\left(\frac{1}{V_s}\right)$$

$$H(s) = \frac{I_f(s)}{V_s(s)} = \left(\frac{1}{L\cdot\left(s + \frac{R}{L}\right)}\right)$$

<---Transfer function.

$$H(s) = \frac{I_f(s)}{V_s(s)} \quad \text{---->} \quad I_f(s) = V_s(s)\cdot H(s)$$

<--- What happens when we set Vs =0?
The forced response, I_f(s) = 0.

So we have reached a **dead end**.
Curent, ie the forced response, equal 0.
But now if we see things the 's' way, the s-plane way we know when we can go for poles and zeros.

Engineers: On the surface, it appears that I(s) must also be zero, but this is not necessarily true **if we are operating at a complex frequency** that is a simple pole of I(s). So its like this, denominator and numerator may both be 0 so that I(s) need not be zero (0). So we took advantage of the s-plane solution method(s).

Because Vs = 0,
there can be found a non 0 value for current operating at a pole of H(s).

$$I_f(s) = V_s(s) \cdot H(s)$$

$$I_f(s) = 0 \cdot H_{pole}(s)$$

Lets solve for the pole in this transfer function:

$$H(s) = \left[\frac{1}{L \cdot \left(s + \frac{R}{L}\right)}\right] \qquad L \cdot \left(s + \frac{R}{L}\right) = 0 \quad \text{When?} \quad s = \sigma + j\ \omega = -\left(\frac{R}{L}\right) + j\ 0\ \omega$$

$$L \cdot \left(\frac{-R}{L} + j\ 0\ \omega + \frac{R}{L}\right) = 0 \quad \text{ok.}$$

Now in that expression we prior looked at:

$$I_f(s) = 0 \cdot H_{pole}(s)$$

$$I_f(s) = 0 \cdot \left[\frac{1}{L \cdot \left[\left(\frac{-R}{L} + j\ 0\right)\omega + \frac{R}{L}\right]}\right] \qquad s = -\left(\frac{R}{L}\right) + j\ 0\ \omega$$

Now we say like the engineer said 'a finite current at this frequency thus represents the natural response' :

$I_f(s) =$ A (Amperes) when $s = -\left(\frac{R}{L}\right) + j\ 0\ \omega$

Where A is an unknown constant. <---*We pick-up from here shortly.*

Impedance of the circuit we started with: $R + sL$

The form we sought should have a decaying exponential function with the time constant (L/R): $e^{-\left(\frac{L}{R}\right)t}$ Thats what we did

For this circuit just so happens its admittance:

$$I_f(s) = \frac{1}{L \cdot \left(s + \frac{R}{L}\right)} = \frac{1}{R + Ls}$$

Expression we had for I_f(s), lets examine it closely. Numerator is 1
Denominator is the circuit series impedance, but its seen as admittance because its 1/Z(s).

$Z(s) = R + Ls \qquad Y(s) = \frac{1}{(R + Ls)}$ <---Admittance.

We pick-up from just before admittance case.

NOW we see that there is a powerful tool in the s-plane method.
We can have a 0 in the numerator, which was what we did in the theory exercise, but got a s = -(R/L) + j0w, that located a pole. Something similar to the plot exercises we had, separating function of sigma and function of w (omega).

Next transform the natural response from frequency domain to time domain response.

'To get this into time domain i_f(t) replace s, R, and L by their values, then reinsert e^st, and take the real part.'

$$i_n(t) = \mathrm{Re}\left(A \cdot e^{-\left(\frac{R}{L}\right)t}\right)$$ Taking the real part; Re.

$$i_n(t) = A \cdot e^{-\left(\frac{R}{L}\right)t}$$ What the form of the function was we took the real part.

We worked the natural response, the forced response is when V(s) is not equal to 0, or v_s(t) is not equal to zero, and with initial conditions we can solve for A the constant.

Complete response: $$i(t) = A \cdot e^{-\left(\frac{R}{L}\right)t} + i_f(t)$$

and A may be determined when the initial conditions are specified for this circuit.

Next the engineers seek to generalise these results,
the results we went thru which of course engineers provided, not my original work!.

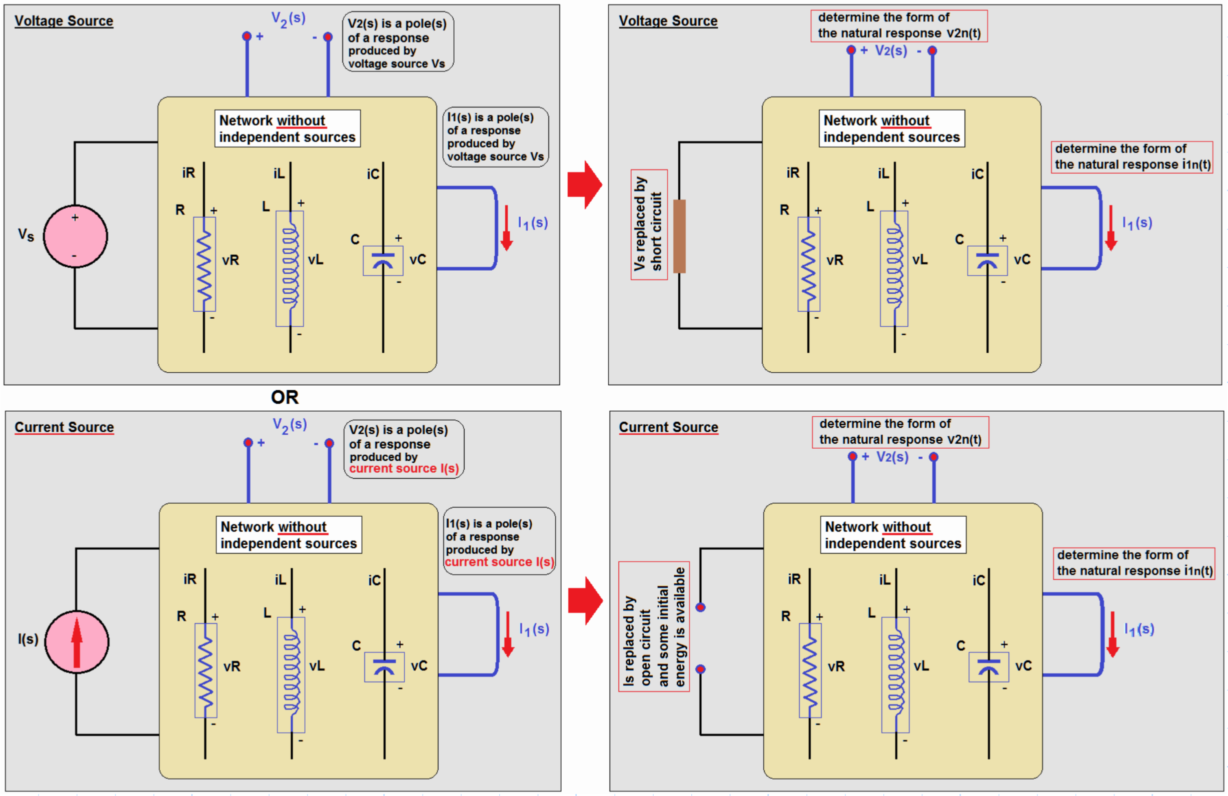
We have some theory then 2 theory-examples.

Then we stop.
End of this part.

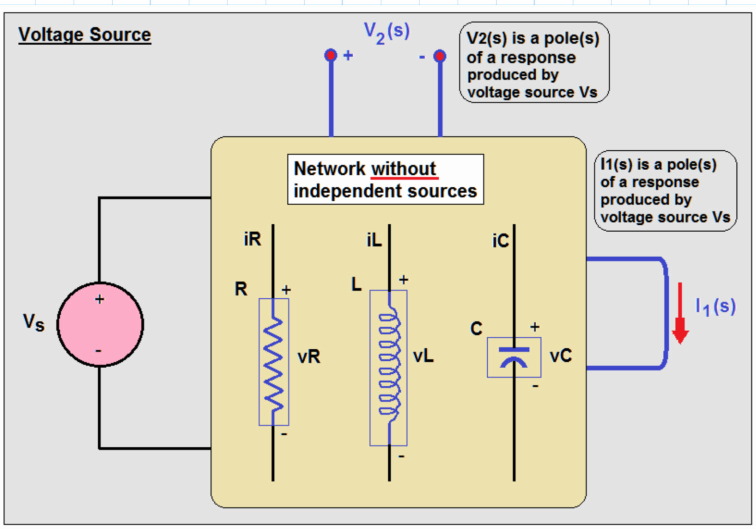
We pick up with section 13.8 in a new part.
Reason? Because I decided or choose to do Op-Amp circuits for section 13.8, it seems unavoidable. Sorry its unavoidable, not seems unavoidable.
That requires a short re-visit or review on Op-Amps, which the next section starts with, and then follows with section 13.8. So we got part D which continues with theory.

Generalised Results For Circuit Solving:

Read the notes in the figures below, ZOOM-IN from Section 13.7 of textbook - Hyat Kemerly.



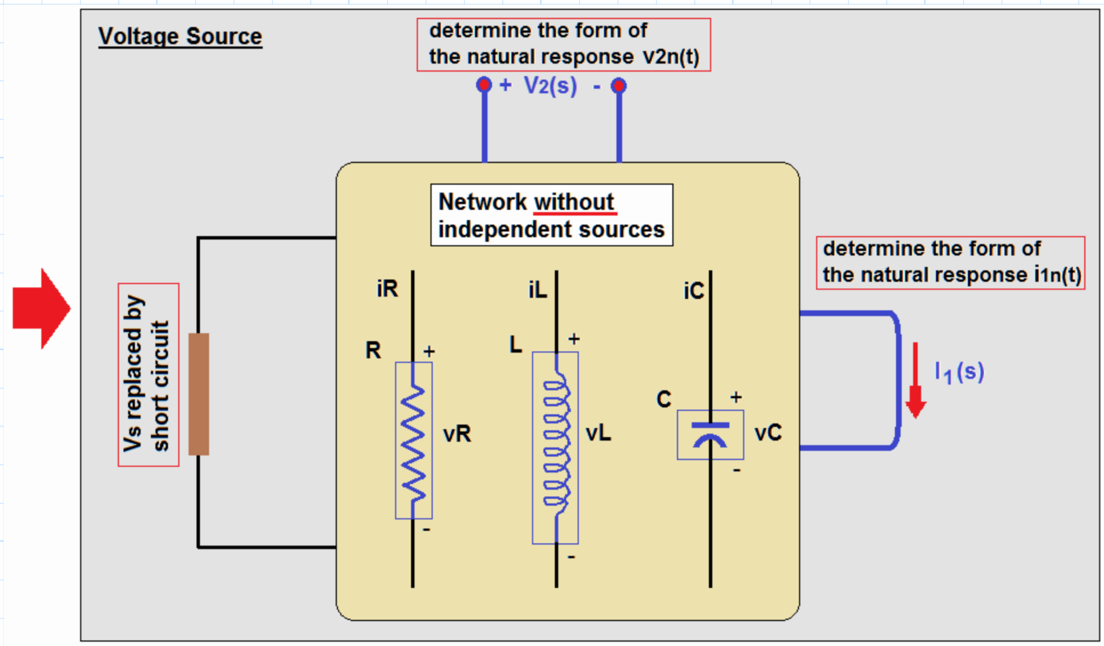
Provided again one by one.



