When saving or printing, disable Automatic Calculation.

$t0\_ := 0 \qquad \tau_{init\_} := time(t0\_) \rightarrow time(0) \qquad \tau_{init\_} = 1.618 \times 10^{9}$

WAVEFORM SPECTRA

Francesco Mezzanino

This worksheet is a collection of some common (and not), signals used in electronics. It deals with the harmonic analysis of periodic signals, satisfying the Dirichlet conditions, determined without any particular artifice to speed up the calculation but using the definition formulas. For each one first, it is plotted a graph, then is calculated its bandwidth in order to do a correct sampling of it. The Fourier harmonics and phase, are plotted in two graphs. Then, the sampled signal is rebuilt with the Shannon interpolation formula. Given the sampled signal, the fft function is applied and plotted to compare the result (first 18 functions only). The previous procedure is repeated for each signal (41).

DATA

DATA

FOURIER

FOURIER

Pulses and Priodic Formulae Only

Periodic Waveforms Periodic Waveforms Periodic Waveforms

Periodic Waveforms Formulae Only

When saving or printing, disable Automatic Calculation.

WAVEFORM SPECTRA

Francesco Mezzanino

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INTRODUCTION

The subscript "gd" is the acronym of "Global Data.xmcd"
The subscript "fs" is the acronym of "Fourier series.xmcd"
The subscript "sl" is the acronym of "Signal List.xmcd"
The subscript "dp" is the acronym of "Dirac Pulse - formulas.xmcd"

INDEX

Periodic Waveforms Frequency Spectra

PERIODIC WAVEFORMS' FREQUENCY SPECTRA

Function parameters description:

BCSA(<u>*Adimensional signal name, relative error, polinomial degree, start time, signal period*</u>)

BCSA stands for "Bandwidth Calculation and Signal Analysis"

The function returns a matrix made of three columns.

The first column contains:

pos. 0: relative error,
pos. 1: bandwidth (adimensional),
pos. 2: the nth. harmonic number corresponding tp the give relative error,
pos. 3: temporary variable,
pos. 4: Parseval,
pos. 5: signal average,
pos. 6: signal r.m.s..

The ***second column contains the coefficients a_k of the Fourier series***,
the ***third column contains the coefficients b_k of the Fourier series***.

TEST Waveforms

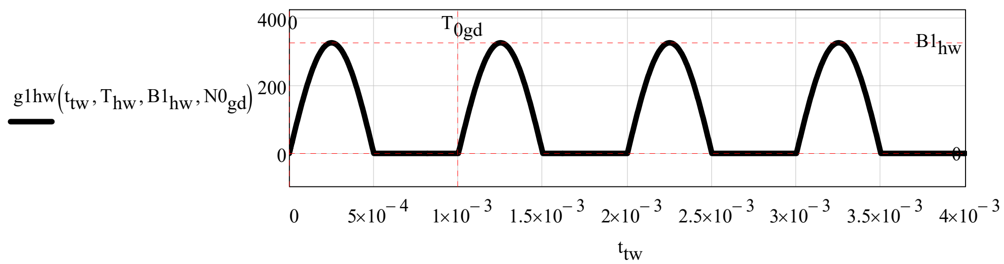
Periodic Waveforms Periodic Waveforms Periodic Waveforms

1) Half wave

Amplitude: $B1_{hw} := 230\cdot\sqrt{2}\cdot V$

$f_{hw} := \dfrac{1}{T_{0gd}}$ $T_{hw} := T_{0gd}$ Angular frequency: $\omega_{hw} := \dfrac{2\cdot\pi}{T_{0gd}}$ $T_{hw} = 1\cdot ms$

Half wave

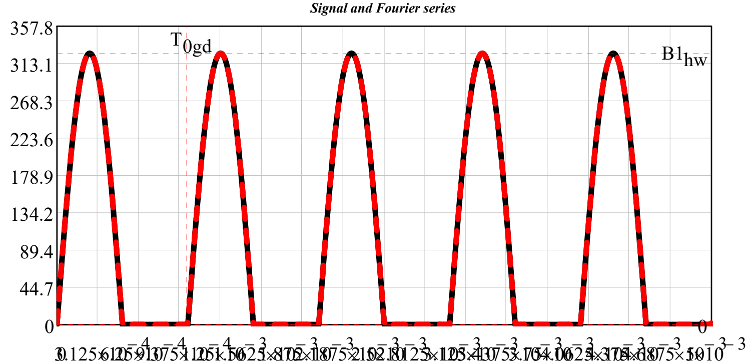


$V_{hw}(t) := g1hw(t, T_{hw}, B1_{hw}, N0_{gd})$ $B1_{hw} = 325.269\,V$

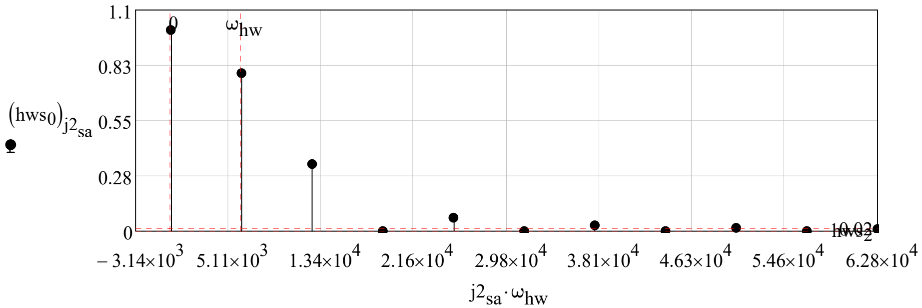
$hws := SPCT(V_{hw}, rt_{gd}, N1\_, 0\cdot sec, T_{hw})$ $N1\_ = 50$

$N_{gd} = 40$ $j2_{sa} := 0..rows(hws_0) - 1$ $(relerr) = 0.1$

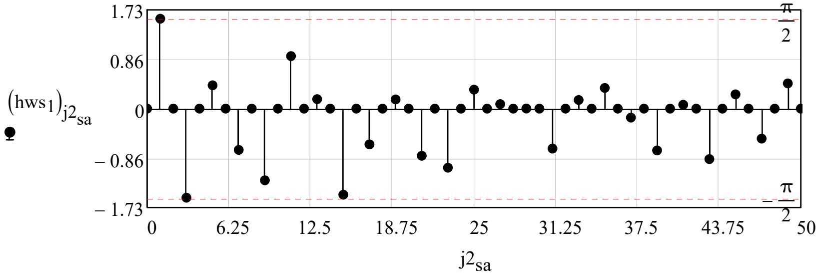
Signal and Fourier series



Signal's Amplitude Spectrum



Phase of the N1_th order Fourier Polynomial



$Bw_{sa} := hws_3 \cdot Hz \qquad Bw_{sa} = 0.019 \cdot MHz$

sampling frequency: $fpt_{so} := 2 \cdot Bw_{sa} \qquad fpt_{so} = 0.038 \cdot MHz$

$relerr := hws_7 \qquad relerr = 10 \cdot \%$

$k := 0..2^8 - 1 \qquad npt_k := \dfrac{k}{fpt_{so}}$

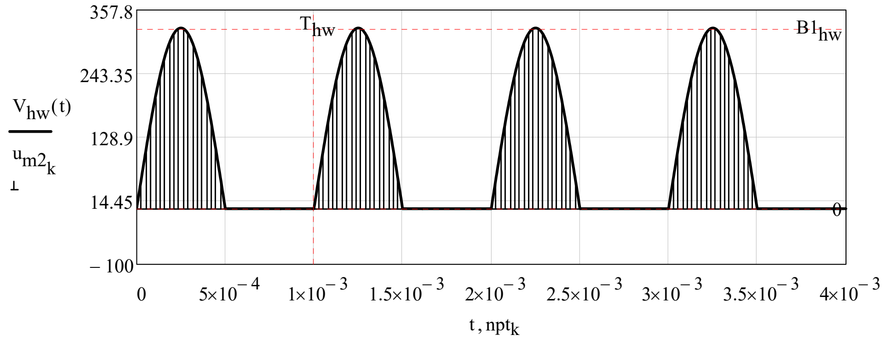
Frequency resolution: $\dfrac{N0_{gd}}{fpt_{so}} \cdot \dfrac{1}{T_{hw}} = 6.737$

Signal sampling: $u_{m2_k} := V_{hw}(npt_k)$

$u_{m2}^T =$

	0	1	2	3	4
0	0	53.538	105.615	154.811	...

Sampling



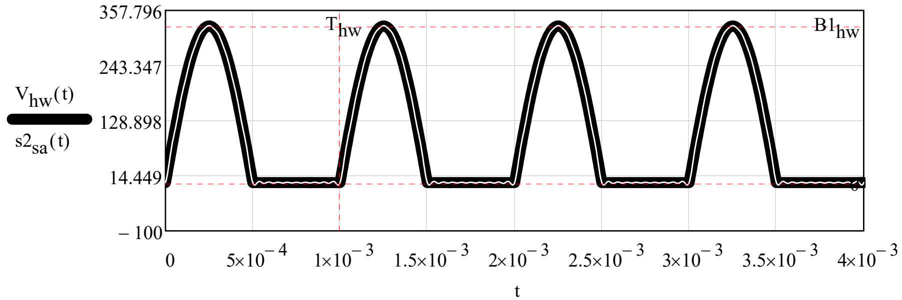
$relerr = 10 \cdot \% \qquad \omega_{bw} := 2 \cdot \pi \cdot Bw_{sa} \qquad \omega_{bw} = 0.119 \cdot \dfrac{Mrads}{sec} \qquad n \cdot \dfrac{\pi}{\omega_{bw}} = n \cdot \dfrac{1}{2 \cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula: $s2_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1} \left(u_{m2_n} \cdot sinc(\omega_{bw} \cdot t - n \cdot \pi) \right) \right] \qquad N0_{gd} - 1 = 255 \qquad u_{m2_{12}} = 297.873$

$rows(u_{m2}) = 256 \qquad relerr = 10 \cdot \%$

Signal Rebuilding from samples

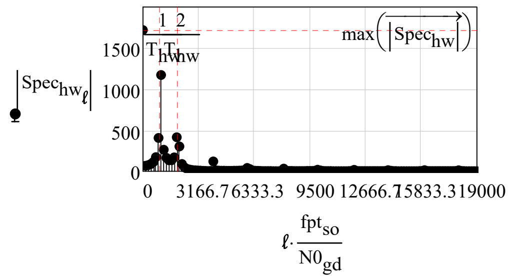


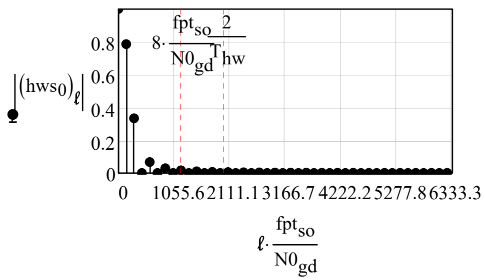
$length(u_{m2}) = 256$

$fpt_{so} = 38 \cdot kHz$

$Spec_{hw} := fft(u_{m2}) \qquad length(Spec_{hw}) = 129$

$$\ell := 0 .. \frac{N0_{gd}}{2} \qquad \frac{N0_{gd}}{2} = 128$$





Periodic Waveforms

2 Half wave filtered (Capacitive)

Max half wave amplitude: $B1_{hw} = 325.269 \cdot V$,

Amplitude of the decreasing exponential for t=0: V_{pp},

Exponential Time constant: $\tau_{hw1} := 2 \cdot T_{0gd}$

Period: , $T_{hw} = 1 \times 10^{3} \cdot \mu s$,

Pulsation: $\omega_{hw} := \frac{2 \cdot \pi}{T_{hw}} = 6.283 \cdot \frac{krads}{sec}$,

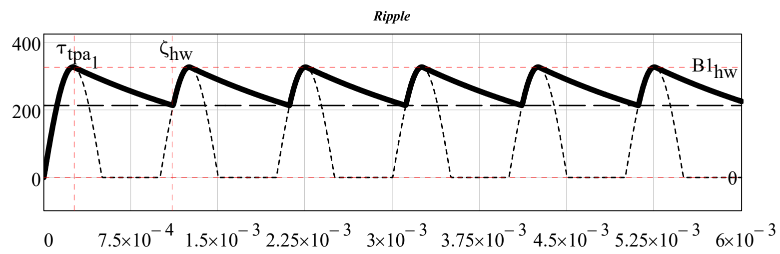
Intersection abscissa between half wave and exponential: ζ (scalar),

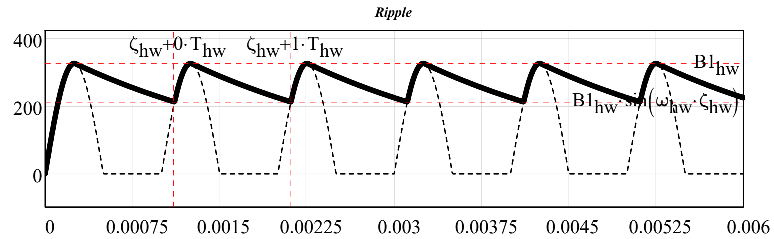
Tangent points abscissas between half wave and exponential: τ_{tpa} (vector)

$$\tau_{tpa_{k_{sl}}} := \frac{atan\left(-\omega_{hw} \cdot \tau_{hw1}\right) + k_{sl} \cdot \pi}{\omega_{hw}}$$

$$V_{tpv} := B1_{hw} \cdot \sin\left(\omega_{hw} \cdot \tau_{tpa_1}\right) \cdot e^{\frac{\tau_{tpa_1}}{\tau_{hw1}}} \qquad V_{tpv} = 369.746\,V$$

$$\zeta_{hw} := Z01\left(\tau_{hw1}, \omega_{hw}, B1_{hw}, V_{tpv}\right)$$

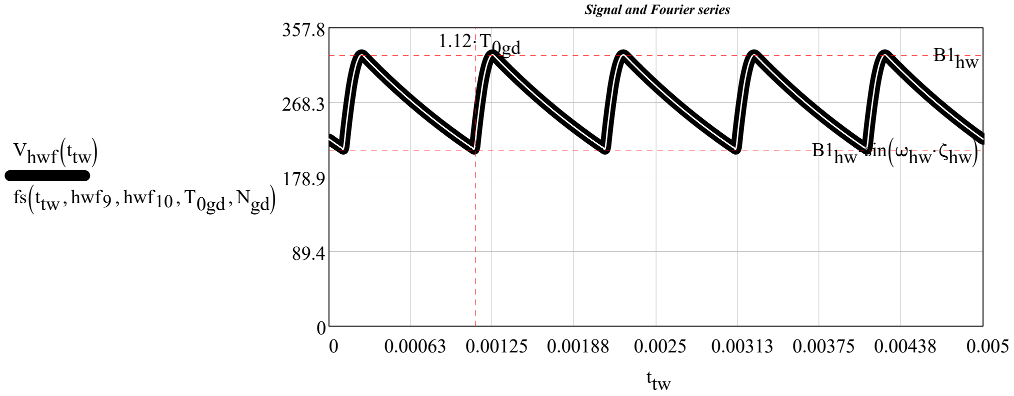


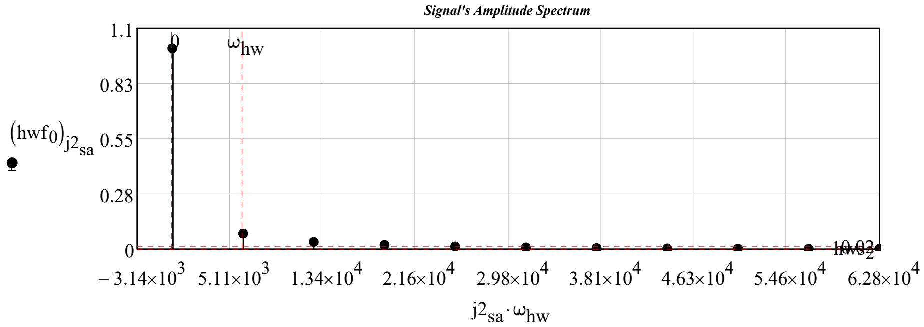


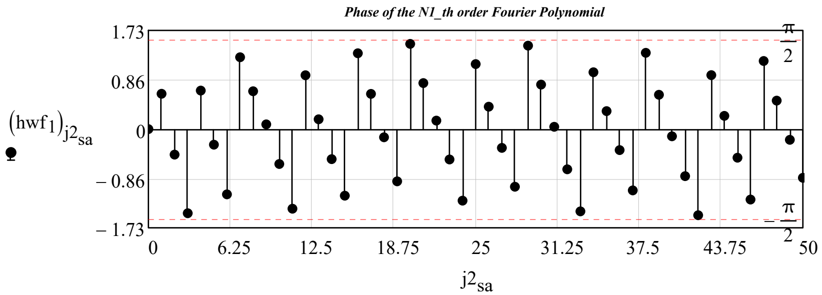
$B1_{hw} = 325.269\,V \qquad V_{hwf}(t) := g2hw\left(t + \tau_{hw1}, \tau_{hw1}, \tau_{tpa}, \zeta_{hw}, \omega_{hw}, B1_{hw}, V_{tpv}, N0_{gd}\right)$

