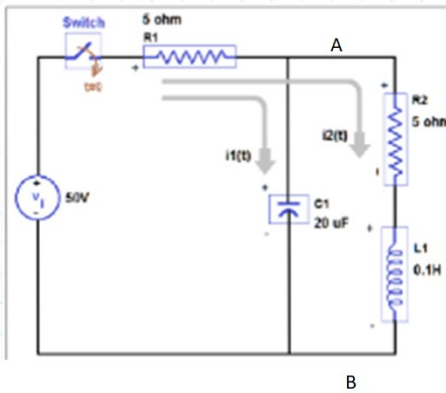


Problem 8.27 (Mesh RLC circuit) :

In the two mesh circuit of the circuit provided, the switch is closed at $t=0$. Find i_1 and i_2 for $t>0$.



$$\begin{aligned}
 R1 &:= 5 && \text{Ohm} \\
 R2 &:= 5 && \text{Ohm} \\
 C1 &:= 20 \cdot 10^{-6} && \text{F} \\
 L1 &:= 0.1 && \text{H} \\
 \frac{1}{C1} &= 50000
 \end{aligned}$$

$$\begin{aligned}
 R1 &:= 5 \ \Omega \\
 R2 &:= 5 \ \Omega \\
 C1 &:= 20 \ \mu\text{F} \\
 L &:= 0.1 \ \text{H} \\
 V_s &:= 50 \ \text{V}
 \end{aligned}$$

$$R1 \cdot C1 = 0.0001 \ \text{s}$$

$$\frac{L}{R1 + R2} = 0.01 \ \text{s}$$

Kirchhoff's Rules

Kirchhoff's first rule—the junction rule. The sum of all currents entering a junction must equal the sum of all currents leaving the junction.

Kirchhoff's second rule—the loop rule. The algebraic sum of changes in potential around any closed circuit path (loop) must be zero.

The voltage drop across $R1$:
$$(V_s - V_A(t)) = (I_1(t) + I_2(t)) \cdot R1 \quad (1)$$

Current through capacitor depends on rate of change of voltage.
$$I_1(t) = C1 \cdot \frac{d}{dt} V_A(t) \quad (2)$$

Voltage and current through inductor
$$V_L = L \cdot \frac{d}{dt} I_2(t) \quad (3)$$

$$V_A(t) = I_2(t) \cdot R2 + L \frac{d}{dt} I_2(t) \quad (3 a)$$

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Set up solution:

$$V_{A_0} := 0 \ \text{V} \quad I_{1_0} := 0 \ \text{A} \quad I_{2_0} := 0 \ \text{A} \quad \text{initial conditions}$$

$$Tm := 5 \ \text{sec} \quad \delta t := \frac{L}{(R1 + R2) \cdot 1000} = 0.00001 \ \text{s} \quad Ns := \frac{Tm}{\delta t} = 5 \cdot 10^5 \quad i := 1..Ns$$

$$T_i := \delta t \cdot i \quad \text{time vector}$$

$$\begin{bmatrix} V_{A_i} \\ I_{1_i} \\ I_{2_i} \end{bmatrix} := \begin{bmatrix} I_{1_{i-1}} \\ V_{A_{i-1}} + \frac{I_{1_{i-1}}}{C1} \cdot \delta t \\ V_s - V_{A_{i-1}} - I_{2_{i-1}} \cdot R1 \\ \frac{R1}{R1} \\ I_{2_{i-1}} + \left(\frac{V_{A_{i-1}} - I_{2_{i-1}} \cdot R2}{L} \right) \cdot \delta t \end{bmatrix}$$

$$\Delta V_A = \frac{I_1}{C} \quad (2)$$

$$I_1 = \frac{V_s - V_A - I_2 \cdot R1}{R1} \quad (1)$$

$$\Delta I_2 = \frac{V_A - I_2 \cdot R2}{L} \quad (3 a)$$

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Answer:

$$i_1(t) = -0.051 e^{-100 t} + 10.05 e^{-9950 t} \quad \text{A.}$$

$$I_{a1}(t) := 10.05 \cdot e^{-9950 \cdot \frac{t}{s}} - 0.051 \cdot e^{-100 \cdot \frac{t}{s}}$$

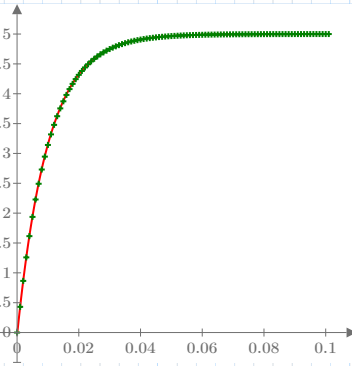
$$i_2(t) = -5.05 e^{-100 t} + 5.00 + 0.05 e^{-9950 t} \quad \text{A.}$$

$$I_{a2}(t) := 5 + 0.05 \cdot e^{-9950 \cdot \frac{t}{s}} - 5.05 \cdot e^{-100 \cdot \frac{t}{s}}$$

$$tt := 0 \text{ s}, 0.001 \text{ s} \dots 0.5 \text{ s}$$

$$ttt := 0 \text{ s}, 0.0001 \text{ s} \dots 0.5 \text{ s}$$

Current through Capacitor

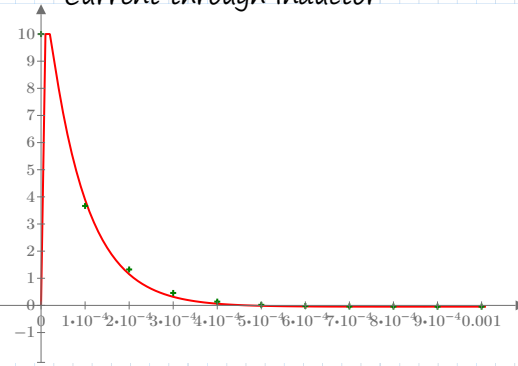


$I_2 \text{ (A)}$
 $I_{a2}(tt)$

$T \text{ (sec)}$

$tt \text{ (sec)}$

Current through Inductor

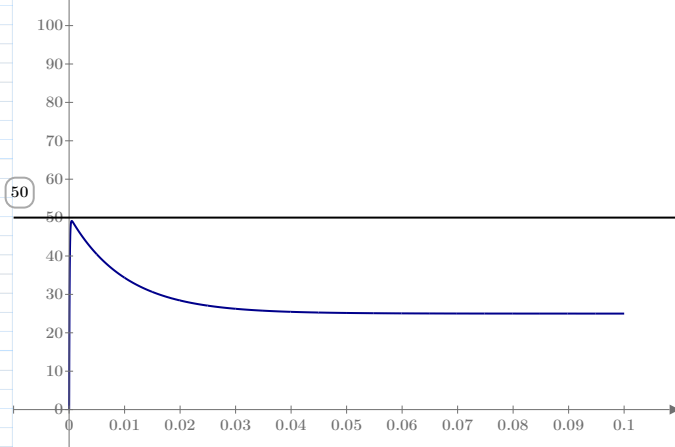


$I_1 \text{ (A)}$
 $I_{a1}(ttt)$

$T \text{ (sec)}$

$ttt \text{ (sec)}$

Voltage at point A



$V_A \text{ (V)}$

$T \text{ (sec)}$

Check DC levels

$$I_{1Ns} = 0.000 \text{ A}$$

$$I_{2Ns} = 5.000 \text{ A}$$

$$V_{ANs} = 25 \text{ V}$$

$$V_s - (I_{1Ns} + I_{2Ns}) \cdot R_1 = 25 \text{ V}$$