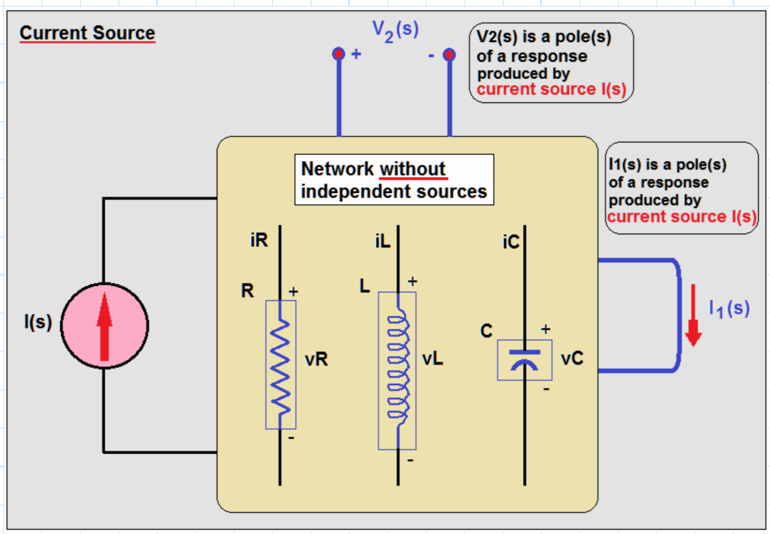
Case: Voltage source.
Resulting with the next figure.

Why did we use V2(s) instead of V1(s) in the upper connection of the circuit?

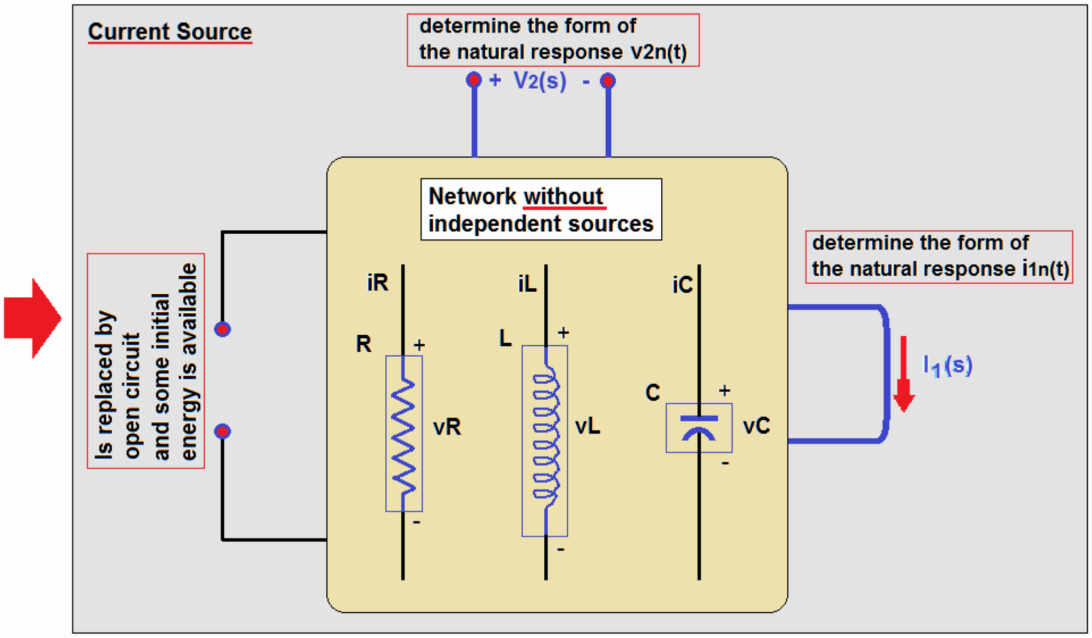
H(s) = V2(s)/V1(s)... this makes V1(s) the source voltage, which may be confusing in the theory section since V1(s) can also be a circuit branch or component voltage.
Thats one thinking on it. Just numbering system.



The poles of the response, I1(s) or V2(s), produced by the voltage source Vs determine the form of the natural response, i_n(t) or v2_n(t), that occurs when Vs is replaced by a short circuit.



Case: Current source.
Resulting with the next figure on next page.



The poles of the response, I1(s) or V2(s), produced by the current source Is determine the form of the natural response, i_n(t) or v2_n(t), that occurs when Is is replaced by an open circuit and some initial energy is available.

Voltage Source Case:
We use the voltage source circuit to build the theory or circuit analysis method. Figure on previous page.

The desired response can be current I1(s) or voltage V2(s).
This response can be described/expressed/determined thru a transfer function that displays **all** the critical frequencies.

$$\frac{V2(s)}{V_s} = H(s) = (k)\cdot\frac{(s-s_1)\,(s-s_3)\,(s-s_5)\,..}{(s-s_2)\,(s-s_4)\,(s-s_6)\,..}$$ <---General expression

Poles occur at: $s = s_2 \quad s_4 \quad s_6 \;...$

For **V2(s)** it's benefit from the transfer function **H(s)** is a possible functional form for the natural response for each of these frequencies **(s2 s4 s6......).**
Reminds me of our recent theory side of things and example we sought to set Vs = 0. Which we realise is making it a? Short circuit.
Now we try/hope to form a mathematical equation, after shorting the input voltage terminals, which represents the natural response.

Why will the equation be an exponential expression?
What ever voltage source type Vs was its no longer there and the current in the circuit with the voltage, thru L and C, can only be? decaying. Because its decaying it has to take the exponential form to express that decay, with a? negative power to the exponent. e^-st decaying, and e^+st increasing.

Joke: If you got another better reason fine put it up on the IEEE journal. It maybe a decade at least on the bright side if I got to it. The limited math I know at this time says it has to be exponential term and the power a negative number (-st). And I asked myself why did the engineers think-up an exponential expression. ***Learner!*** *Always Check With Your Local Engineer.*

Voltage response: $v_{2n}(t) = A_2 e^{s_2 t} + A_4 e^{s_4 t} + \ldots$

How we solved for each A prior:

1). We had a voltage source Vs at t=0 was removed.
2). The voltage in a particular path or branch of the circuit will be this voltage Vs its magnitude at time t=0.
3). V(-t) = V(0) = V(t+).
4). Same for current source.
5). Then we use that information for setting up equations and solving for the coefficients of A.

Remember this was one of the methods but this is electric circuits to be watchful of all conditions of the circuit.

Engineers: ....where each A must be evaluated in terms of the initial conditions (including the initial value of any voltage source applied at the input terminals).

Next the desired current response in the voltage source circuit.

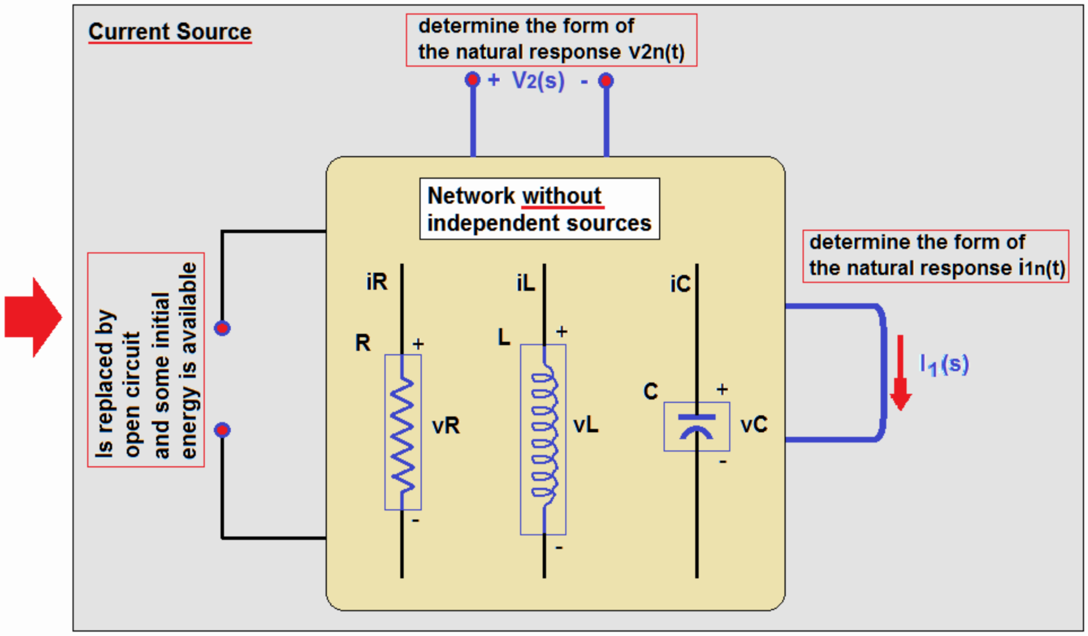
Similarly we seek the? Poles of the transfer function. However, of course, the transfer function will be different because we now have I(s) for numerator, with same Vs shorted.

Transfer function: $\frac{I_1(s)}{V_s} = H(s) = (k) \cdot \frac{(s-s_1)(s-s_3)(s-s_5)\ldots}{(s-s_2)(s-s_4)(s-s_6)\ldots}$

Poles occur at: $s = s_2 \quad s_4 \quad s_6 \ldots$

Current response: $i_{1n}(t) = B_2 e^{s_2 t} + B_4 e^{s_4 t} + \ldots$ Used B instead of A, same idea the coefficients.