$hwf := SPCT\left(V_{hwf}, rt_{gd}, N1\_, 0 \cdot sec, T_{0gd}\right) \qquad N1\_ = 50$

$j2_{sa} := 0 .. rows\left(hws_0\right) - 1$







$Bw_{sa} := hwf_3 \cdot Hz$

$Bw_{sa} = 0.02 \cdot MHz$

sampling frequency: $fpt_{so} := 2 \cdot Bw_{sa}$ $\quad fpt_{so} = 0.04 \cdot MHz$

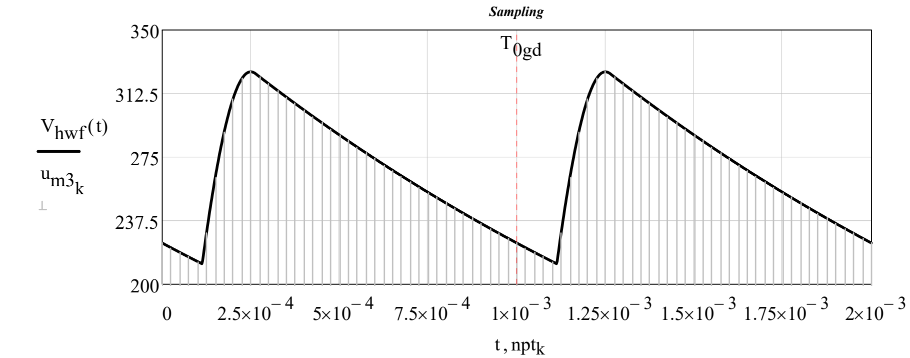
$npt_k := \frac{k}{fpt_{so}}$

Frequency resolution: $\frac{N0_{gd}}{fpt_{so}} \cdot \frac{1}{T_{0gd}} = 6.4$

$u_{m3_k} := V_{hwf}(npt_k)$

$u_{m3}^T =$

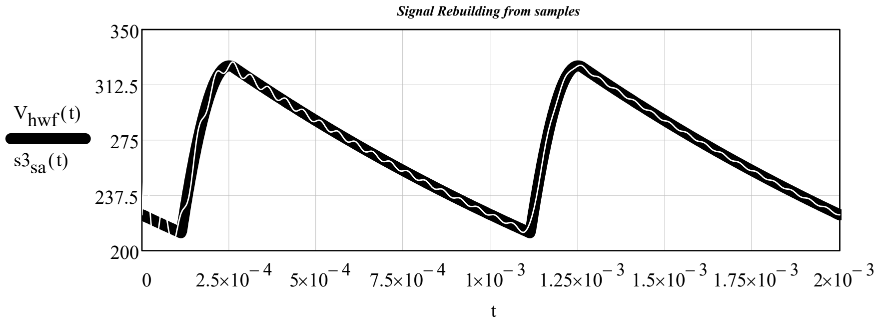
	0	1	2	3	4	5	6
0	224.262	221.476	218.725	216.008	213.325	230	...



relerr = 10·% $\quad \omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa}$ $\quad \omega_{bwr} = 0.126 \cdot \frac{Mrads}{sec}$ $\quad n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula: $s3_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1} \left(u_{m3_n} \cdot sinc\left(\omega_{bwr} \cdot t - n \cdot \pi \right) \right) \right]$ $\quad N0_{gd} - 1 = 255$ $\quad$ relerr =



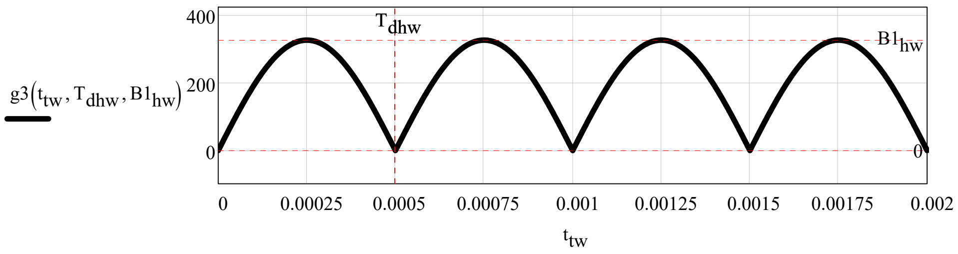
Symbol frequency:

Periodic Waveforms

3 Double Half wave

$$T_{dhw} := \frac{T_{hw}}{2} \qquad \omega_{dhw} := \frac{\pi}{T_{dhw}} \qquad g3(t_{sl}, T_{dhw}, B1_{hw}) := \frac{B1_{hw}}{V}\cdot\left|\sin\left(\frac{2\cdot\pi}{T_{hw}}\cdot t_{sl}\right)\right|$$

Double Half wave



Dirichlet conditions

A periodic function s(t)=s(t+T), can be expressed by the Fourier series provided that (Dirichlet conditions):
(1) it is discontinuous and presents a finite number of discontinuities in the period T;
(2) has a limited average value in the period T;
(3) it has a finite number of maximum positive or negative.
If these conditions are met, the considered function can be developed in Fourier series in trigonometric form.

The Dirichlet conditions apply to:

1) signals of energy for which holds: $\int_{-\infty}^{\infty} \left(\left|s_{fs}(t)\right|\right)^2 dt < \infty$,

2) power signals for which holds: $\lim_{T\to\infty}\left[\frac{1}{T}\cdot\int_{-T}^{T}\left(\left|s_{fs}(t)\right|\right)^2 dt\right] < \infty$

Fourier series definition

$$s_{fs}(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty}\left(a_k \cos(\omega\cdot k\cdot t) + b_k\cdot\sin(\omega\cdot k\cdot t)\right)$$

The coefficients are defined as follows:

$$\frac{a_0}{2} = A_{fs} = \frac{1}{T}\cdot\int_{t_0}^{t_0+T} s_{fs}(t)\, dt$$

$$a_k = \frac{2}{T}\cdot\int_{t_0}^{t_0+T} s_{fs}(t)\cdot\cos(\omega\cdot k\cdot t)\, dt$$

$$b_k = \frac{2}{T}\cdot\int_{t_0}^{t_0+T} s_{fs}(t)\cdot\sin(\omega\cdot k\cdot t)\, dt$$

$$B1_{hw} = 325.269\,V \qquad s_{fs}(t) := \frac{B1_{hw}}{V}\cdot\left|\sin\left(\frac{2\cdot\pi}{T_{hw}}\cdot t\right)\right|$$

$$T_{dhw} := T_{dhw} \qquad t := t \qquad T_{hw} := T_{hw}$$

$$\frac{a_0}{2} = A_{fs} = \frac{2}{T_{hw}}\cdot\frac{B1_{hw}}{V}\cdot\int_0^{\frac{T_{hw}}{2}} \sin\left(\frac{2\cdot\pi}{T_{hw}}\cdot t\right) dt = \frac{2\cdot B1_{hw}}{\pi\cdot V}$$

$$a_k = \frac{4}{T_{hw}}\cdot\frac{B1_{hw}}{V}\cdot\int_0^{\frac{T_{hw}}{2}} \sin\left(\frac{2\cdot\pi}{T_{hw}}\cdot t\right)\cdot\cos\left(\frac{2\cdot\pi}{T_{hw}}\cdot k\cdot t\right) dt = \frac{2\cdot B1_{hw}\cdot(\cos(\pi\cdot k)+1)}{-\pi\cdot V\cdot\left(k^2-1\right)}$$

$$b_k = \frac{4}{T_{hw}}\cdot\frac{B1_{hw}}{V}\cdot\int_0^{\frac{T_{hw}}{2}} \sin\left(\frac{2\cdot\pi}{T_{hw}}\cdot t\right)\cdot\sin\left(\frac{2\cdot\pi}{T_{hw}}\cdot k\cdot t\right) dt = -\frac{2\cdot B1_{hw}\cdot\sin(\pi\cdot k)}{\pi\cdot V\cdot\left(k^2-1\right)}$$

$$s_{fs}(t) = \frac{2\cdot B1_{hw}}{\pi\cdot V}\cdot\left[1 + \sum_{k=1}^{\infty}\left[\frac{(\cos(\pi\cdot k)+1)}{-\left(k^2-1\right)}\cos(\omega\cdot k\cdot t) + \frac{\sin(\pi\cdot k)}{\left(k^2-1\right)}\cdot\sin(\omega\cdot k\cdot t)\right]\right] \qquad \cos[k\cdot(\pi+\omega\cdot t)] = (-1)^k\cdot\cos(k\cdot$$

$$\frac{2\cdot B1_{hw}}{\pi\cdot V}\cdot\left[1 + \sum_{k=1}^{\infty}\frac{\cos(\omega\cdot k\cdot t) + \cos[k\cdot(\pi+\omega\cdot t)]}{1-k^2}\right] = \frac{2\cdot B1_{hw}}{\pi\cdot V}\cdot\left[1 + \sum_{k=1}^{\infty}\frac{\cos(\omega\cdot k\cdot t) + (-1)^k\cdot\cos(k\cdot\omega\cdot t)}{1-k^2}\right]$$

$$\frac{2\cdot B1_{hw}}{\pi\cdot V}\cdot\left[1 + \sum_{k=1}^{\infty}\frac{\cos(\omega\cdot k\cdot t) + (-1)^k\cdot\cos(k\cdot\omega\cdot t)}{1-k^2}\right] = \frac{2\cdot B1_{hw}}{\pi\cdot V}\cdot\left[1 + \frac{\pi\cdot\cos(\omega\cdot t)\cdot i}{2} + \sum_{k=2}^{\infty}\frac{\left[1+(-1)^k\right]\cdot\cos(k\cdot\omega\cdot t)}{1-k^2}\right]$$

$$\lim_{k\to 1^+}\frac{\left[1+(-1)^k\right]\cdot\cos(k\cdot\omega\cdot t)}{1-k^2} \to \frac{\pi\cdot\cos(\omega\cdot t)\cdot i}{2} \qquad \lim_{k\to 1^-}\frac{\left[1+(-1)^k\right]\cdot\cos(k\cdot\omega\cdot t)}{1-k^2} \to \frac{\pi\cdot\cos(\omega\cdot t)\cdot i}{2}$$

$$s_{dhw}(t) = \frac{2\cdot B1_{hw}}{\pi\cdot V}\cdot\left[1 + \frac{\pi\cdot\cos\left(\frac{2\cdot\pi}{T_{hw}}\cdot t\right)\cdot i}{2} + \sum_{k=2}^{\infty}\frac{\left[1+(-1)^k\right]\cdot\cos\left(k\cdot\frac{2\cdot\pi}{T_{hw}}\cdot t\right)}{1-k^2}\right]$$

$$s_{dhw}(t) := \frac{2\cdot B1_{hw}}{\pi\cdot V}\cdot\left[1+\frac{\pi\cdot\cos\left(\frac{2\cdot\pi}{T_{hw}}\cdot t\right)\cdot i}{2}+\sum_{k=2}^{100}\frac{\left[1+(-1)^k\right]\cdot\cos\left(k\cdot\frac{2\cdot\pi}{T_{hw}}\cdot t\right)}{1-k^2}\right]$$

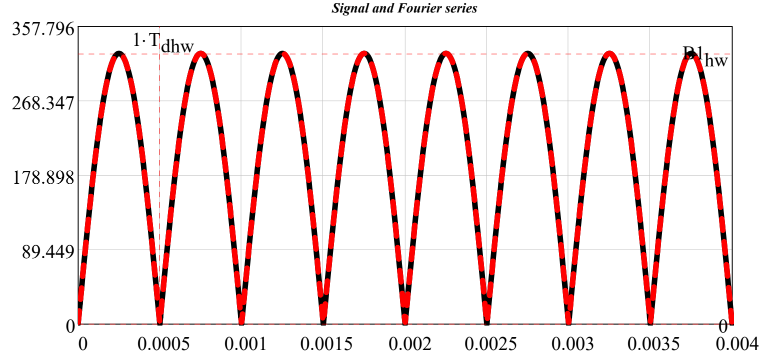
$s_{dhw}(0) = 2.05 + 325.269i$ $\quad \left|s_{dhw}(0)\right| = 325.276$ $\quad s_{dhw}\left(\frac{T_{dhw}}{2}\right) = 325.249$

$B1_{hw} = 325.269\,V$ $\quad V_{dhw}(t) := g3\left(t, T_{dhw}, B1_{hw}\right)$ $\quad \omega_{sa} := \omega_{dhw}$ $\quad 2\cdot\frac{B1_{hw}}{\pi\cdot V} = 207.07$