Solve for B coefficients using? Initial conditions: i(-0) = i(0) = i(0+).



Current source circuit case (figure above):

Similarly and the transfer functions are:

$$\frac{I1(s)}{I_s} = H(s) = (k)\cdot\frac{(s-s_1)(s-s_3)(s-s_5)\ldots}{(s-s_2)(s-s_4)(s-s_6)\ldots}$$

Poles occur at: $s = s_2 \quad s_4 \quad s_6 \ldots$ <--- poles determind the natural response.

Current response: $i_{1n}(t) = A_2 e^{s_2 t} + A_4 e^{s_4 t} + \ldots$

$$\frac{V2(s)}{I_s(s)} = H(s) = (k)\cdot\frac{(s-s_1)(s-s_3)(s-s_5)\ldots}{(s-s_2)(s-s_4)(s-s_6)\ldots}$$

Poles occur at: $s = s_2 \quad s_4 \quad s_6 \ldots$ <--- poles determind the natural response.

Voltage response: $v_{2n}(t) = B_2 e^{s_2 t} + B_4 e^{s_4 t} + \ldots$

Short Talk: So far I managed to follow the steps. You probably better than me. There are some situations when we do not have Vs and Is in the circuit. Hard to imagine, but we seen this when the switch is opened at t=0, circuit got charged at t<0, at t=0 switch opened, t>0 circuit is supplied by L and C. Can we not work it as initial conditions with the transfer function at t<0? **Don't know**, lets follow the engineers method. *So its getting a little complex for me, this is the skill under the same section and then 2 corresponding examples.*

Something is different, sort of an addition, to what we already know.
Slight adjustments, which at first reading I missed till later when I got it.
So, you may do better here. There are two cases which will be presented with an example each.

Hyat & Kemerly page 361:

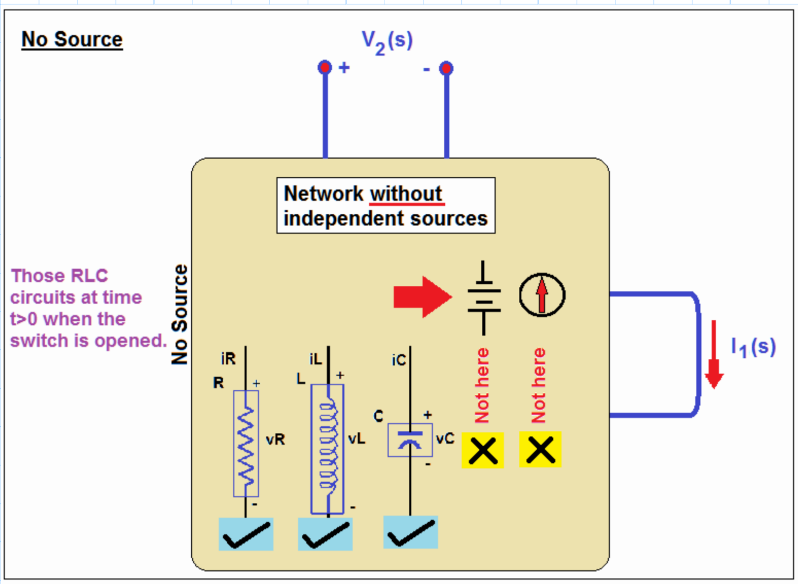
If the network already contains a source, Vs or Is, that source may be set equal to zero and another source inserted at a more convenient point.

Something new and different, so we may add a new source, and we learn later it has to be strategically placed in the circuit.

Hyat & Kemerly page 360:

If the natural response is desired for a network that does not contain any independent sources, then a source Vs or Is may be inserted at any convenient point, restricted only by the condition that the original network is obtained when the source is killed.

...that the original network is obtained when the ONLY inserted source is killed. There was no source in the circuit, no external source like we showed prior in the figures, and here we added the first source. See updated figure below no source before 1st source was added. WHERE DO WE ADD THE SOURCE? Later.



No independent source present external to circuit. Within the area shown cream color there is no independent source either.

Hyat & Kemerly page 360 and 361:

The corresponding transfer function is then determined and its poles specify the natural frequencies. Note that the same frequencies must be obtained for any of the many source locations possible.

We add the source, we proceed to get the transfer function, but where ever the source is placed in the circuit the transfer function must be the same, plus remember we were told when this source is killed, removed and maybe made 0, the original circuit is obtained.

Now the sentences in the paragraph in the same sequence as in the text book, just one was placed earlier to remove confusion.

If the natural response is desired for a network that does not contain any independent sources, then a source Vs or Is may be inserted at any convenient point, restricted only by the condition that the original network is obtained when the source is killed. The corresponding transfer function is then determined and its poles specify the natural frequencies. Note that the same frequencies must be obtained for any of the many source locations possible. If the network already contains a source, Vs or Is, that source may be set equal to zero and another source inserted at a more convenient point. - Hyat & Kemerly.

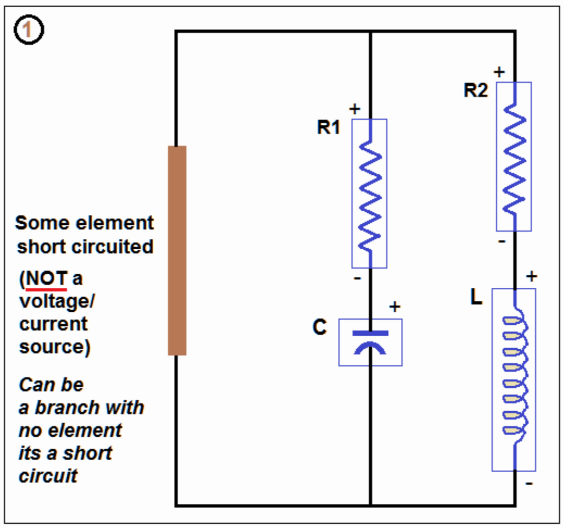
2 Cases:

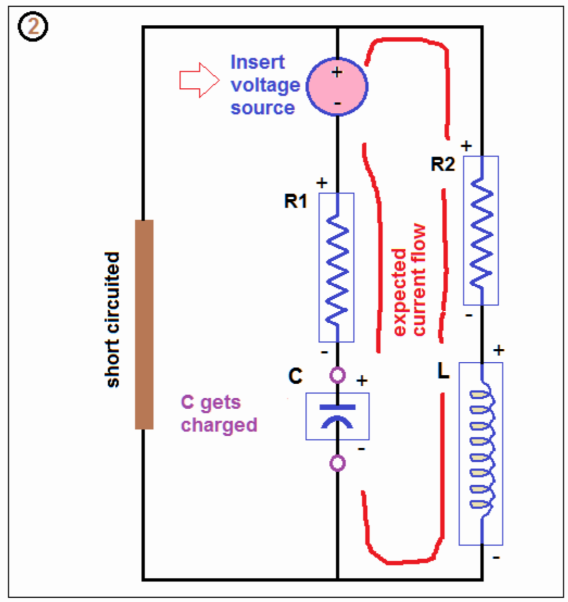
Before we can start with the examples, we need to take into consideration 2 cases in context to this skill. First is the electric circuit network contains 2 or more parts that are isolated from each other, we see a figure on this. This is on the electrical circuitry. Second is the transfer function has the same values for multiple poles, so here we got example s2=s4=s8, three ploes with same values. This is tackled best using Laplace-Transform method not attempted here you, not me, you catch up with it in Laplace. However, note here the critically damped RLC circuits, *we attempted in previous chapter*, possess a double pole, and the form of the natural response provides a clue by which the astute engineer might extrapolate and guess the results that you shall obtain later. Extrapolate what? The poles maybe? Thats the only unknown we seek after getting s we proceed to the 'form of the natural response expression' and here we plug in the values of s.

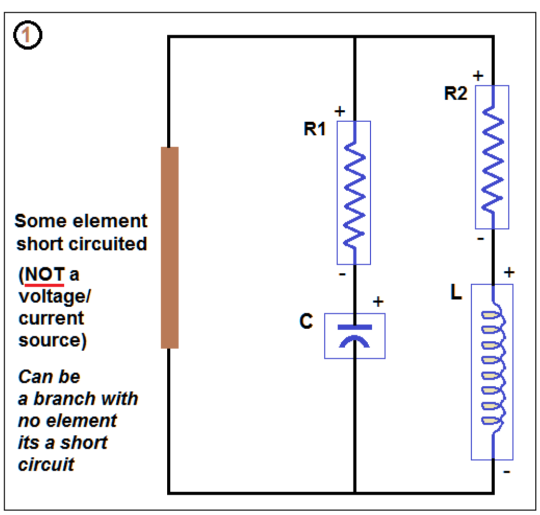
Attempt to show the case 1 split into 2 circuits on next page with respect to the circuitry. We have one without external source and other with external source. *WHERE DO WE ADD THE SOURCE? Later.<--- This is next.*

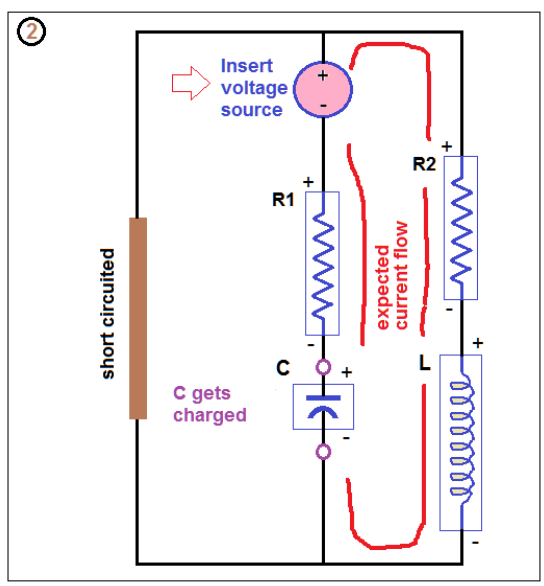
Case 1 A: Without external source and place source in network (branch).

Parallel combination of 3 networks (branches). R1 in series with C, R2 in series with L, and a short circuit. We cannot insert a voltage source at R1 in series with C branch (network) because the R2 and L branch will not get current. Instead place it at R2 L branch. Ok, something like that on where to place the source.