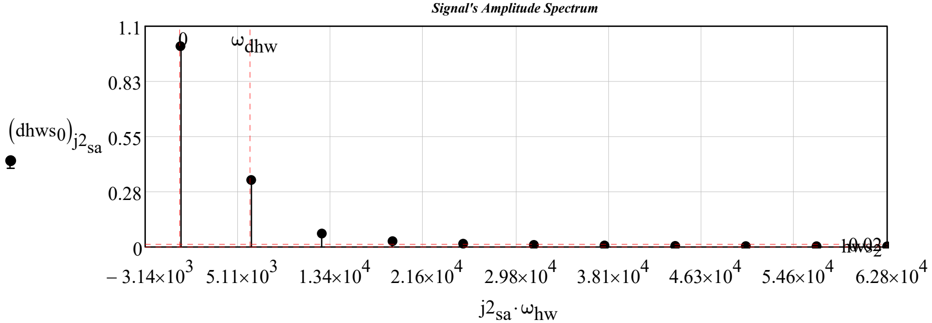
$dhws := SPCT\left(V_{dhw}, rt_{gd}, N1\_, 0\cdot sec, T_{dhw}\right)$ $\quad N1\_ = 50$

$j2_{sa} := 0 .. rows\left(hws_0\right) - 1$

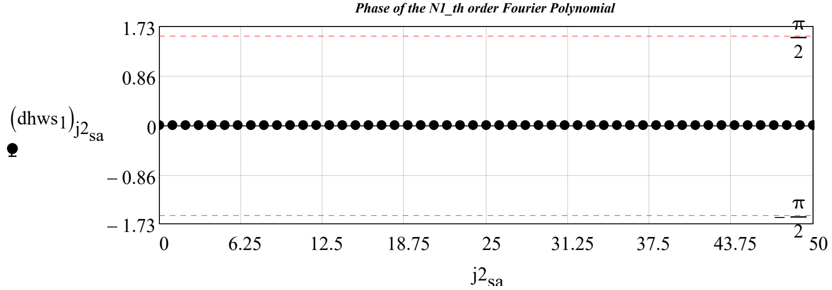
Signal and Fourier series



Signal's Amplitude Spectrum



Phase of the N1_th order Fourier Polynomial



$Bw_{sa} := dhws_3\cdot Hz$

$Bw_{sa} = 0.03\cdot MHz$

sampling frequency: $\quad fpt_{so} := 2\cdot Bw_{sa}$ $\quad fpt_{so} = 0.06\cdot MHz$

$k := 0 .. 2^8 - 1$ $\quad npt_k := \frac{k}{fpt_{so}}$

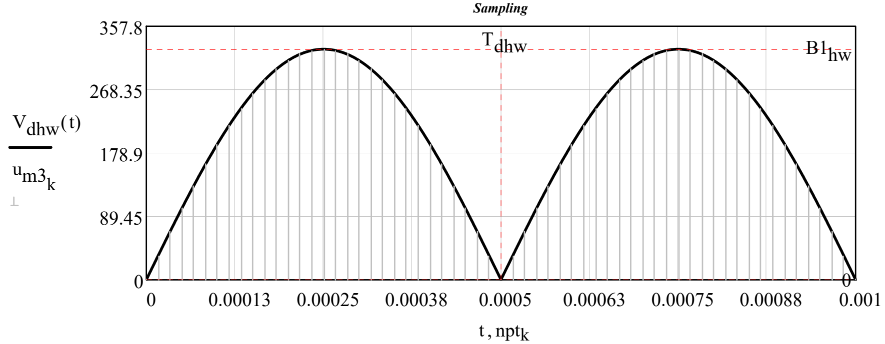
Frequency resolution: $\quad \frac{N0_{gd}}{fpt_{so}}\cdot\frac{1}{T_{dhw}} = 8.533$

$u_{m3_k} := V_{dhw}\left(npt_k\right)$

$u_{m3}^T =$

	0	1	2	3	4
0	0	34	67.627	100.514	...

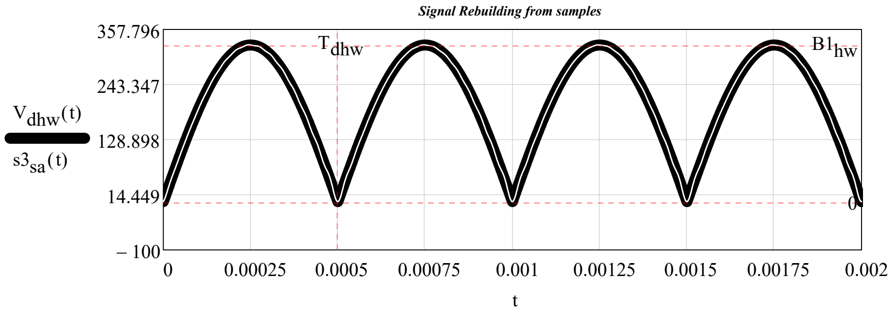
Sampling

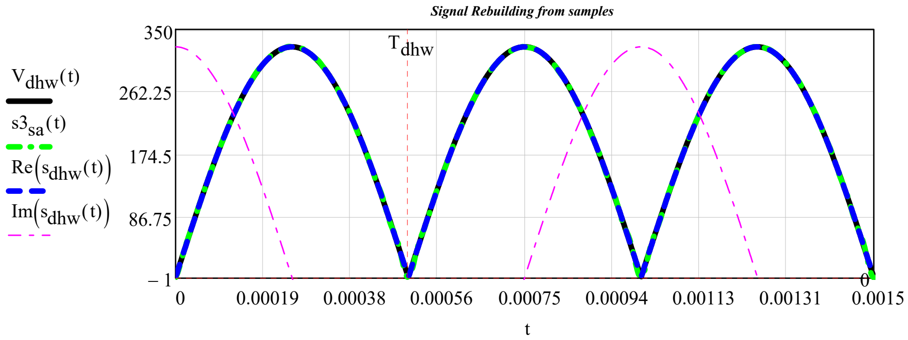


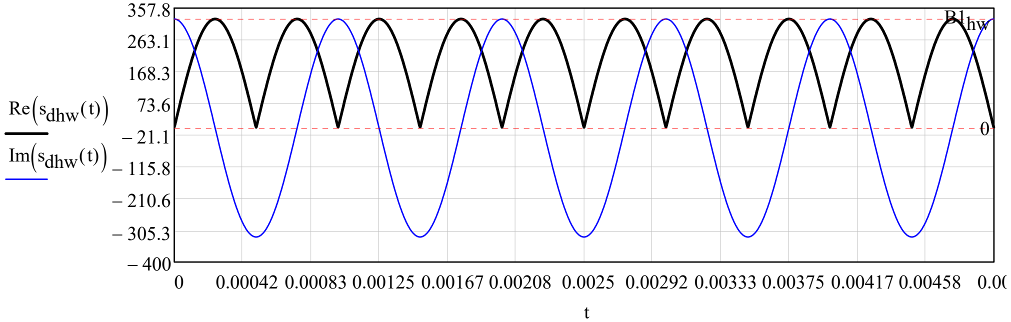
$relerr = 10\cdot\%$ $\quad \omega_{bwr} := 2\cdot\pi\cdot Bw_{sa}$ $\quad \omega_{bwr} = 0.188\cdot\frac{Mrads}{sec}$ $\quad n\cdot\frac{\pi}{\omega_{bwr}} = n\cdot\frac{1}{2\cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula:: $\quad s3_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1}\left(u_{m3_n}\cdot sinc\left(\omega_{bwr}\cdot t - n\cdot\pi\right)\right)\right]$ $\quad N0_{gd} - 1 = 255$ $\quad relerr =$





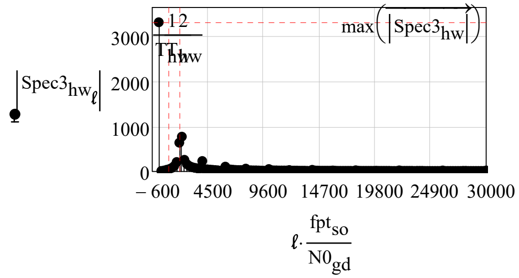


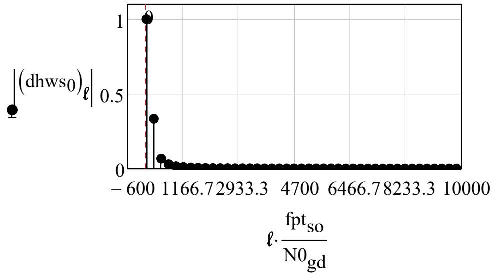
$\text{length}(u_{m2}) = 256$

$fpt_{so} = 60 \cdot kHz$

$Spec3_{hw} := \text{fft}(u_{m3}) \quad \text{length}(Spec3_{hw}) = 129$

$\ell := 0 .. \frac{N0_{gd}}{2} \quad \frac{N0_{gd}}{2} = 128$



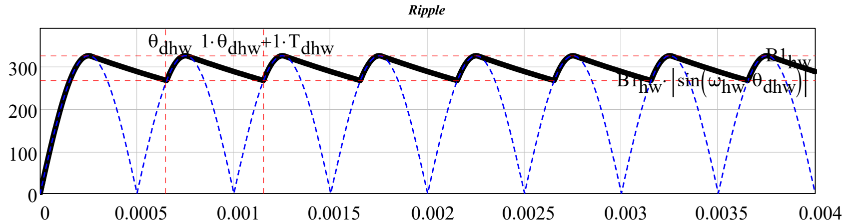


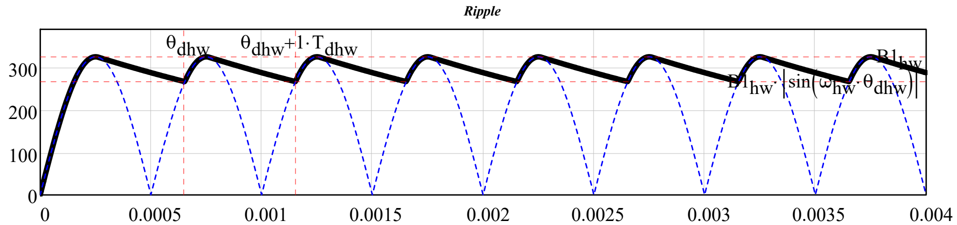
Periodic Waveforms

4 Double Half wave filtered

$$V_{ppt} := B1_{hw} \cdot \sin\left(\omega_{hw} \cdot \tau_{tpa_1}\right) \cdot e^{\frac{\tau_{tpa_1}}{\tau_{hw1}}} \qquad \theta_{dhw} := Z1\left(\tau_{hw1}, \omega_{hw}, B1_{hw}, V_{ppt}\right)$$

$$V_{ppt} = 369.746\,V \qquad rip1 = \frac{\frac{B1_{hw}}{V} - \frac{B1_{hw}}{V} \cdot \left|\sin\left(\omega_{hw} \cdot \theta_{dhw}\right)\right|}{\frac{B1_{hw}}{V}}$$

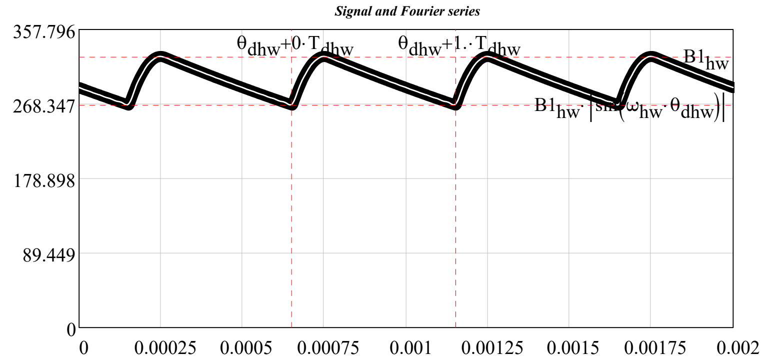


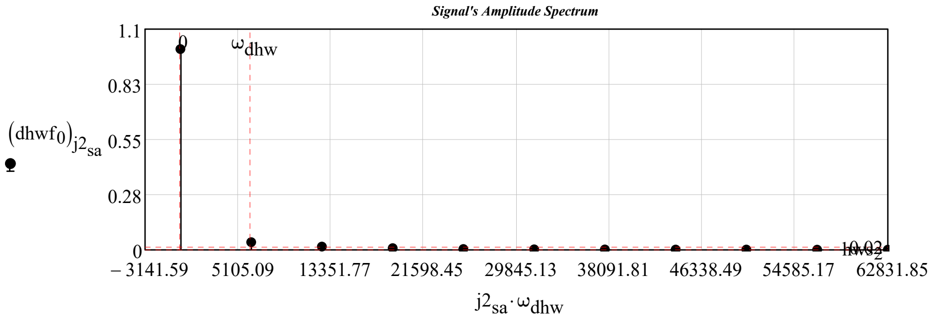


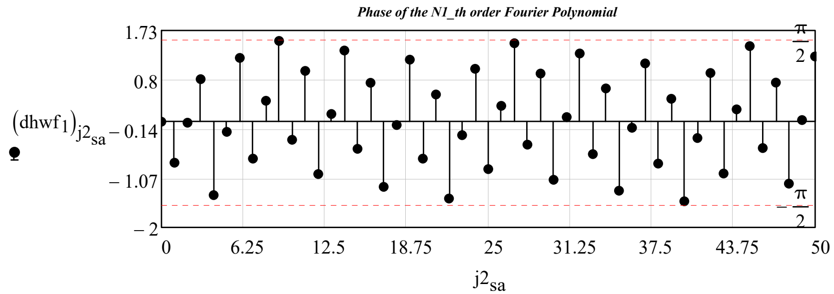
$$B1_{hw} = 325.269\,V \qquad V_{dhwf}(t) := g4\left(t + \tau_{hw1}, \tau_{hw1}, \tau_{tpa}, \theta_{dhw}, \omega_{hw}, B1_{hw}, V_{ppt}, N0_{gd}\right)$$

$$\tau_{hw1} = 2 \times 10^{-3}\,s \qquad dhwf := SPCT\left(V_{dhwf}, rt_{gd}, 50, 0 \cdot sec, T_{dhw}\right)$$

$$j2_{sa} := 0 .. rows\left(dhws_0\right) - 1$$







$$Bw_{sa} := dhwf_3 \cdot Hz$$

$$Bw_{sa} = 0.036 \cdot MHz$$

sampling frequency: $fpt_{so} := 2 \cdot Bw_{sa} \qquad fpt_{so} = 0.072 \cdot MHz$

$$npt_k := \frac{k}{fpt_{so}}$$

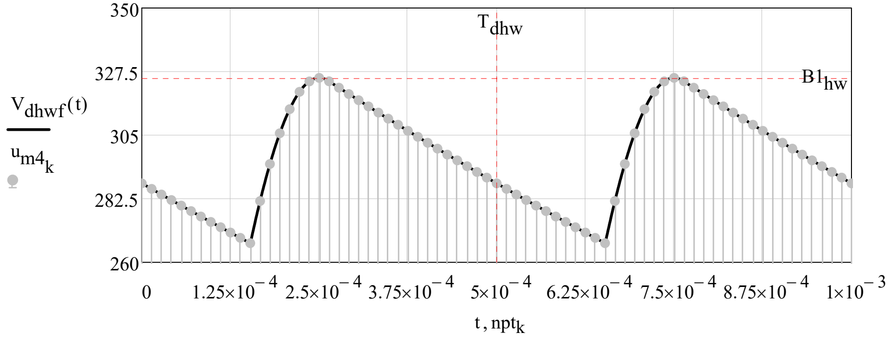
Frequency resolution: $\frac{N0_{gd}}{fpt_{so}} \cdot \frac{1}{T_{dhw}} = 7.111$

$$u_{m4_k} := V_{dhwf}\left(npt_k\right)$$

$u_{m4}^{T} =$

	0	1	2	3	4	5	6
0	287.958	285.966	283.987	282.021	280.07	278.131	...

Sampling

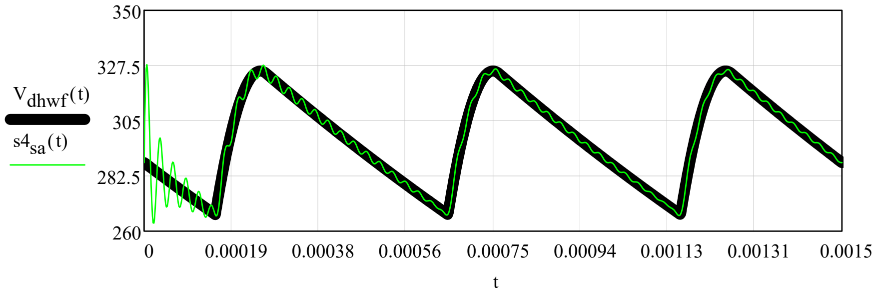


$relerr = 10 \cdot \%$ $\qquad \omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa}$ $\qquad \omega_{bwr} = 0.226 \cdot \frac{Mrads}{sec}$ $\qquad n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula: $\quad s4_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1} \left(u_{m4_n} \cdot sinc\left(\omega_{bwr} \cdot t - n \cdot \pi\right)\right) \right]$ $\qquad N0_{gd} - 1 = 255 \quad relerr = 10 \cdot \%$

Signal Rebuilding from samples

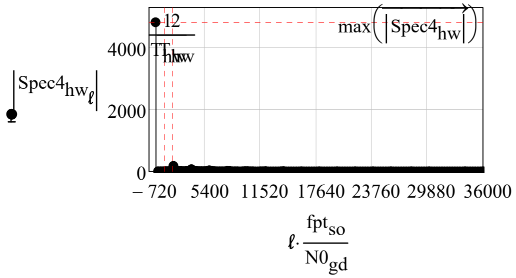


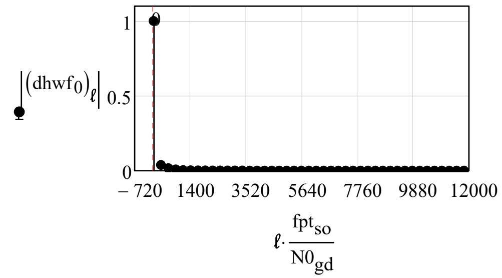
$length\left(u_{m4}\right) = 256$

$fpt_{so} = 72 \cdot kHz$

$Spec4_{hw} := fft\left(u_{m4}\right) \qquad length\left(Spec4_{hw}\right) = 129$

$\ell := 0 .. \frac{N0_{gd}}{2} \qquad \frac{N0_{gd}}{2} = 128$





TEST Waveforms

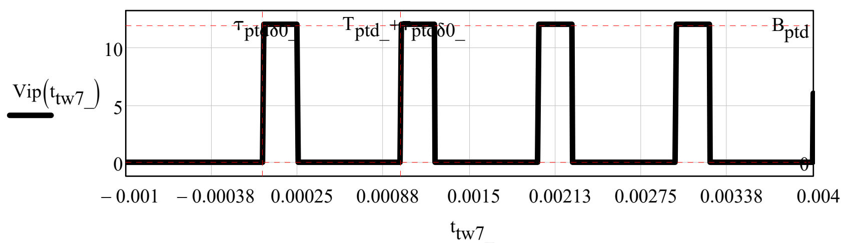
Periodic Waveforms

5 Voltage Pulse Train

Data " pulse train data"

$Vip(t) := Vip1\left(t, T_{ptd\_}, \tau_{ptd\delta0\_}, \delta_{ptd\_}, B_{ptd}, N0_{gd}\right)$

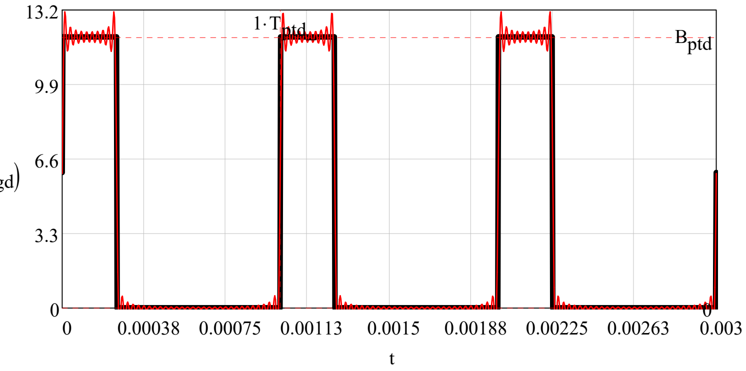
Voltage Pulse Train



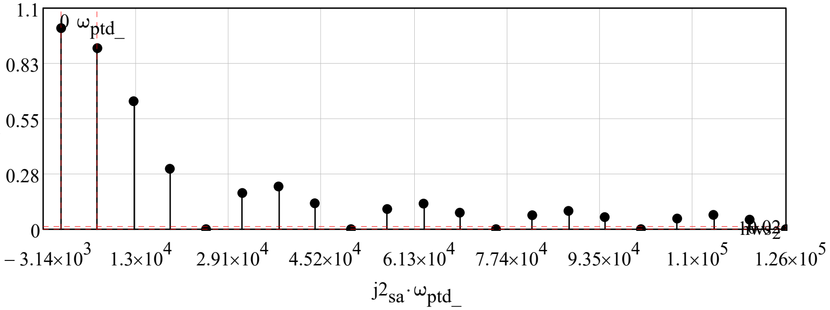
$Vpt := SPCT\left(Vip, rt_{gd}, N1\_, 0 \cdot sec, T_{ptd\_}\right) \qquad N1\_ = 50$

$j2_{sa} := 0 .. rows\left(dhws_0\right) - 1 \qquad \omega_{ptd\_} = 6.283 \times 10^{-3} \cdot \frac{Mrads}{s}$

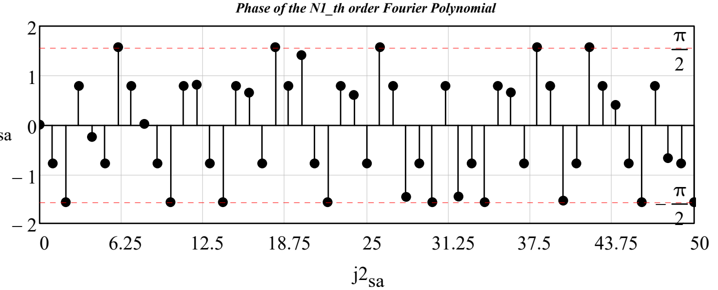
Signal and Fourier series



Signal's Amplitude Spectrum



Phase of the N1_th order Fourier Polynomial



$Bw_{sa} := Vpt_3 \cdot Hz$

$Bw_{sa} = 0.048 \cdot MHz$

sampling frequency: $fpt_{so} := 2 \cdot Bw_{sa}$ $\quad fpt_{so} = 0.096 \cdot MHz$

$npt_k := \dfrac{k}{fpt_{so}}$

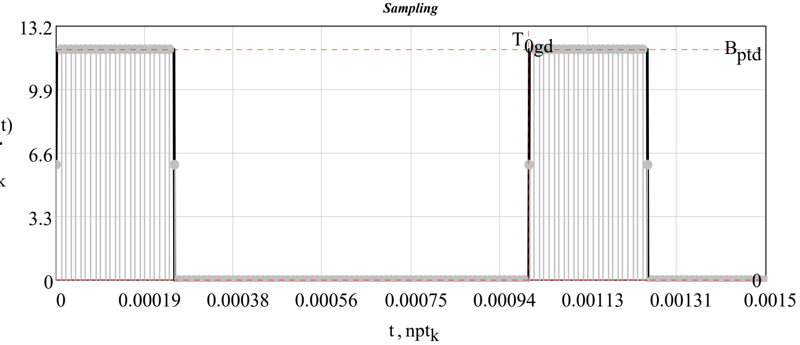
Frequency resolution: $\dfrac{N0_{gd}}{fpt_{so}} \cdot \dfrac{1}{T_{0gd}} = 2.667$

$u_{m5_k} := Vip(npt_k)$

$u_{m5}^{T} =$

	0	1	2	3	4	5	6	7	8	9
0	6	12	12	12	12	12	12	12	12	...

Sampling

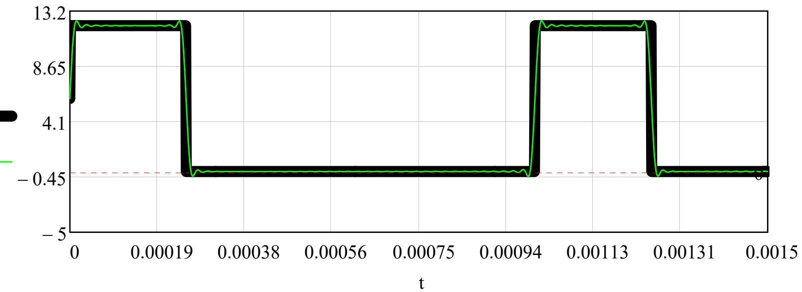


relerr = 10·% $\quad \omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa} \quad \omega_{bwr} = 0.302 \cdot \dfrac{Mrads}{sec} \quad n \cdot \dfrac{\pi}{\omega_{bwr}} = n \cdot \dfrac{1}{2 \cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula: $s5_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1} \left(u_{m5_n} \cdot sinc(\omega_{bwr} \cdot t - n \cdot \pi)\right)\right] \quad N0_{gd} - 1 = 255 \quad relerr = 10 \cdot \%$

Signal Rebuilding from samples

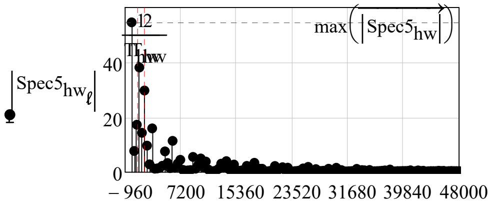


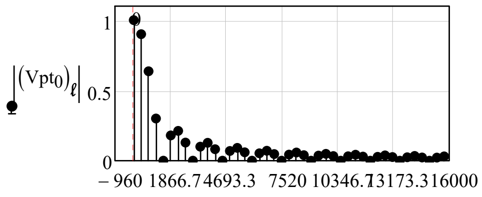
$length(u_{m5}) = 256$

$fpt_{so} = 96 \cdot kHz$

$Spec5_{hw} := fft(u_{m5}) \quad length(Spec5_{hw}) = 129$

$$\ell := 0 .. \frac{N0_{gd}}{2} \qquad \frac{N0_{gd}}{2} = 128$$





TEST Waveforms

Periodic Waveforms

6 RF Pulse Train

Data " rf pulse data"

Step amplitude............................: $V_{rfpd} := B_{ptd}, \; V_{rfpd} = 12 \cdot V$

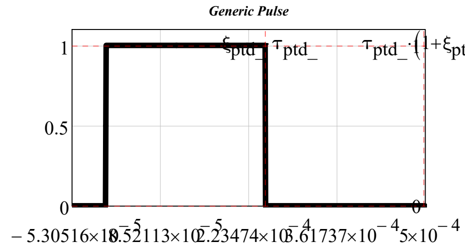
Signal frequency.........................: $f_{rfpd} := 30 \cdot f_{ptd\_} \;, \quad f_{ptd\_} = 1 \times 10^3 \frac{1}{s}$

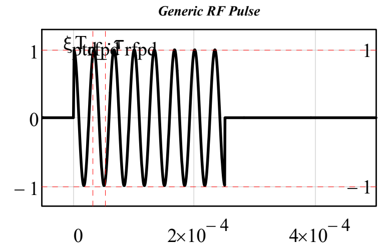
Signal period..............................: $T_{rfpd} := \frac{1}{f_{rfpd}} \;, \qquad T_{ptd\_} = 1 \times 10^{-3} s$

Signal angular frequency...............: $\omega_{rfpd} := 2 \cdot \pi \cdot f_{rfpd} \qquad \omega_{rfpd} = 0.188 \cdot \frac{Mrads}{sec},$

time constant.............................: $\tau_{rfpd} := \frac{10}{\omega_{rfpd}}, \; \tau_{rfpd} = 53.052 \cdot \mu s$

Rising edge delay ....: $\tau_{\delta rfpd} := 0 \cdot ns, \; \xi_{ptd\_} \cdot \tau_{ptd\_} = 250 \cdot \mu s, \; \xi_{ptd\_} = 1 \quad , \tau_{ptd\_} = 250 \cdot \mu s$

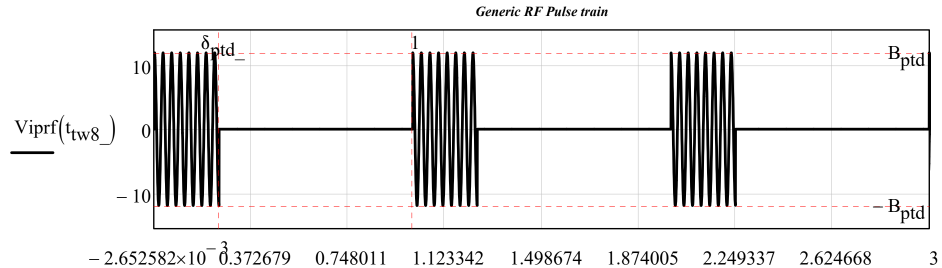




Average value: $v_{ptmrfsl} := B_{ptd} \cdot \delta_{ptd\_}$

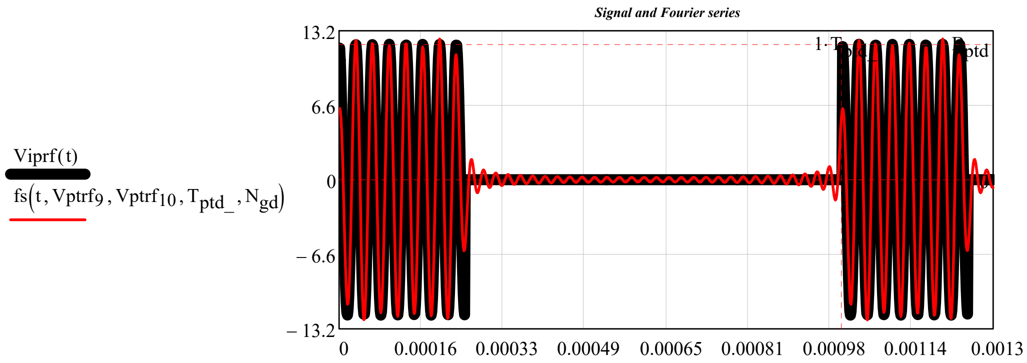
$$t_{tw8\_} := -1 \cdot \tau_{ptd\_}, -1 \cdot \tau_{ptd\_} + \frac{4 \cdot T_{ptd\_} + \tau_{ptd\_}}{8000} .. 4 \cdot T_{ptd\_}$$