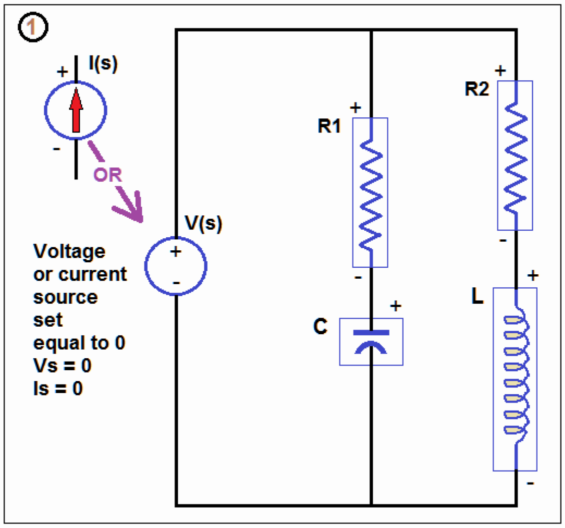


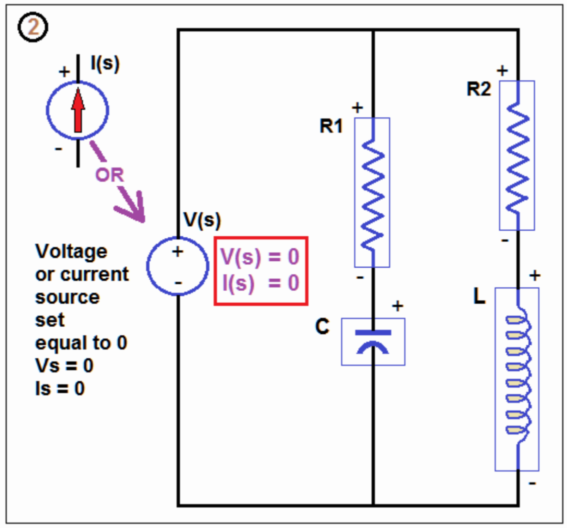


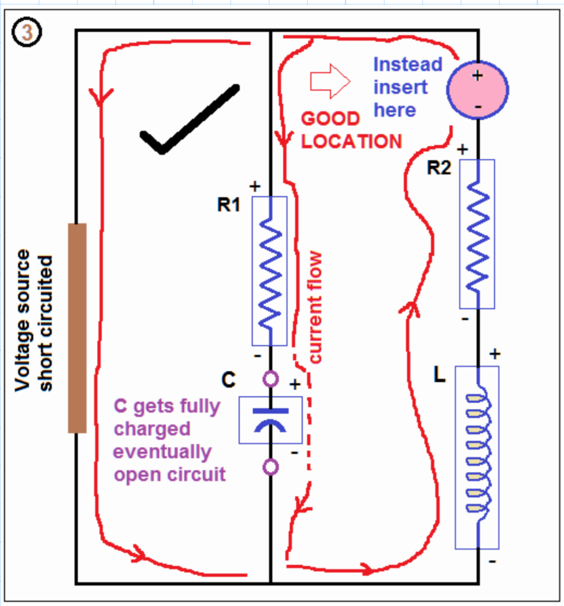


Case 1 B: With external source and place source in network (branch).

Like case 1A but we add the external source and a source in the branch (network).

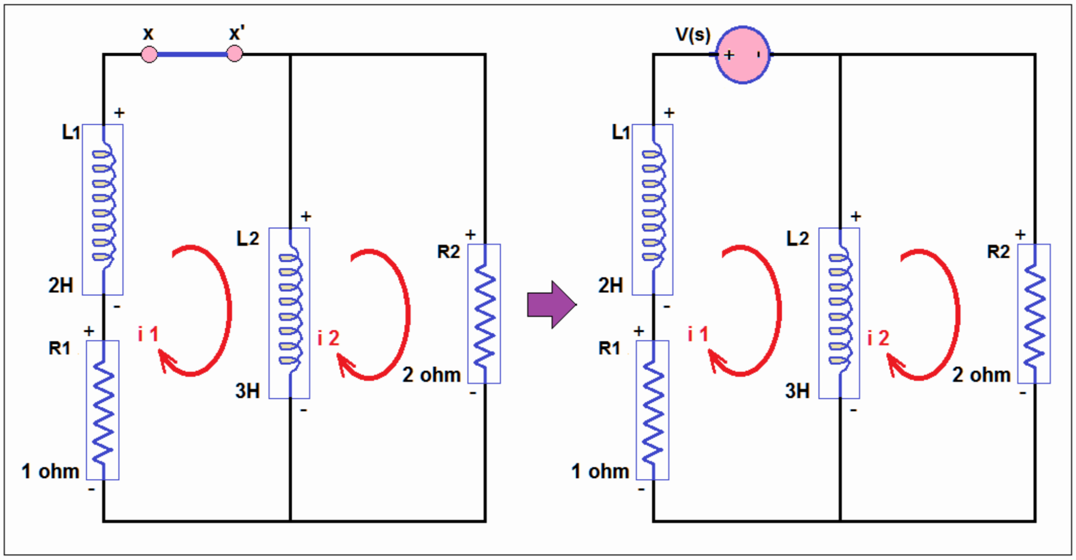






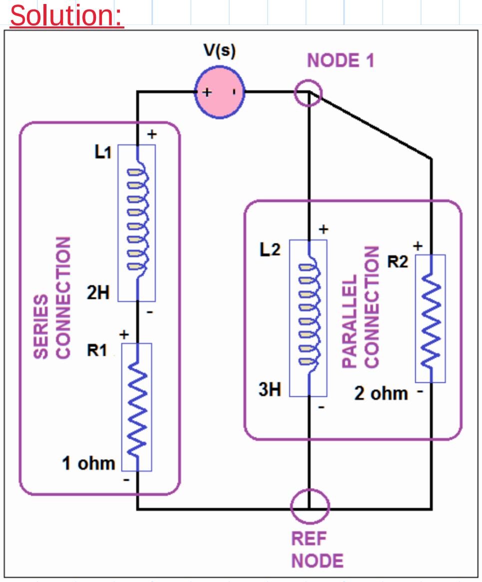
This was my understanding of it. Next we got the engineer examples. These I hope break it down for me leaving me without guess work. Thats usually my understanding of examples, sometimes you get non-engineers acting like....#1's and these dont know what they are doing in industry and in publishing. Fortunately Hyat and Kemmerly wrote this one for me! *You got a better book you hold on to it, here NOT interested.*

Example 1 RL Circuit Without Independent Source:



Problem:
The circuit to the left is source free.
Given intial conditions: i1(0) = i2(0) = 11A.
Install a voltage source Vs between points x and x'. Circuit to the right.
Find expressions for i1 and i2 for t>0.
Find the transfer function H(s) = I1(s)/Vs. Which also happens
to be the input admittance seen by the voltage source Vs.

Solution:



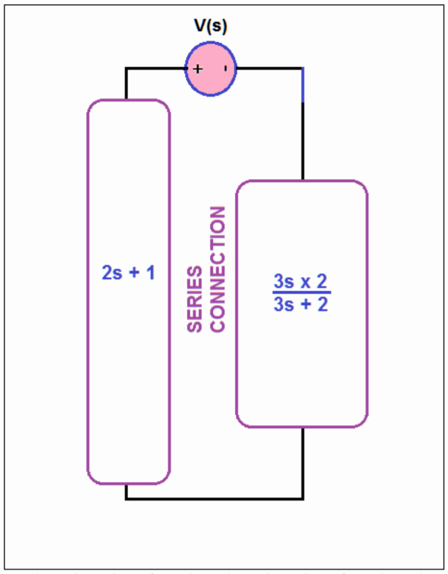
Our circuit now looks like this.

We need to calculate the impedance seen by the voltage source.

It has to be impedance seen by voltage source because our transfer function starts with H(s) = Vs/Z(s).

Vs in the denominator requires the circuit impedance Z(s) reflect on Vs.

R1	=	1	L1	=	2 s
R2	=	2	L2	=	3 s



$$H(s) = \frac{V_s}{Z_{eq}}$$

$$Z_{parallel} = \frac{(3\,s \cdot 2)}{3\,s+2}$$

$$Z_{series} = 1+2\,s$$

$$Z_{eq} = (1+2\,s) + \frac{(3\,s \cdot 2)}{3\,s+2}$$

$$H(s) = \frac{V_s}{Z_{eq}} = \frac{V_s}{(1+2\,s) + \frac{(3\,s \cdot 2)}{3\,s+2}}$$

Multiply by numerator and denominator by (3s + 2):

$$H(s) = \frac{(3\,s+2)\cdot V_s}{(1+2\,s)\,(3\,s+2) + \frac{(3\,s+2)\cdot(3\,s\cdot 2)}{(3\,s+2)}}$$

$$= \frac{(3\,s+2)\cdot V_s}{(1+2\,s)\,(3\,s+2)+(6\,s)} = \frac{(3\,s+2)\cdot V_s}{\left(3\,s+2+6\,s^2+4\,s\right)+(6\,s)}$$

$$H(s) = \frac{(3\,s+2)\cdot V_s}{\left(6\,s^2+13\,s+2\right)}$$

Sorting the numerator: $(3\,s+2)$ divide by 3 = $s+\frac{2}{3}$

We have unity coefficient for s. Same for denominator now.

$$\frac{1}{3}\cdot\left(6\,s^2+13\,s+2\right) = \left(2\,s^2+\frac{13}{3}\,s+\frac{2}{3}\right)$$

$$H(s) = \frac{\left(s+\frac{2}{3}\right)}{\left(2\,s^2+\frac{13}{3}\,s+\frac{2}{3}\right)}$$

Next divide by 2 the numerator and denominator.

$$H(s) = \frac{\left(\frac{1}{2}\right)\left(s+\frac{2}{3}\right)}{\left(s^2+\frac{13}{6}s+\frac{2}{6}\right)}$$

Numerator is ok since s is unity coefficient with multiplier (1/2), we can have H(s) = k(Numerator/Denominator). k here is 1/2.

<--- Fix the denominator factor it.

$$\left(s^2+\frac{13}{6}s+\frac{2}{6}\right) = \left(s^2+2\frac{1}{6}+\frac{2}{6}\right) = (s+2)\left(s+\frac{1}{6}\right) = s^2+\frac{1}{6}s+2s+\frac{2}{6}$$

$$= s^2+2\frac{1}{6}s+\frac{2}{6}$$

Done, by my own? I wouldn't had got it. Maybe the engineer picked one where he had the factor sorted first. I will remember that when I am writing a book.

Continuing

$$H(s) = \frac{\left(\frac{1}{2}\right)\left(s+\frac{2}{3}\right)}{(s+2)\left(s+\frac{1}{6}\right)}$$

<--- We got zeros and poles. We moslty are concerned with pole. So its a exponential expression why? Because the source is NOT in the circuit, placed a fake one just so to help in circuit analysis steps.

Poles: $(s+2)\left(s+\frac{1}{6}\right)$

$s1 = -2 \qquad s2 = \frac{-1}{6}$

$$i(t) = Ae^{-2t} + Be^{-\left(\frac{1}{6}\right)t}$$

The general form of the current i(t) expression based on poles.

We have to work each mesh/loop current in a full set of calculation each, like we did for RLC circuit in previous part.

Given intial conditions: i1(0) = i2(0) = 11A.

$i_1(0) = 11$ and $i_2(0) = 11$ Proceed with i1(t) first.

$$i_1(t) = Ae^{-2t} + Be^{-\frac{1}{6}t}$$

$11 = A + B$ at t=0 Equation A (As in textbook)

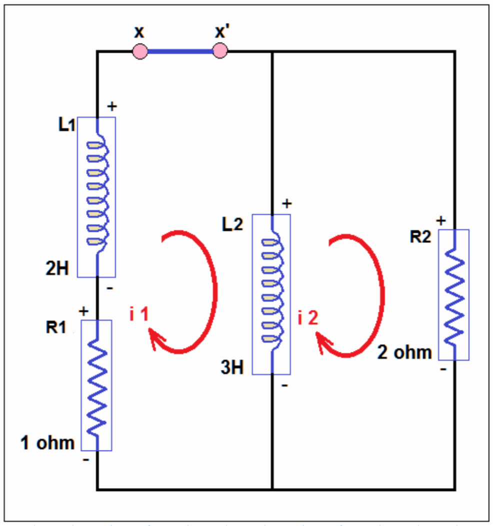
Next step? Remember, we done a differention and we got results, starts on page 43 Part 3A.

$$i_1(t) = Ae^{-2t} + Be^{-\frac{1}{6}t}$$ <--- Differentiate RHS

$$\frac{d(i_1(t))}{dt} = -2\,Ae^{-2t} - \frac{1}{6}\,Be^{\left(-\frac{1}{6}\right)t}$$

$$i_1(0) = 11$$ <--- Differentiate LHS.

$$\frac{di_1(0)}{dt} =$$ Solve this.



We use this circuit layout.

Mesh i1:

$$R1\,i_1 + L1\left(\frac{di_1}{dt}\right) + L2\left(\left(\frac{di_1}{dt}\right) - \left(\frac{di_2}{dt}\right)\right) = 0$$

Mesh i2:

$$R2\,i_1 + L2\left(-\left(\frac{di_1}{dt}\right) + \left(\frac{di_2}{dt}\right)\right) = 0$$

Mesh i1:

$$R1\,i_1 + (L1 + L2)\cdot\left(\frac{di_1}{dt}\right) - L2\left(\frac{di_2}{dt}\right) = 0$$

$$(L1 - L2)\cdot\left(\frac{di_1}{dt}\right) + L2\left(\frac{di_2}{dt}\right) = -R1\,i_1(0) = -1\cdot 11 = -11$$

$$L1\cdot\left(\frac{di_1}{dt}\right) + L2\left(\frac{di_1}{dt} - \frac{di_2}{dt}\right) = -11$$ Eq 1

Mesh i2:

$$R2\,i_1 + L2\left(-\left(\frac{di_1}{dt}\right) + \left(\frac{di_2}{dt}\right)\right) = 0$$

$$L2\left(-\frac{di_1}{dt} + \frac{di_2}{dt}\right) = -R2\,i_1(0) = -2\cdot 11 = -22$$ Eq 2

We place the 2 equations here:

$$L1 \cdot \left(\frac{di_1}{dt}\right) + L2\left(\frac{di_1}{dt} - \frac{di_2}{dt}\right) = -11 \quad \text{Eq 1}$$

$$L2\left(-\frac{di_1}{dt} + \frac{di_2}{dt}\right) = -22 \quad \text{Eq 2}$$

Add eq 1 and 2 :

$$L1 \cdot \left(\frac{di_1}{dt}\right) = (-11 - 22) = -33$$

$$\left(\frac{di_1}{dt}\right) = -\frac{33}{L1}$$

$$\frac{di_1}{dt} = -\frac{33}{2}$$

<--- This is what we were looking for
this value gets placed in the equation.

$$\frac{d\left(i_1(t)\right)}{dt} = -2\ Ae^{-2t} - \frac{1}{6}\ Be^{\left(-\frac{1}{6}\right)t}$$

$$-\frac{33}{2} = -2\ Ae^{-2t} - \frac{1}{6}\ Be^{\left(-\frac{1}{6}\right)t}$$

We work this equaion

$$-\frac{33}{2} = -2\ A - \frac{1}{6}\ B \quad \text{at } t=0 \quad \text{Eq B}$$

We got 2 equations 2 unknowns lets solve them.

$$11 = A + B \quad \text{Eq A}$$

$$-\frac{33}{2} = -2\ A - \frac{1}{6}\ B \quad \text{Eq B}$$

Eq A times 2:

$$22 = 2\ A + 2\ B \quad \text{Eq C}$$

Eq C add to eq B:

$$22 - \frac{33}{2} = 0\ A + \left(2\ B - \frac{1}{6}\ B\right) = \frac{11}{6}\ B$$

$$\frac{44-33}{2} = \frac{11}{6}B$$

$$\frac{11}{2} = \frac{11}{6}B$$

$$B = \left(\frac{6}{11}\right)\cdot\left(\frac{11}{2}\right) = 3$$ Answer.

$11 = A + B$ Substitute B=3

$11 = A + 3$

$A = 11 - 3 = 8$ Answer.

I made some time consuming errors on the mesh equations, and it was my sign on the resistor voltage that was incorrect. Plus the values in the steps were so close that it took time to catch the error. Seemed like something was opposite thats what I meant by values of the solution were not matching up, so to fix I looked for an opposite. That took time almost to build some empires on this planet! Troubling. So now thats fixed.

Now on creating the time domain function for i1.

$$i(t) = Ae^{-2t} + Be^{-\left(\frac{1}{6}\right)t}$$ The general form of the current i(t) expression based on poles.

$$i1(t) = 8e^{-2t} + 3e^{-\left(\frac{1}{6}\right)t}$$ Answer.

Next on the steps to getting the time domain function for i2(t).

We solved for di1(t)/dt we do the same for di2(t)/dt. The equation form i(t) = A e ^-2t + Be^-6t, this is used again we need the 2nd equation, from differentiation term, to solve for i2(t).

$$i_2(0) = 11$$

Mesh i2:

$$L2\left(-\frac{di_1}{dt} + \frac{di_2}{dt}\right) = -R2\ i_2(0)$$

$$L2\left(-\frac{di_1}{dt} + \frac{di_2}{dt}\right) = -2\cdot 11 = -22 \quad \text{Eq 2}$$

Substitue values to solve for di2/dt.

$$3\left(-\left(-\frac{33}{2}\right)\right)+3\left(\frac{di_2}{dt}\right) = -22 \quad \text{--->} \quad 3\left(\frac{di_2}{dt}\right) = -22-\frac{99}{2}$$

$$\left(\frac{di_2}{dt}\right) = \frac{1}{3}\cdot\left(-22-\left(\frac{99}{2}\right)\right)$$

$$\frac{di_2}{dt} = -\frac{22}{3}-\left(\frac{33}{2}\right) = \frac{-(22\cdot 2)-(33\cdot 3)}{6} = \frac{-143}{6}$$

$$\frac{d\left(i_2(t)\right)}{dt} = -2\ Ae^{-2t}-\frac{1}{6}\ Be^{\left(-\frac{1}{6}\right)t}$$

$$\frac{-143}{6} = -2\ A-\frac{1}{6}\ B \qquad \text{Eq D} \qquad \text{<--- When t=0}$$

Now we solve for the coefficients of A and B for i2(t).
We have the 1st equation already done.

$$11 = A+B \qquad \text{Eq A}$$

$$\frac{-143}{6} = -2\ A-\frac{1}{6}\ B \qquad \text{Eq D}$$

Eq A times 2 then add to eq D.

$$22 = 2\ A+2\ B \qquad \text{Eq A}$$

$$\frac{-143}{6} = -2\ A-\frac{1}{6}\ B \qquad \text{Eq D}$$

$$22\cdot 6 = 132$$

Add I got:

$$\frac{132-143}{6} = 0+\frac{11}{6}\ B$$

$$-\frac{11}{6} = \frac{11}{6}\ B$$

$$B = -1$$

$$11 = A+B \qquad \text{Eq A, substitute and solve for A}$$

$$A = 11-B = 11-(-1) = 12$$

$$A = 12 \qquad B = -1$$

Now on creating the time domain function for i2.

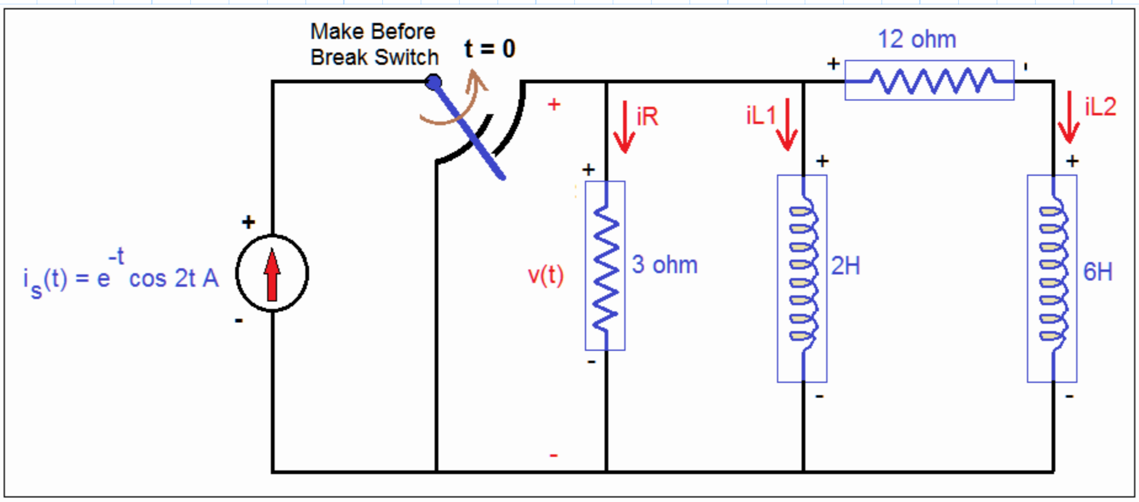
$i(t) = Ae^{-2t} + Be^{-\left(\frac{1}{6}\right)t}$ The general form of the current i(t) expression based on poles.