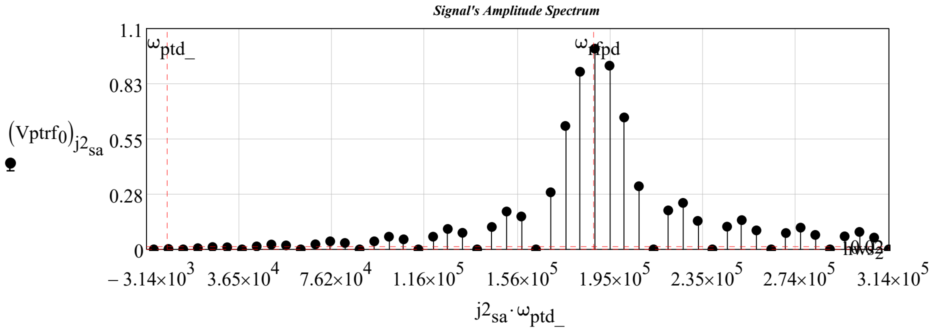
$$Viprf(t) := v_{ptrf}\left(t, T_{ptd\_}, \tau_{\delta rfpd}, \delta_{ptd\_}, \omega_{rfpd}, \frac{V_{rfpd}}{V}, N0_{gd}\right)$$

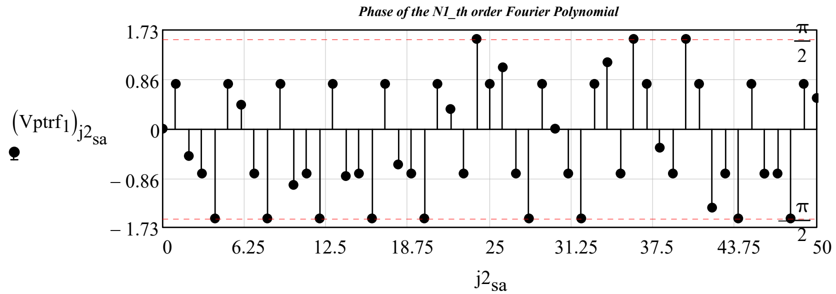


$$Vptrf := SPCT(Viprf, rt_{gd}, N1\_, 0 \cdot sec, T_{ptd\_}) \qquad N1\_ = 50$$

$N_{gd} = 40 \qquad \omega_{ptd\_} = 6.283 \cdot \frac{krads}{s} \quad j2_{sa} := 0 .. rows(dhws_0) - 1 \qquad \omega_{rfpd} = 188.496 \cdot \frac{krads}{s}$







$Bw_{sa} := Vptrf_3 \cdot Hz$

$Bw_{sa} = 0.048 \cdot MHz$

sampling frequency: $fpt_{so} := 2 \cdot Bw_{sa}$ $\quad fpt_{so} = 0.096 \cdot MHz$

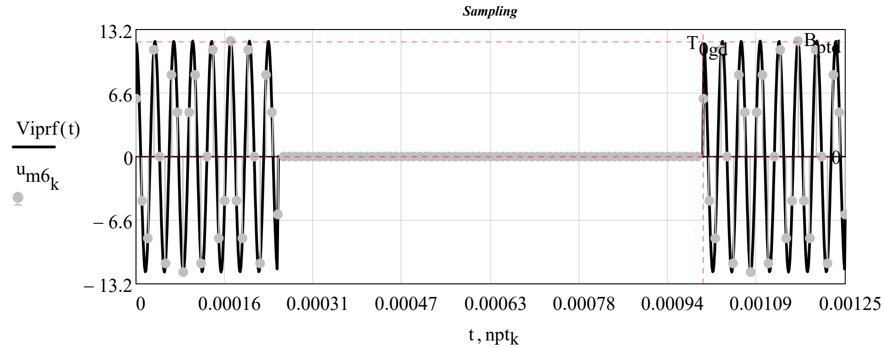
$$npt_k := \frac{k}{fpt_{so}}$$

Frequency resolution: $\frac{N0_{gd}}{fpt_{so}} \cdot \frac{1}{T_{0gd}} = 2.667$

$$u_{m6_k} := Viprf(npt_k)$$

$u_{m6}^T =$

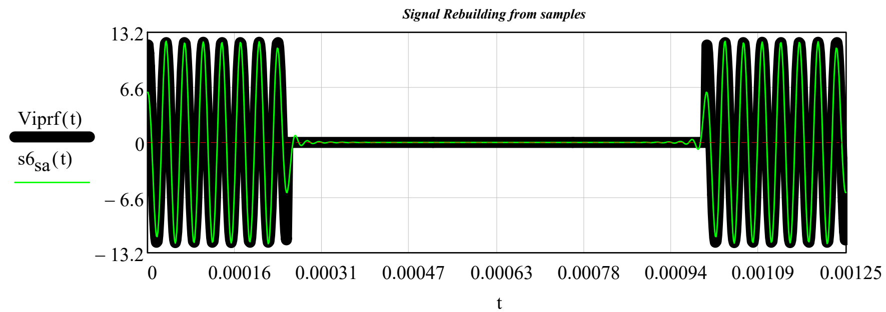
	0	1	2	3	4
0	6	-4.592	-8.485	11.087	...



relerr = 10·% $\quad \omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa}$ $\quad \omega_{bwr} = 0.302 \cdot \frac{Mrads}{sec}$ $\quad n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula: $s6_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1} \left(u_{m6_n} \cdot \mathrm{sinc}(\omega_{bwr} \cdot t - n \cdot \pi)\right)\right]$ $\quad N0_{gd} - 1 = 255$ $\quad$ relerr = 10·%

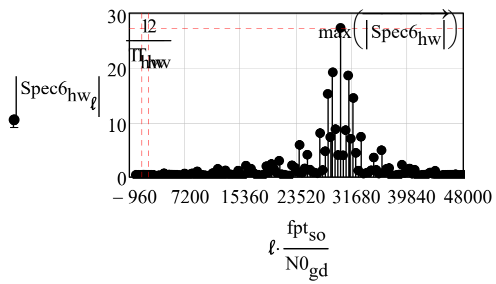


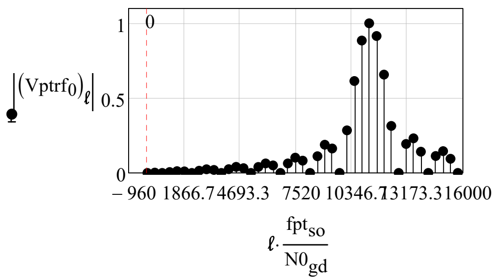
$length(u_{m6}) = 256$

$fpt_{so} = 96 \cdot kHz$

$Spec6_{hw} := fft(u_{m6})$ $\quad length(Spec6_{hw}) = 129$

$\ell := 0 .. \frac{N0_{gd}}{2}$ $\quad \frac{N0_{gd}}{2} = 128$





Periodic Waveforms

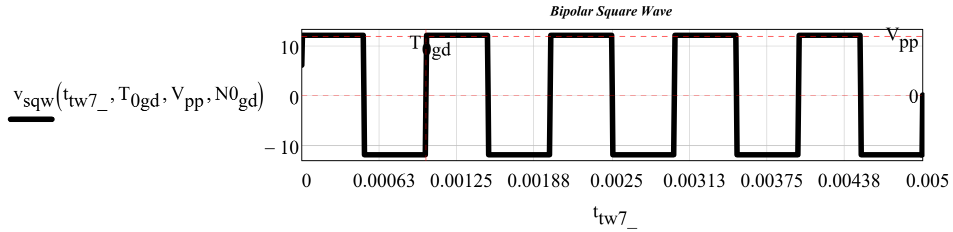
7 Bipolar Square Wave

Data file " pulse train data.xmcd"

Signal amplitude: $V_{pp} = 12 \cdot V$

Square wave period: $T_{0gd} = 1 \times 10^{6} \cdot ns$

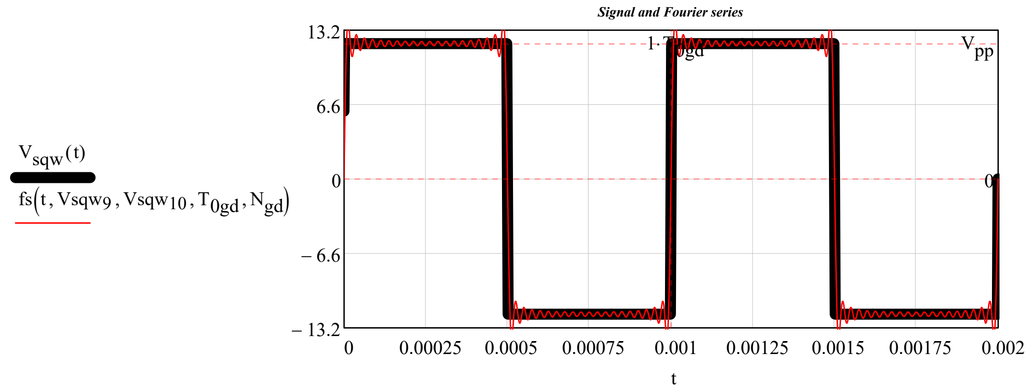
$\omega_{ptd\_} = 6.283 \times 10^{-6} \cdot \frac{Grads}{sec}$



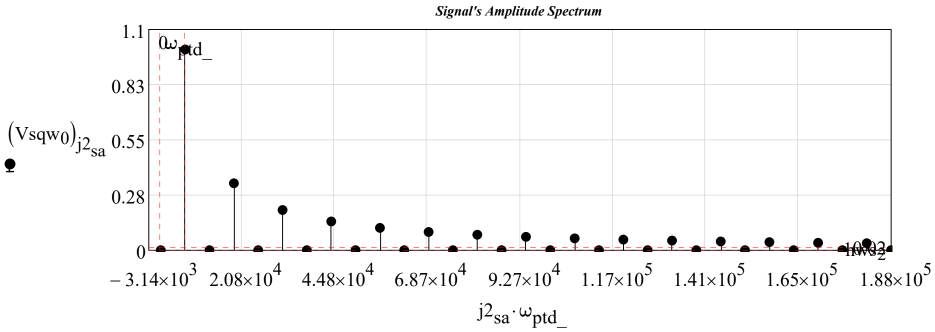
$$V_{sqw}(t) := \frac{v_{sqw}(t, T_{0gd}, V_{pp}, N0_{gd})}{V}$$

$Vsqw := SPCT(V_{sqw}, rt_{gd}, N1\_, 0 \cdot sec, T_{0gd})$ $N1\_ = 50$

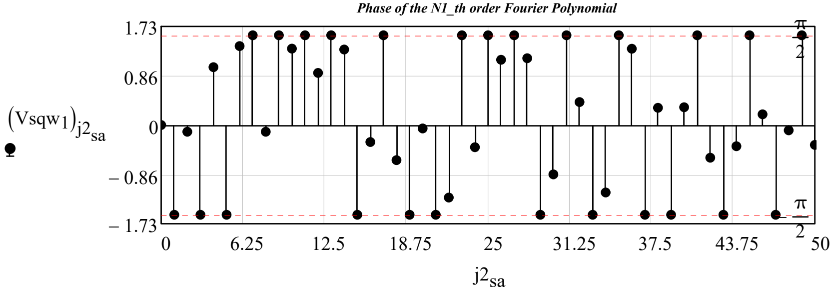
$j2_{sa} := 0 .. rows(dhws_0) - 1$ $\omega_{ptd\_} = 6.283 \times 10^{-3} \cdot \frac{Mrads}{s}$



Signal's Amplitude Spectrum



Phase of the N1_th order Fourier Polynomial



$Bw_{sa} := Vsqw_3 \cdot Hz$

$Bw_{sa} = 0.048 \cdot MHz$

sampling frequency: $fpt_{so} := 2 \cdot Bw_{sa}$ $\qquad fpt_{so} = 0.096 \cdot MHz$

$$npt_k := \frac{k}{fpt_{so}}$$

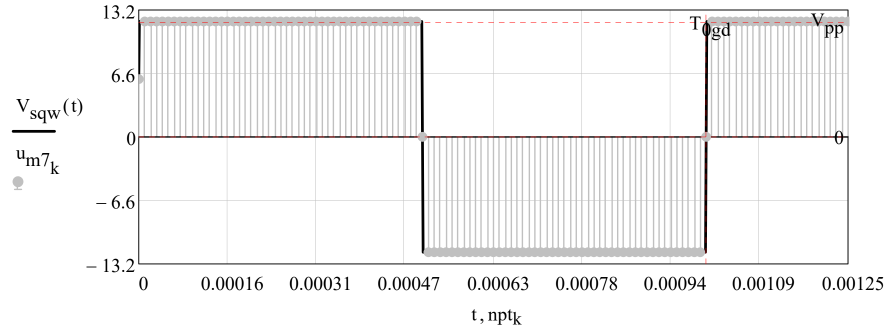
Frequency resolution: $\dfrac{N0_{gd}}{fpt_{so}} \cdot \dfrac{1}{T_{0gd}} = 2.667$

$$u_{m7_k} := V_{sqw}(npt_k)$$

$u_{m6}{}^T =$

	0	1	2	3	4
0	6	-4.592	-8.485	11.087	...