$i2(t) = 12\,e^{-2t} - 1\,e^{-\left(\frac{1}{6}\right)t}$

$i2(t) = 12\,e^{-2t} - e^{-\left(\frac{1}{6}\right)t}$ Answer.

Comments: Again I made some sign (-ve/+ve) errors and that was adequate time to bring down some major historical empires.

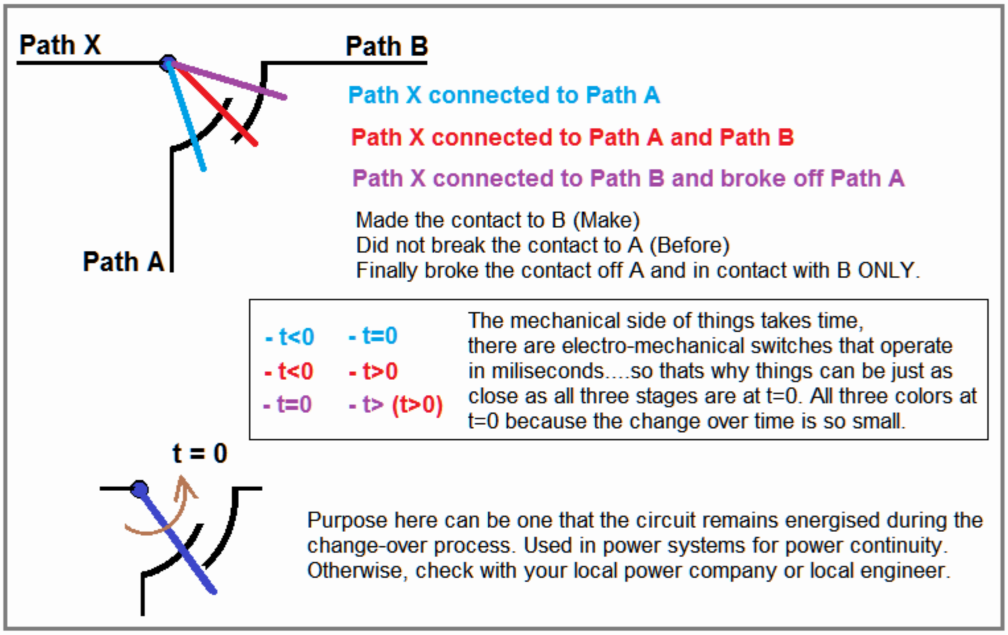
Example 2 RL Circuit With Added Independent Source:



Problem:
The make before break switch ensures that all currents and voltages to the right of the switch are initially at zero. Logical because the swith had not made contact to the right when t<0.

When switch moves-up voltage across the 3 Ohm resistor is to be found?

First whats a make before break switch? See figure below. My idea here, you check with your engineer/lecturer. I see this switch not needed here because at t=0 we do not want to energise the right side of the circuit, which the engineers said want it to be zero for voltage and current. With the make and break at t=0 the right and left side are exposed to the same source for a short instant before the left is separated off. Maybe that time interval is negligible, so why have it. The usual switch would do.



Lets put that figure aside and start with the solution. *I was not going to check on the internet.*

Solution:

This circuit has a source, its a current source (left side of switch).

We are told we have a forced response due to the current source and a natural response.

I do not now yet at this time why we have a natural response because the source is not removed. Natural response is due to no source present and the circuit discharges. Maybe the thinking on the swtich is not correct lets find out! *All said and done I am not the engineer here.*

Whats the first thing we need to do?

Form the expression for impedance, then form the transfer function, then seek the poles and zeros,..........

$R1 = 3 \qquad R2 = 12 \qquad L_1 = 2\ s \qquad L_2 = 6\ s$

$Z_1 = 12 + 6\ s$

$$Z_2 = \frac{(12+6\ s)\cdot 2\ s}{(12+6\ s)+2\ s} = \left(\frac{24\ s + 12\ s^2}{12 + 8\ s}\right)$$

$$Z_{eq} = \frac{3\cdot\left(\frac{24\ s+12\ s^2}{12+8\ s}\right)}{3+\left(\frac{24\ s+12\ s^2}{12+8\ s}\right)}$$

<---Monster! This a little difficult to sort things out, I tried some, I could not come up with a descent factor.

The engineer resorted to the defintion of parallel resistance instead. They did this. The basic one, which I said no need waste time further thats what? I will do!

$$\frac{1}{Z_{eq}} = \frac{1}{\left(\frac{1}{Z_1}\right)} + \frac{1}{\left(\frac{1}{Z_2}\right)} + \frac{1}{\left(\frac{1}{Z_3}\right)} \ \ldots\ldots\ \frac{1}{\left(\frac{1}{Z_N}\right)}$$

$Z_1 = 12 + 6\ s$

$$Z_2 = \frac{1}{2\ s} + \frac{1}{12 + 6\ s}$$ This is a parallel connection.

$$Z_{eq} = \frac{1}{3} + \frac{1}{2\ s} + \frac{1}{6\ s + 12}$$

We got this down but the hard part is factoRING---> Used in calculation below.

$$\frac{6\ s}{2\ s+3} = \frac{6\ s}{2\ s} + \frac{6\ s}{3} = 3 + 2\ s$$

$$= \frac{\left(\frac{6\ s}{2\ s+3}\right)\cdot(6\ s+12)}{\left(\frac{6\ s}{2\ s+3}\right)+(6\ s+12)} = \frac{(6\ s)\cdot(6\ s+12)}{(6\ s)+(2\ s+3)\ (6\ s+12)}$$

$$= \frac{\left(36\ s^2 + 72\ s\right)}{(6\ s)+\left(12\ s^2+24\ s+18\ s+36\right)} = \frac{\left(36\ s^2+72\ s\right)}{\left(12\ s^2+48\ s+36\right)}$$

$$= \frac{\left(3\ s^2+6\ s\right)}{\left(1\ s^2+4\ s+3\right)}$$

$$Z_{eq} = \frac{3\ s\,(s+2)}{(s+1)\ (s+3)}$$

Ok we can live with this.
What do I think? Sure they had factors fixed first.
Then worked the circuit. Puts the Math back in business!
You got your principles I got my guesses - Karl Bogha.

$$H(s) = \frac{V(s)}{I(s)} = \frac{3\,s\,(s+2)}{(s+1)\,(s+3)}$$

Zeros: $s = 0 \quad s = -2$
Poles: $s = -1 \quad s = -3$
At this time we are interested in poles.

From the poles, thus far, on the natural response we can say its an exponential expression for the form of the natural response.

$$v_n(t) = Ae^{0\,t} + Be^{-2\,t} = A + Be^{-2\,t}$$

$$v_n(t) = Ae^{-t} + Be^{-3\,t}$$

This is cleaver one on how to find the forced response. *I remember something like this in a previous example/exercise, where we set V(s) to a zero to solve for I(s). Maybe not.*

$$H(s) = \frac{V(s)}{I(s)} = \frac{3\,s\,(s+2)}{(s+1)\,(s+3)}$$

$$V(s) = H(s) \cdot I(s)$$

$i_s(t) = e^{-t} \cos(2\,t)$ <---Equivalent frequency domain?

$\sigma = -1$

$\omega = 2\,j\,\omega$

$s = -1 + j2$

Where in terms of s (sigma+jw) will s be for I(s) to equal 1?

When t = 0 i_s(t) ? $i_s(t) = e^{-t} \cos(2\,t) = e^{-0} \cos(0) = 1$

Ok we sorted where it equal 1 for frequency domain too.
And here for the frequency domain what is s equal to?

$I(s) = Ae^{(-1+2\,j\,\omega)\,t}$ I(s) = 1 where? at s = -1+2j. <--- Here.

$1 = 1\,e^{(-1+2\,j\,\omega)\,t}$

Next step set V(s) equal I(s) multiplied to the input impedance Z_eq? Eventually.

$V(s) = I(s)\,Z_{eq}(s)$ <--- We substitute s=-1+2j in the transfer function H(s) and I(s).

$$H(s) = \frac{V(s)}{I(s)}$$

$$V(s) = I(s)\ H(s)$$

Here we do not multiply I(s) to H(s).
RATHER, we substitute s for -1+2j.
AND I(s) = 1 in time domain when t=0.
Discussion: So, thats why we said I(s) = 1 so it dont change nothing in terms of proportional value of V(s)? NO. It could be i(t) at t=0 may equal 153, just so happens 1 here. Does raise question if it need be 1 with respect to coefficient. Reasonable question. Just so happen 1, it can be 234.

Now I see the smartness of the complex plane method revealing itself in the transfer function.

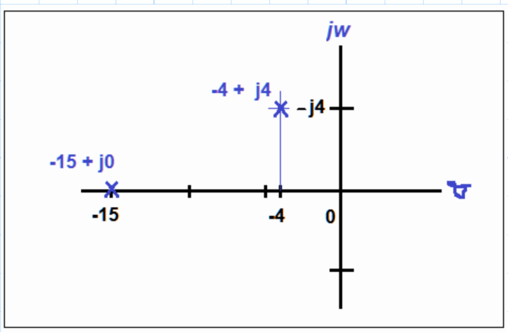
$$H(s) = \frac{V(s)}{I(s)} = \frac{3\ s\ (s+2)}{(s+1)\ (s+3)}$$

$$s = -1 + j2$$

$$V(s) = I(s)\ H(s) = \frac{3\ (-1+j2)\ (-1+j2+2)}{(-1+j2+1)\ (-1+j2+3)}$$

$$= \frac{3\ (-1+j2)\ (1+j2)}{(j2)\ (2+j2)} = \frac{3\ \left(-1-j2+j2+j^2\ 4\right)}{\left(j4+j^2\ 4\right)}$$

$$V(s) = \frac{3\ (-1-4)}{(j4-4)} = \frac{-15+j0}{(-4+j4)}$$



<--- V(s) numerator and denominator shown in figure.
It is **NOT** the vector between these two points we seek. I say this based on the answer's 45 deg angle, and I did a check. You check.
Next step is a phasor division.

Convert to phasors: $-15 + j0 = 15\angle 0$ deg

Could been 180 deg? 180 deg is flat, for the calculation its 0 deg, relative to the x-axis.

$$-4 + j4 = \left|\sqrt{(-4)^2 + (4)^2}\right| = 5.657$$

$$\text{ang}(-4 + j4) = \text{atan}\left(\frac{-4}{4}\right) = -45 \text{ deg}$$

$$V(s) = \frac{15\angle 0}{5.657\angle -45} = 2.652\angle 45 \text{ deg} \quad \text{OR} \quad 1.875\cdot\sqrt{2}\angle 45$$

$V(s) = 1.875 \cdot \sqrt{2} \angle 45$ Engineers used this form for V(s) here.
Sqrt(2) is a? Magic Number.

Sqrt(2)? Makes some calculations easier. Shows a little smarter edge. Better Smarter Engineer!
Its more abstract mystical sets you apart from the typical engineer - Karl Bogha.

Now we can convert this to the time domain.

Magnitude: $1.875 \cdot \sqrt{2}$

Phase angle: 45 deg

Our current time domain form will set the precedence or lets say
set the form our voltage will take, we know this from previous work.

$i_s(t) = e^{-t} \cos(2\ t)$ <---Current in time domain

$\sigma = -1$

$\omega = 2\ j\ \omega$

$s = -1 + j2$

$v_f(t) = 1.875 \cdot \sqrt{2} \cdot e^{-1\ t} \cos(2\ t + 45)$ Answer.

Remember: This is vf because we worked with a source (current source i_s(t)).

Complete response v(t): $v(t) = v_n(t) + v_f(t)$

$$v(t) = Ae^{-t} + Be^{-3\ t} + 1.875 \cdot \sqrt{2} \cdot e^{-t} \cos(2\ t + 45)$$

So next the electrical engineers want to solve for? A and B.

Most times for me going thru the thinking engineer step seems difficult,
my mind latches on to the last thing I done OR what that usual thing was, and
in this case I drew a blank. When I checked the textbook it was AGAIN the initial
condition step, setting t=0. Can you blame me? Maybe.

I(s) = 1, we say the current is 1A flowing out of the source. Since its a
sinusoidal expression the peak or maxium would be 1A. Would they had
meant the effecitve value or RMS? Usually they make known if it were RMS.
Either case we say the magnitude is 1A.

When the switch closes, make before break, there is 1A flowing in the Resistor 3 ohm.
The voltage v(t) is measured across this resistor's terminals. So? v_R1(0) = 1x3=3V.

$v_{R1}(0) = 1\ A \cdot 3\ Ohm = 3\ V$ This goes in the next expression.

Next we set t=0 for the v(t) expression, then evaluate it.

$$v(t) = Ae^{-t} + Be^{-3t} + 1.875\cdot\sqrt{2}\cdot e^{-t}\cos(2t+45)$$

$$v(0) = 3 = Ae^{-0} + Be^{-3\,0} + 1.875\cdot\sqrt{2}\cdot e^{-t}\cos(2\,0+45)$$

$$v(0) = 3 = A + B + 1.875\cdot\sqrt{2}\cdot\cos(45\ \text{deg})$$

$$\cos(45\ \text{deg}) = 0.707 \qquad \frac{1}{\sqrt{2}} = 0.707$$

$$v(0) = 3 = A + B + \frac{1.875\cdot\sqrt{2}}{\sqrt{2}}$$

Very common for the EE engineers to use Sqrt(2), and we saw here why. ***Elegant*** *is the reason I said, you may have your reason for it which I trust happens to be true - Karl Bogha.*

That was one equation above we are looking for the other equation.
We differentiate the v(t) term.

$$v(t) = Ae^{-t} + Be^{-3t} + 1.875\cdot\sqrt{2}\cdot e^{-t}\cos(2t+45)$$

$$\frac{d(v(t))}{dt} = -1\ Ae^{-t} - 3\ Be^{-3t} + \blacksquare \ \ldots$$

...the remaining term to differentiate $\left(1.875\cdot\sqrt{2}\right)\cdot e^{-t}\cdot\cos(2t+45)$

$$\frac{d}{dt}\left(e^{-t}\cdot\cos(2t+45)\right) = e^{-t}\cdot\frac{d}{dt}(\cos(2t+45)) + \cos(2t+45)\cdot\left(-e^{-t}\right)$$

$$\frac{d}{dt}(\cos(2t+45)) = -2\sin(2t+45)$$

$$= -2e^{-t}\cdot\sin(2t+45) - e^{-t}\cdot\cos(2t+45)$$

$$= -e^{-t}\cdot(2\cdot\sin(2t+45) + \cos(2t+45))$$

$$= \left(1.875\cdot\sqrt{2}\right)\cdot\left(-e^{-t}\cdot(2\cdot\sin(2t+45) + \cos(2t+45))\right)$$

When t=0:

$$= \left(1.875\cdot\sqrt{2}\right)\cdot\left(-e^{-0}\cdot(2\cdot\sin(2\,0+45) + \cos(2\,0+45))\right)$$

$$= \left(1.875\cdot\sqrt{2}\right)\cdot(-1\cdot(2\cdot\sin(45) + \cos(45)))$$

$$= \left(1.875\cdot\sqrt{2}\right)\cdot\left(-1\cdot\left(2\cdot\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\right)\right)$$

$$= \left(1.875 \cdot \sqrt{2}\right) \cdot \left(-1 \cdot \left(2 \cdot \left(\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\right)\right)$$

$$= \left(1.875 \cdot \sqrt{2}\right) \cdot \left(-\left(\sqrt{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\right)$$

$$= (1.875) \cdot (-(2) - (1))$$

$$= (1.875) \cdot (-3)$$

$$= -5.625$$

Returning to whole expression

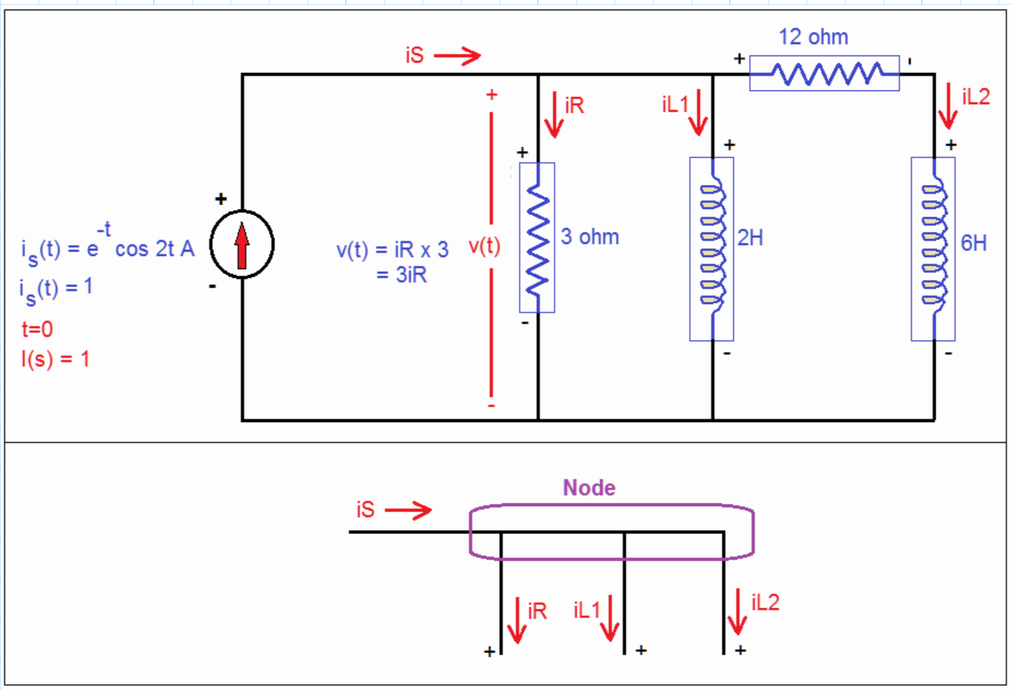
$$\frac{d(v(t))}{dt} = -1\ Ae^{-t} - 3\ Be^{-3t} - 5.625$$

... and setting t = 0 for the remaining terms.

$$\frac{d(v(t))}{dt} = -A - 3\ B - 5.625$$

We are almost near our 2nd equation but we need a value for? d(v(t))/dt.

Review and work the figure below. *Fix it if you find errors.*



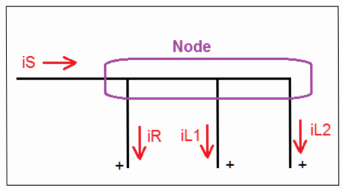
I prefer to use the engineers wording here from the textbook on the next step, the general sketch of it from previous figure.

The initial value of this rate of change, (dv/dt), is obtained by analysing the circuit. However, those rates of change which are most easily found are the derivatives of the inductor currents, for v = L(di/dt), and the initial values of the inductor voltages should not be difficult to find. We therefore express the response v(t) in terms of the resistor current,

$$v(t) = 3\, i_R$$ voltage across 3 Ohm resistor

and then apply Kirchoff's current law

$$i_s = i_R + i_{L1} + i_{L2}$$



Cleaver step involved next.
We have v(t) ie the resistor 3 ohm volate is 3i_R.
We turn that expression into voltage but multiply 3 to the other terms as well.

$$3\, i_s = 3\, i_R + 3\, i_{L1} + 3\, i_{L2}$$ Nothing changed relationship wise between one term and the next.

However, $3\, i_R$ is the voltage expression we sought across 3 Ohm resistor.

$$3\, i_s = v(t) + 3\, i_{L1} + 3\, i_{L2}$$ Next re-arrange

$$v(t) = 3\, i_s - 3\, i_{L1} - 3\, i_{L2}$$ This is what textbook has.