Sampling

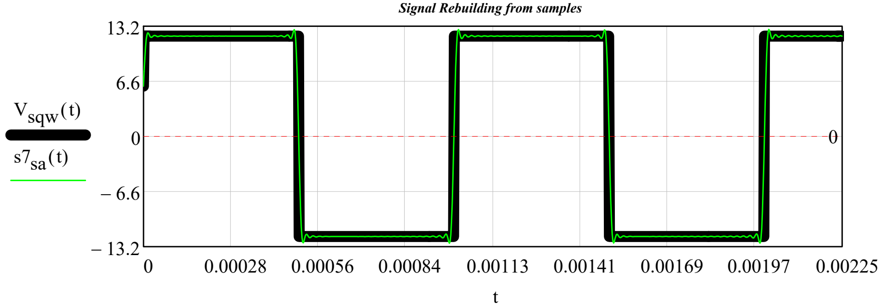


relerr = 10·% $\qquad \omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa}$ $\qquad \omega_{bwr} = 0.302 \cdot \dfrac{Mrads}{sec}$ $\qquad n \cdot \dfrac{\pi}{\omega_{bwr}} = n \cdot \dfrac{1}{2 \cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula: $s7_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1} \left(u_{m7_n} \cdot sinc(\omega_{bwr} \cdot t - n \cdot \pi)\right)\right]$ $\qquad N0_{gd} - 1 = 255$ $\qquad$ relerr = 10·%

Signal Rebuilding from samples

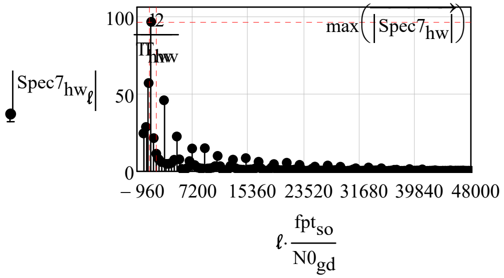


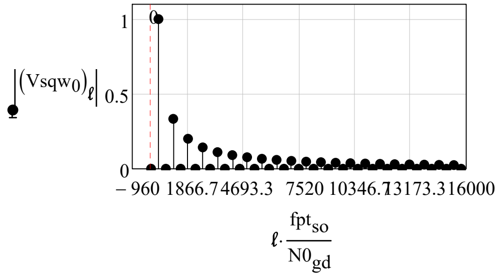
$length(u_{m7}) = 256$

$fpt_{so} = 96 \cdot kHz$

$Spec7_{hw} := fft(u_{m7})$ $\qquad length(Spec7_{hw}) = 129$

$\ell := 0 .. \dfrac{N0_{gd}}{2}$ $\qquad \dfrac{N0_{gd}}{2} = 128$





TEST Waveforms

Periodic Waveforms

8 Bipolar Square Wave 1

Pulse train data

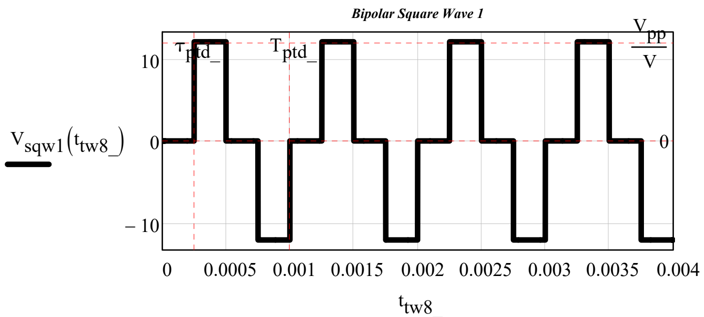
Signal amplitude: $V_{pp} = 12\,V$

Square wave period: $T_{0gd} = 1 \times 10^{6} \cdot ns$ $\xi_{twsl} := \xi_{ptd\_}$

$\omega_{ptd\_} = 6.283 \times 10^{-6} \cdot \frac{Grads}{sec}$ $\tau_{\delta sl} := -\tau_{ptd\_} \cdot (1 - \xi_{twsl})$

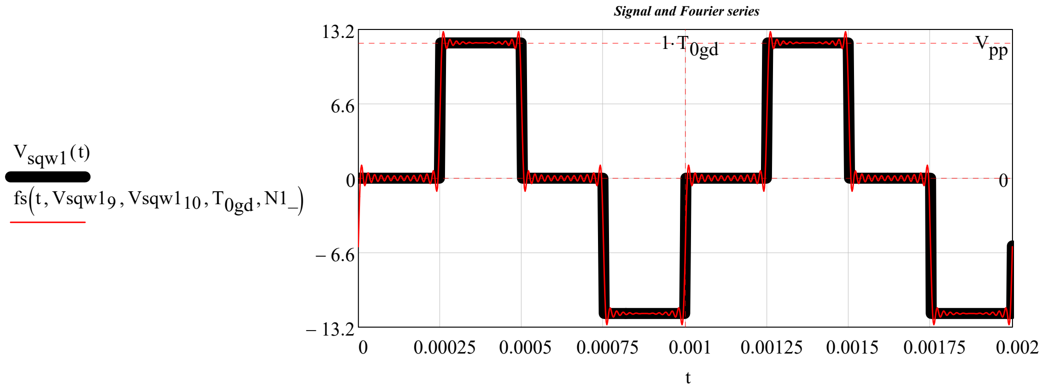
$\tau_{ptd\_} = 250 \cdot \mu s$

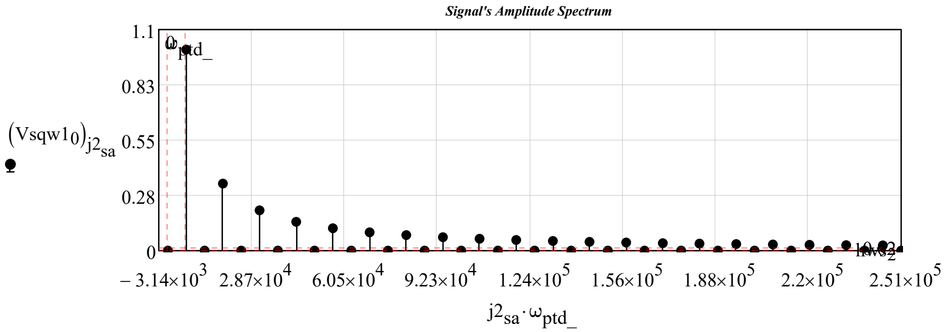
$$V_{sqw1}(t) := V_6(t, \tau_{\delta sl}, \tau_{ptd\_}, T_{0gd}, V_{pp}, N0_{gd})$$

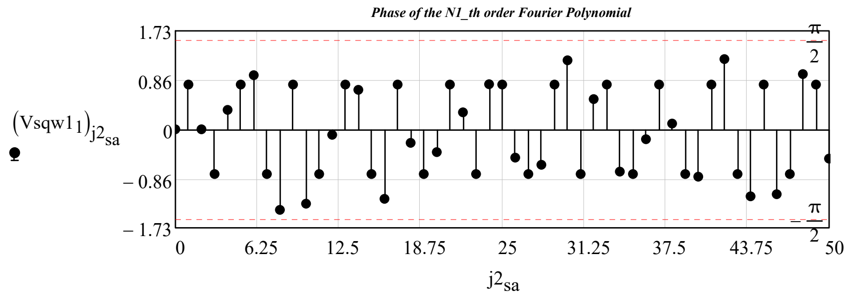


$Vsqw1 := SPCT(V_{sqw1}, rt_{gd}, N1\_, \tau_{ptd\_}, T_{ptd\_})$ $N1\_ = 50$

$j2_{sa} := 0 .. rows(Vsqw1_0) - 1$ $\omega_{ptd\_} = 6.283 \cdot \frac{krads}{s}$







$Bw_{sa} := Vsqw1_3 \cdot Hz$

$Bw_{sa} = 0.048 \cdot MHz$

sampling frequency: $fpt_{so} := 2 \cdot Bw_{sa}$ $fpt_{so} = 0.096 \cdot MHz$

$npt_k := \frac{k}{fpt_{so}}$

Frequency resolution: $\frac{N0_{gd}}{fpt_{so}} \cdot \frac{1}{T_{0gd}} = 2.667$

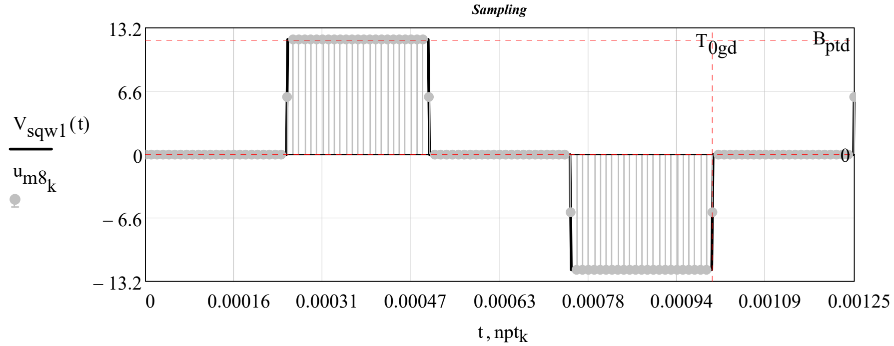
$u_{m8_k} := V_{sqw1}(npt_k)$

$u_{m8}^{T} =$

	0	1	2	3	4	5	6	7	8	9

$u_{m8} =$

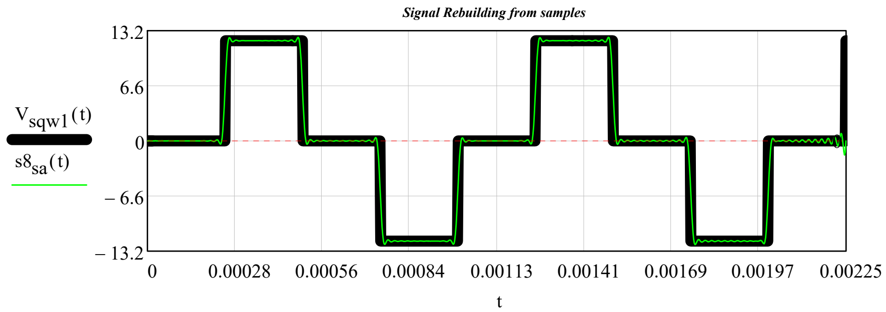
0	0	0	0	0	0	0	0	0	0	...



relerr = 10·% $\omega_{bwr} := 2\cdot\pi\cdot Bw_{sa}$ $\omega_{bwr} = 0.302\cdot\frac{Mrads}{sec}$ $n\cdot\frac{\pi}{\omega_{bwr}} = n\cdot\frac{1}{2\cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula: $s8_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1}\left(u_{m8_n}\cdot sinc\left(\omega_{bwr}\cdot t - n\cdot\pi\right)\right)\right]$ $N0_{gd} - 1 = 255$ relerr = 10·%

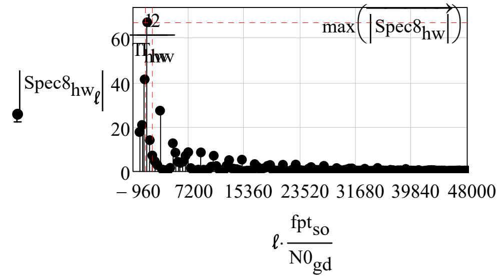


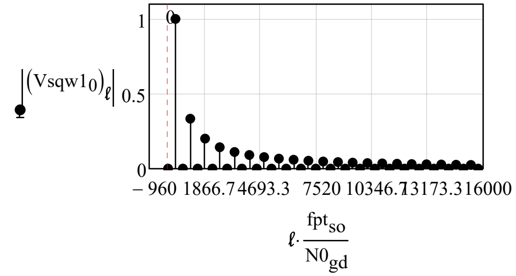
$length(u_{m8}) = 256$

$fpt_{so} = 96\cdot kHz$

$Spec8_{hw} := fft(u_{m8})$ $length(Spec8_{hw}) = 129$

$\ell := 0..\frac{N0_{gd}}{2}$ $\frac{N0_{gd}}{2} = 128$





TEST Waveforms

Periodic Waveforms

9 Staircase 1 Voltage Pulse Train

Description of the Function's parameters:

$v_{stcp}(t_{sl}, period, signal\_amplitude, number\_of\_steps, max\_number\_of\_periods)$

$v_{stc}(t_{sl}, step\_length, signal\_amplitude, number\_of\_steps, max\_number\_of\_periods)$

Period: $T_{stcpt} := (m1_{steps} + 1)\cdot T_{1stpl\_}\cdot 2$

Duty Cycle: $\delta_{stcpt} := \frac{m1_{steps}\cdot T_{1stpl\_}}{T_{stcpt}}$

Staircase frequency: $f_{stcpt} := \frac{1}{T_{stcpt}}$

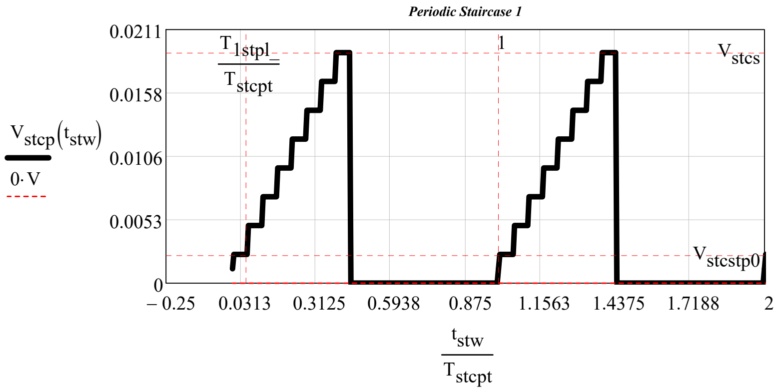
$\omega_{stcpt} := 2\cdot\pi\cdot f_{stcpt}$ $\omega_{1stpl} := \frac{2\cdot\pi}{T_{1stpl\_}}$

Number of periods shown: $n_p := 20$

$$v_{stcptasl} := \frac{V_{stcs}}{2\cdot m1_{steps}\cdot(m1_{steps}+1)}\cdot\sum_{k=1}^{m1_{steps}}(m1_{steps} - k + 1) = 4.8\cdot mV$$

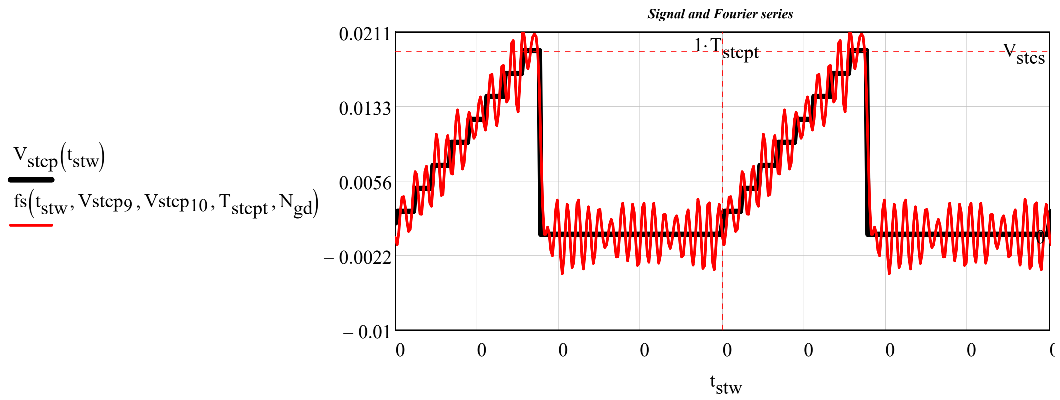
$$t_{stw} := 0\cdot T_{stcpt}, 0\cdot T_{stcpt} + \frac{10\cdot T_{stcpt}}{2000} .. 10\cdot T_{stcpt}$$

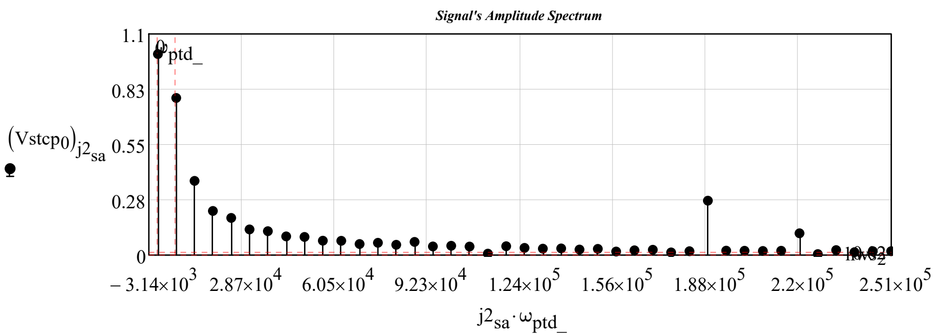
Dimensionless function: $V_{stcp}(t) := \frac{v_{stcp}(t, T_{stcpt}, V_{stcs}, m1_{steps}, N0_{gd})}{V}$

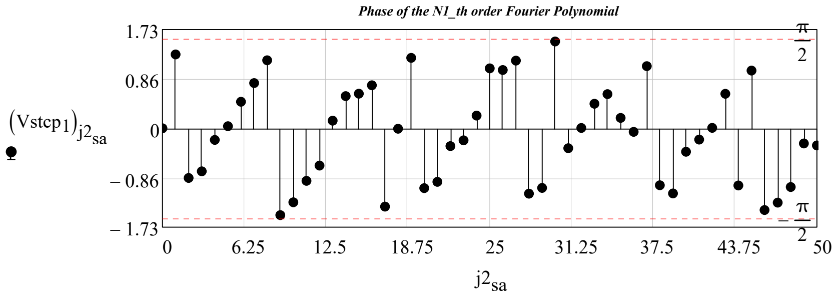


$$Vstcp := SPCT\left(V_{stcp}, rt_{gd}, N1\_, 0\cdot s, T_{stcpt}\right)$$

$$j2_{sa} := 0..rows\left(Vstcp_0\right) - 1 \qquad \omega_{ptd\_} = 6.283\times 10^{-3}\cdot\frac{Mrads}{s}$$







$Bw_{sa} := Vstcp_3\cdot Hz$

$Bw_{sa} = 7.2\cdot MHz$

sampling frequency: $fpt_{so} := 2\cdot Bw_{sa}$ $\qquad fpt_{so} = 14.4\cdot MHz$

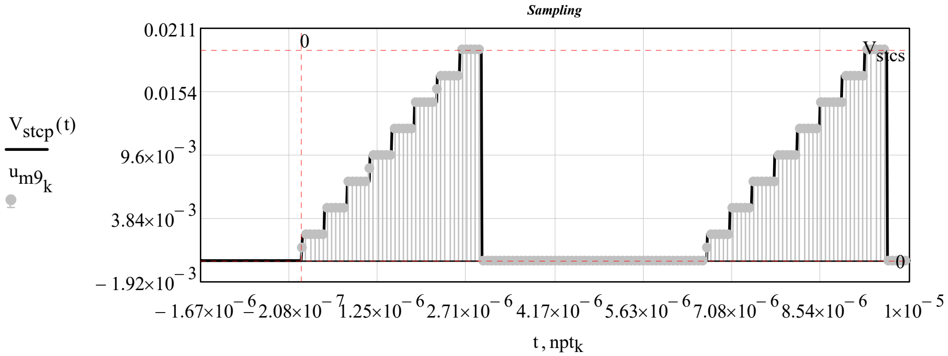
$$npt_k := \frac{k}{fpt_{so}}$$

Frequency resolution: $\dfrac{N0_{gd}}{fpt_{so}}\cdot\dfrac{1}{T_{stcpt}} = 2.667$

$$u_{m9_k} := V_{stcp}\left(npt_k\right)$$

$u_{m9}^{T} =$

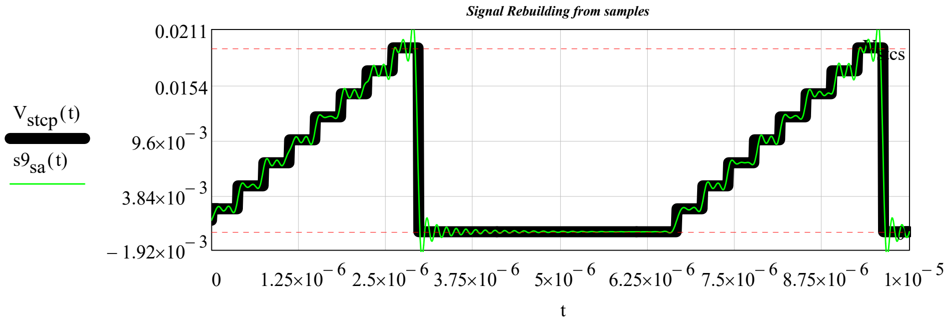
	0	1	2	3	4	5	6
0	1.2·10^-3	2.4·10^-3	2.4·10^-3	2.4·10^-3	2.4·10^-3	2.4·10^-3	...



$relerr = 10\cdot\%$ $\qquad \omega_{bwr} := 2\cdot\pi\cdot Bw_{sa}$ $\qquad \omega_{bwr} = 45.239\cdot\dfrac{Mrads}{sec}$ $\qquad n\cdot\dfrac{\pi}{\omega_{bwr}} = n\cdot\dfrac{1}{2\cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula: $$s9_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1}\left(u_{m9_n}\cdot sinc\left(\omega_{bwr}\cdot t - n\cdot\pi\right)\right)\right] \qquad N0_{gd} - 1 = 255 \qquad relerr = 10\cdot\%$$

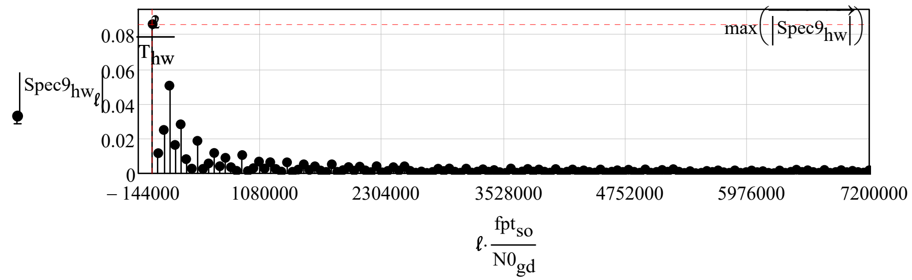


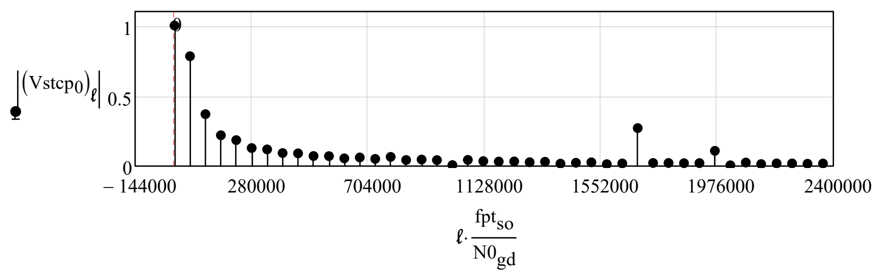
$$\mathrm{length}(u_{m9}) = 256$$

$$fpt_{so} = 1.44 \times 10^{4} \cdot \mathrm{kHz}$$

$$Spec9_{hw} := \mathrm{fft}(u_{m9}) \qquad \mathrm{length}(Spec9_{hw}) = 129$$

$$\ell := 0 .. \frac{N0_{gd}}{2} \qquad \frac{N0_{gd}}{2} = 128$$





TEST Waveforms

Periodic Waveforms

10 Staircase 2 Voltage Pulse Train

Description of the Function's parameters:

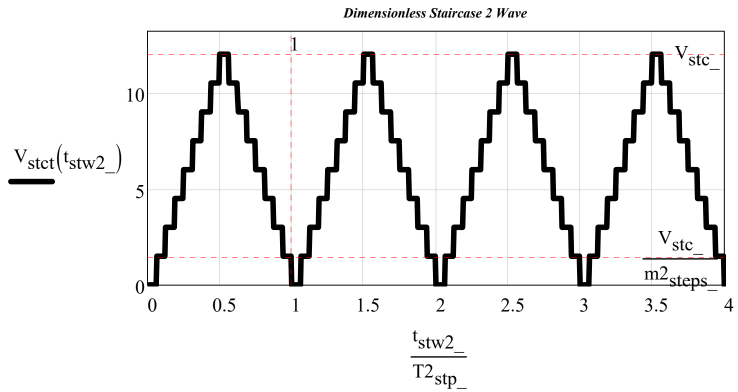
v_{stct}(time, period , max_amplitude, number_of_steps, max_number_of_periods)

v_{stcc}(t_{sl}, step_length, signal_amplitude, number_of_steps, number_max_of_periods)

For data, see "*staircase 2 pulse data*"

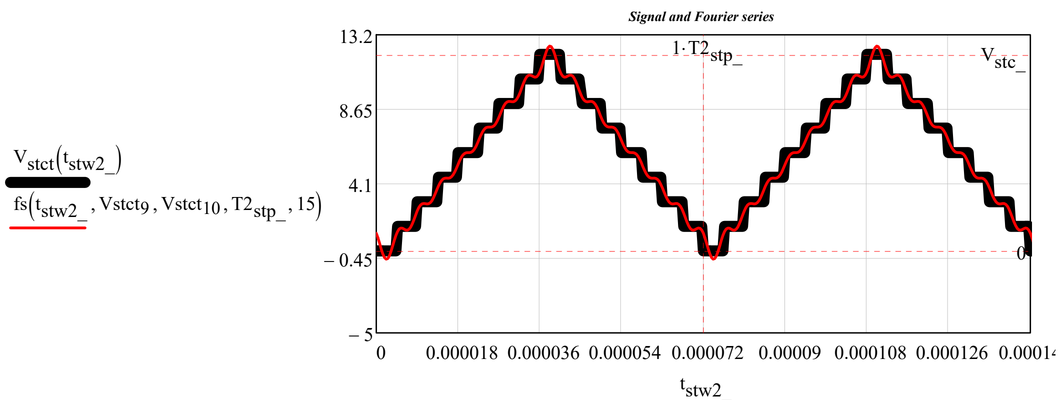
$$t_{stw2\_} := 0 \cdot T2_{stp\_}, 0 \cdot T2_{stp\_} + \frac{10 \cdot T2_{stp\_}}{2000} .. 10 \cdot T2_{stp\_}$$

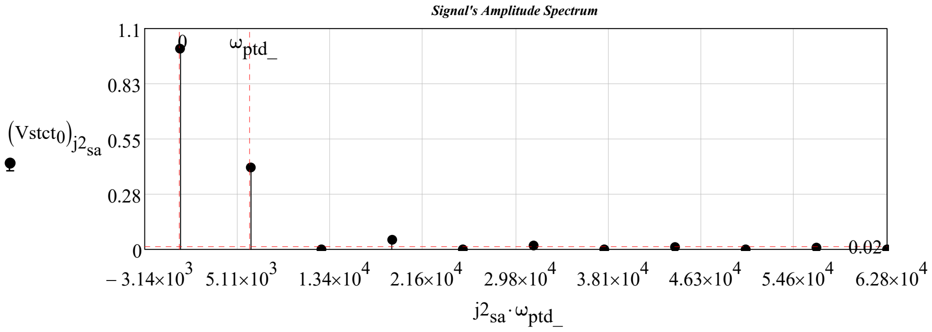
$$V_{stct}(t) := \frac{v_{stct}(t, T2_{stp\_}, V_{stc\_}, m2_{steps\_}, N0_{gd})}{V}$$

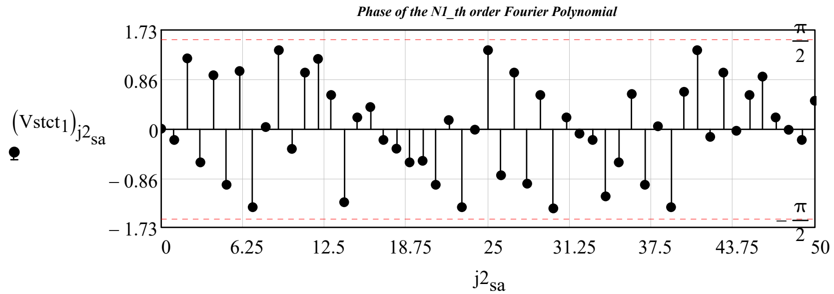


$$Vstct := \mathrm{SPCT}(V_{stct}, rt_{gd}, N1\_, 0 \cdot s, T2_{stp\_}) \qquad N1\_ = 50$$

$$j2_{sa} := 0 .. \mathrm{rows}(Vstct_0) - 1 \qquad \omega_{ptd\_} = 6.283 \times 10^{-3} \cdot \frac{\mathrm{Mrads}}{s}$$







$Bw_{sa} := Vstct_3 \cdot Hz$

$Bw_{sa} = 0.667 \cdot MHz$

sampling frequency: $fpt_{so} := 2 \cdot Bw_{sa}$ $\qquad fpt_{so} = 1.333 \cdot MHz$

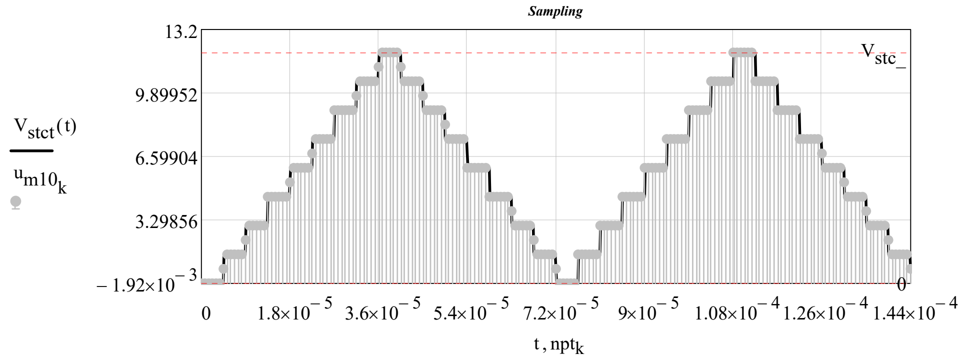
$$npt_k := \frac{k}{fpt_{so}}$$

Frequency resolution: $\dfrac{N0_{gd}}{fpt_{so}} \cdot \dfrac{1}{T2_{stp\_}} = 2.667$

$$u_{m10_k} := V_{stct}\left(npt_k\right)$$

$u_{m10}{}^{T} =$

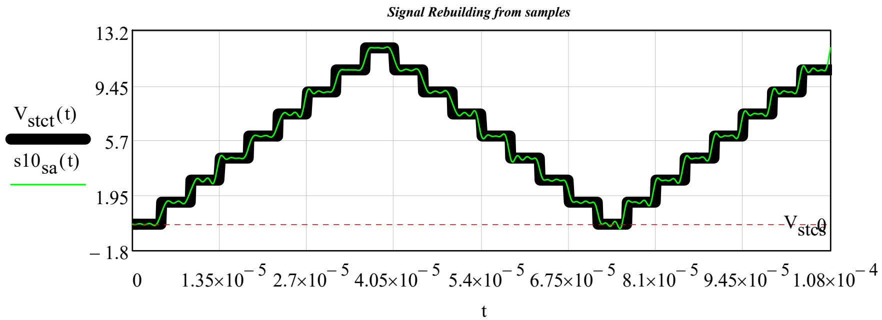
	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0.75	1.5	...