Now we take the derivative of the expression above, since we know inductor voltage is L(di/dt), so taking the derivative of the current i terms.

$$\frac{dv(t)}{dt} = 3\,\frac{di_s}{dt} - 3\,\frac{i_{L1}}{dt} - 3\,\frac{i_{L2}}{dt}$$

The LHS term dv/dt is the value we seek to make our 2nd equation good.
Cleaver. I wouldnt be able to think this up myself, and have no idea how much time it took the engineers to come up with this in the early days.

In the above expression there is one term we can take advantage of and work with it, its di_s/dt, the derivative of the source current. Since, we have the expression for source current i_s(t).

$$i_s(t) = e^{-t}\cos(2\, t)$$

$$\frac{di_s(t)}{dt} = e^{-t}\cdot(-\sin(2\,t)) - 1\,e^{-t}\cdot(\cos(2\,t))$$

$$= -e^{-t}\sin(2\,t) - e^{-t}\cdot(\cos(2\,t))$$

When t = 0

$$= \quad -1\,(0) - 1\,(1)$$

$$\frac{di_s(t)}{dt} = -1$$

Now we calculate for the 3(di_s/dt) at t=0: $3\,\frac{di_s}{dt} = 3\cdot(-1)$

$$= -3$$

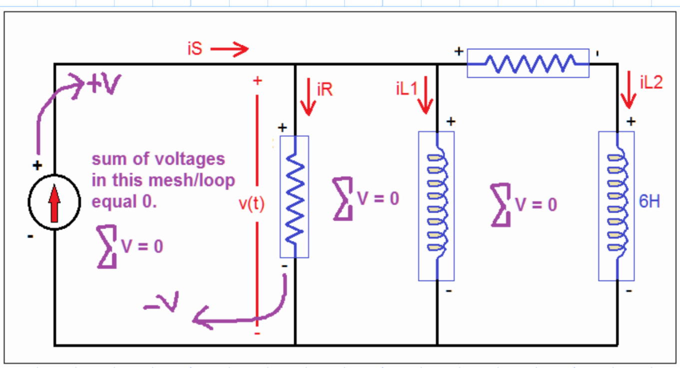
What is the unit for this term dv/dt ? V/s.
It looks like something else but the expression represents a relationship at the node, which is current, represented in V/s.

$$\frac{dv(t)}{dt} = 3\,\frac{di_s}{dt} - 3\,\frac{i_{L1}}{dt} - 3\,\frac{i_{L2}}{dt}$$

$$3\,\frac{di_s}{dt} = -3V$$

Next what is $3\,\frac{i_{L1}}{dt}$ equal?

The voltage across all the parallel branches is the same at t=0.
What could this be? -3V. So we may say vL1(0) = -3V. NOT exactly.
First apply the Sum of Voltages in a mesh/loop = 0.
So here it tells us its not -3V
RATHER +3V, so -3V + 3V = 0.
Tricky.



Since $v_{L1}(t) = L1\left(\frac{di_{L1}}{dt}\right)$

We have (di_L1/dt) we have L1, substitute them in for vL1.

$$v_{L1}(0) = 3\ V$$

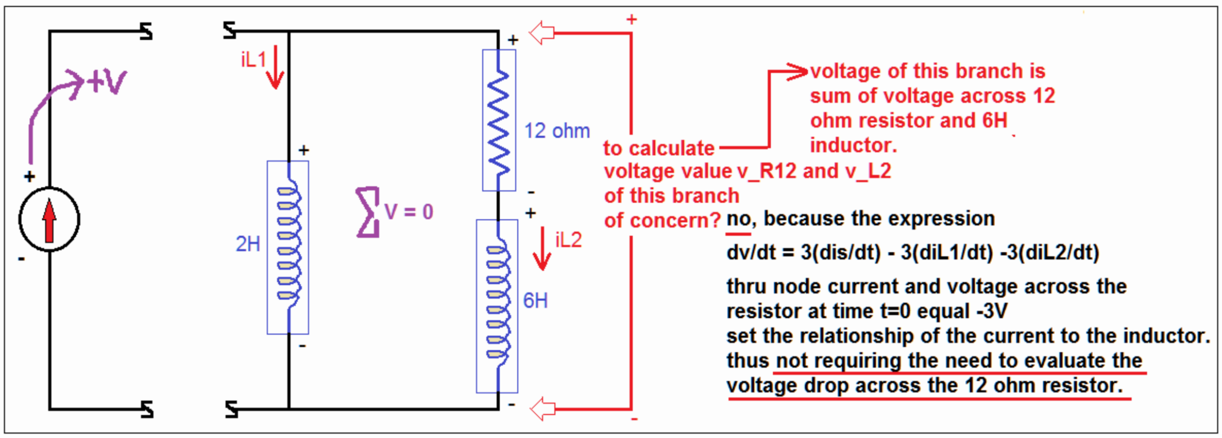
$$\left(\frac{di_{L1}}{dt}\right) = \frac{v_{L1}(t)}{L1} = \frac{3}{2}$$

Twists N Turns, Yes. What we were we looking for? $3\left(\frac{i_{L1}}{dt}\right)$

$$3\left(\frac{i_{L1}}{dt}\right) = 3\cdot\left(\frac{3}{2}\right) = 3\cdot\left(\frac{3}{2}\right) = 4.5 \quad V/s$$

We can intepret this as 3(di_L1/dt) is (3/2) of the initial voltage 3V/s across the 2H inductor, or its equal to -4.5V/s (-3di_L1/dt).

Before I proceed, there is this voltage drop across the 12 ohm resistor in the last branch where L2 is located, do I need to find that drop then adjust for the voltage across the inductor L2? NO.



Discussion: Read the discussion or explanation in the figure above.
Key point is at t=0, so for L(di/dt) time t is at t=0. Here we expect that to be the voltage across the 3 ohm resistor. Voltage across the 12 ohm resistor has to gradually increase from t=0 and here its near 0V. So we neglect it.

Next the last term in the expression : $\frac{dv(t)}{dt} = 3\frac{di_s}{dt} - 3\frac{i_{L1}}{dt} - 3\frac{i_{L2}}{dt}$

Solve for: $3\frac{i_{L2}}{dt}$

Since $v_{L2}(t) = L2\left(\frac{di_{L2}}{dt}\right)$

We have (di/dt) we have L2, substitute them in for vL2.

$$v_{L2}(t) = 3 \text{ V}$$

$$\left(\frac{di_{L2}}{dt}\right) = \frac{v_{L2}(t)}{6} = \frac{3}{6} = \frac{1}{2}$$

$$3\left(\frac{i_{L2}}{dt}\right) = 3\cdot\left(\frac{1}{2}\right) = \left(\frac{3}{2}\right) = 1.5 \quad V/s$$

We can intepret this as 3(di_L2/dt) is (1/2) of the initial voltage 3V/s across the 6H inductor, or its equal to - 1.5V/s (-3di_L2/dt).

How did we get here?

$$\frac{d(v(t))}{dt} = -A - 3\ B - 5.625$$ We were solving for d(v(t))/dt.

So we plug in the voltage (V/s) terms for dv(t)/dt.

$$3\,\frac{di_s}{dt} = -3\,V/s \qquad 3\left(\frac{i_{L1}}{dt}\right) = 4.5\quad V/s \qquad 3\left(\frac{i_{L2}}{dt}\right) = 1.5\quad V/s$$

$$\frac{dv(t)}{dt} = 3\,\frac{di_s}{dt} - 3\,\frac{i_{L1}}{dt} - 3\,\frac{i_{L2}}{dt}$$

$$\frac{dv(t)}{dt} = -3 - (4.5) - (1.5) = -9 \quad V/s$$

$$\frac{d(v(t))}{dt} = -9 = -A - 3\ B - 5.625$$

$$-3.375 = -A - 3\ B$$

Now we have the 2nd equation.

Some may say lenghty because of the explanation needed for myself, true. I say I dont want this on my test. Its a lots of memorisation on whats involved, memory is a requirement for university tests, skills test, most universities rely heavily on memorisation.
RAM and ROM and RECALL and CALCULATE....FORGET IT.
You may ***KEEP CALM AND CARRY ON ENGINEER!***
- Karl Bogha.

$$v(0) = 3 = A + B + \frac{1.875\cdot\sqrt{2}}{\sqrt{2}}$$ The 1st equation.

$$3 = A + B + \frac{1.875\cdot\sqrt{2}}{\sqrt{2}}$$

$$3 - 1.875 = 1.125 = A + B$$

Place the 2 equation next to each other and solve for A and B.

$$1.125 = A + B \qquad \text{Eq 1}$$

$$-3.375 = -A - 3\ B \qquad \text{Eq 2}$$

$$-2.25 = -2\ B \qquad \text{Add eq 1 and 2}$$

$$B = \frac{-2.25}{-2} = 1.125$$ Substitute in eq 1 for A

$$1.125 = A + 1.125$$

$$A = 0$$

Finally can we fix the time domain expression for v(t)?
Lets give it a try, we got answers so far matching the textbook.

$$v(t) = Ae^{-t} + Be^{-3t} + 1.875 \cdot \sqrt{2} \cdot e^{-t} \cos(2t + 45)$$

We plug in for A and B:

$$v(t) = 0\,e^{-t} + 1.125\,e^{-3t} + 1.875 \cdot \sqrt{2} \cdot e^{-t} \cos(2t + 45)$$

$$v(t) = 1.125\,e^{-3t} + 1.875 \cdot \sqrt{2} \cdot e^{-t} \cos(2t + 45)$$ Answer.

Final mention by the engineers their last paragraph in chapter 13 of the textbook.

The process which we must pursue to evaluate the amplitude coefficients of the natural response is a detailed one, except those cases where the initial values of the desired response and its derivatives are obvious. However, we should not lose sight of the ease and rapidity with which the form of the natural response can be obtained. - *Hyat & Kemerly page 364 of 4th edition.*

This brings to end most of chapter 13 from Engineering Circuit Analysis titled Complex Frequency.

Part 3B comes to end here and picks up with Part 3C where Schaums Solved Problems (Partial Solutions) and Unsolved Supplementary Problems.

Part 3D will start with Op-Amps, then the only remaining section 13.8 of Chapter 13 (Hyat & Kemerly) and sections of Chapter 8 in Schaums.
Followed by some relevant solved examples.

Apologies for any errors and omissions.

End of Part 3B.