relerr = 10·% $\qquad \omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa} \qquad \omega_{bwr} = 4.189 \cdot \dfrac{Mrads}{sec} \qquad n \cdot \dfrac{\pi}{\omega_{bwr}} = n \cdot \dfrac{1}{2 \cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula: $s10_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1} \left(u_{m10_n} \cdot sinc\left(\omega_{bwr} \cdot t - n \cdot \pi\right)\right)\right]$ $\qquad N0_{gd} - 1 = 255 \qquad relerr = 10 \cdot \%$

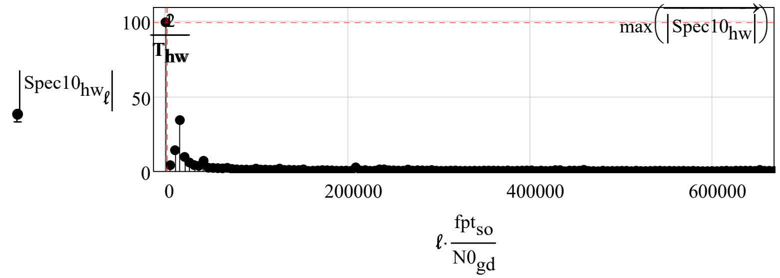


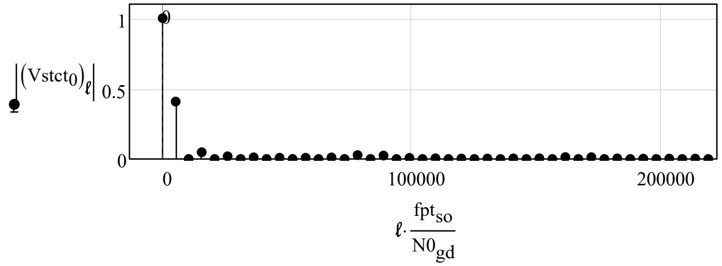
$length\left(u_{m10}\right) = 256$

$fpt_{so} = 1.333 \times 10^3 \cdot kHz$

$Spec10_{hw} := fft\left(u_{m10}\right) \quad length\left(Spec10_{hw}\right) = 129$

$$\ell := 0 .. \frac{N0_{gd}}{2} \qquad \frac{N0_{gd}}{2} = 128$$





TEST Waveforms

Periodic Waveforms

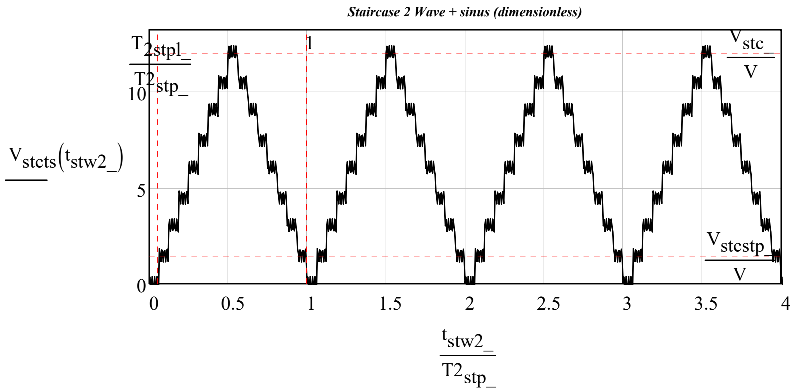
11 Staircase 2 Voltage Pulse Train + sinus

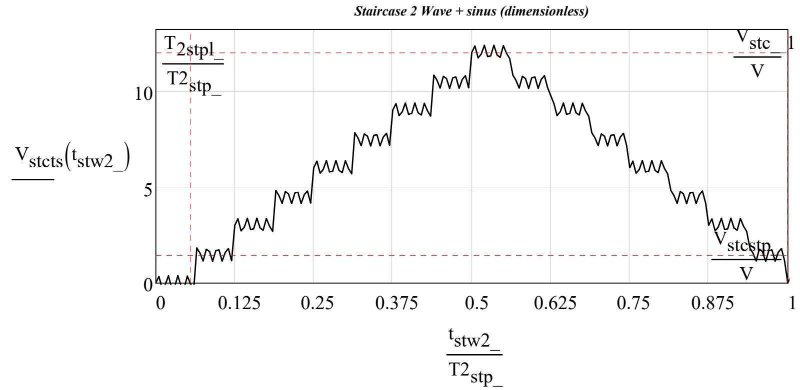
Description of the Function's parameters:

$Vstcsin(t_{s1}, period, max\_amplitude, number\_of\_steps, max\_number\_of\_periods, Number\_of\_periods)$

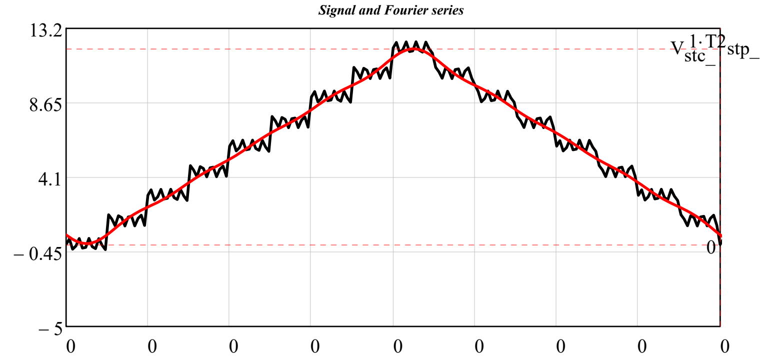
For data, see *" staircase 2 pulse data"*

$$V_{stcts}(t) := Vstcsin(t, T2_{stp\_}, V_{stc\_}, m2_{steps\_}, N0_{gd})$$



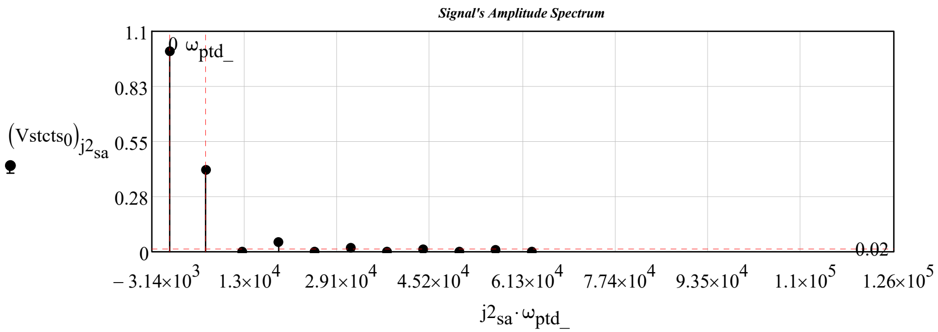


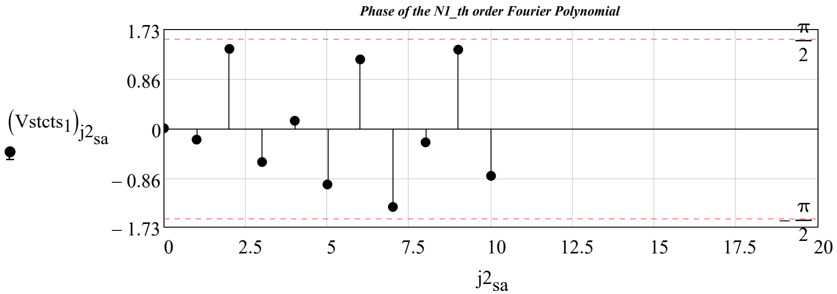
$N1\_ := 10 \qquad Vstcts := SPCT(V_{stcts}, rt_{gd}, N1\_, 0 \cdot s, T2_{stp\_})$



$N1\_ = 10$

$j2_{sa} := 0 .. rows(Vstcts_0) - 1 \qquad \omega_{ptd\_} = 6.283 \cdot \frac{krads}{s}$





$Bw_{sa} := Vstcts_3 \cdot Hz$

$Bw_{sa} = 0.111 \cdot MHz$

sampling frequency: $fpt_{so} := 2 \cdot Bw_{sa}$ $\quad fpt_{so} = 0.222 \cdot MHz$

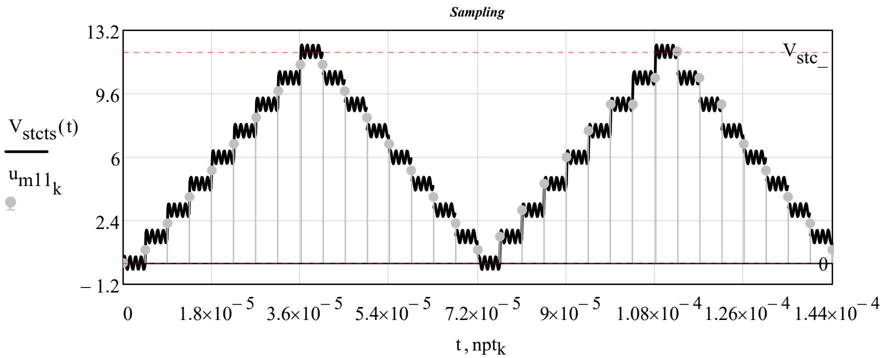
$$npt_k := \frac{k}{fpt_{so}}$$

Frequency resolution: $\frac{N0_{gd}}{fpt_{so}} \cdot \frac{1}{T2_{stp\_}} = 16$

$u_{m11_k} := V_{stcts}(npt_k)$

$u_{m11}^T =$

	0	1	2	3	4
0	0	0.75	2.25	3.75	...

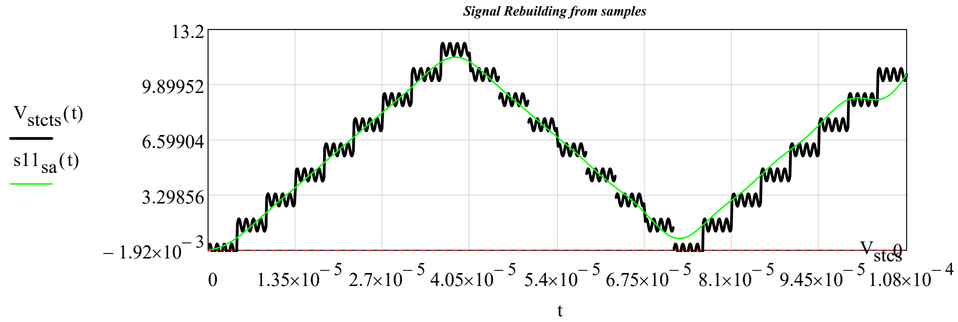


relerr = 10·% $\quad \omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa} \quad \omega_{bwr} = 0.698 \cdot \frac{Mrads}{sec} \quad n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula: $s11_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1} \left(u_{m11_n} \cdot \mathrm{sinc}(\omega_{bwr} \cdot t - n \cdot \pi)\right)\right]$ $\quad N0_{gd} - 1 = 255 \quad$ relerr = 10·%

N1_ = 10

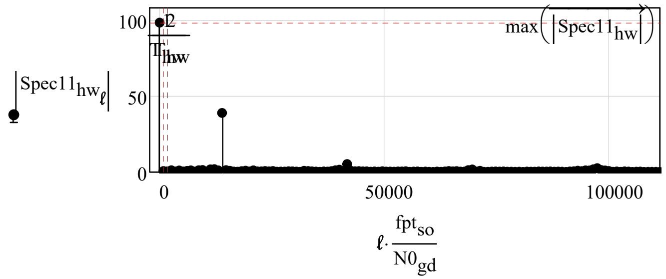


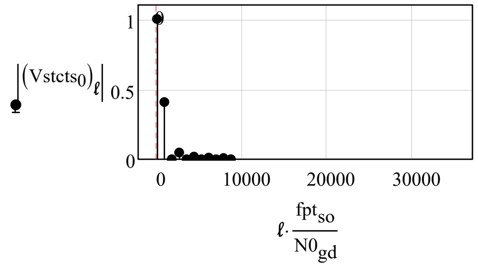
$\mathrm{length}(u_{m11}) = 256$

$fpt_{so} = 222.222 \cdot kHz$

$Spec11_{hw} := \mathrm{fft}(u_{m11}) \quad \mathrm{length}(Spec11_{hw}) = 129$

$\ell := 0 .. \frac{N0_{gd}}{2} \quad \frac{N0_{gd}}{2} = 128$





TEST Waveforms

Periodic Waveforms

12 Staircase 3 Voltage Pulse Train

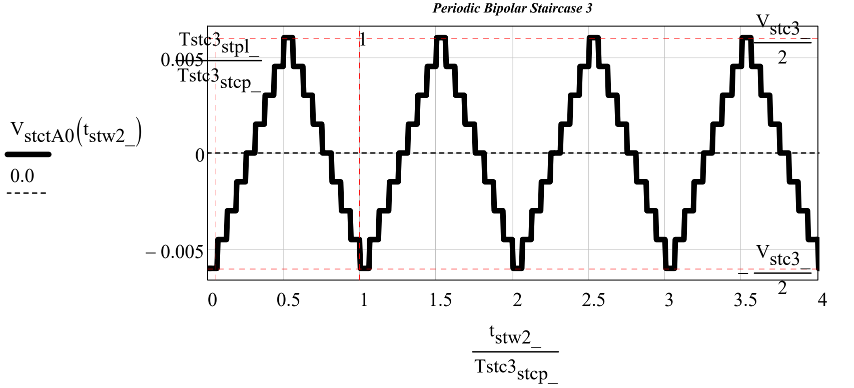
Description of the Function's parameters: $v_{stct}(t_{sl}, period, step\_amplitude, number\_of\_steps, max\_number\_of\_periods)$

$: v_{stctA0}[t_{sl}, (period, step\_amplitude, number\_of\_steps, max\_number\_of\_periods)]$

You can find the data in "*staircase 3 pulse data*"

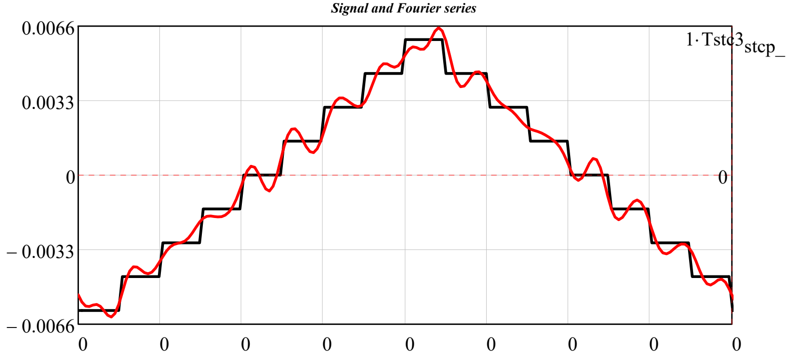
$$V_{stctA0}(t) := \frac{v_{stctA0}(t, Tstc3_{stcp\_}, V_{stc3\_}, mstc3_{steps\_}, N0_{gd})}{V} \qquad N1\_ := 25$$

Periodic Bipolar Staircase 3



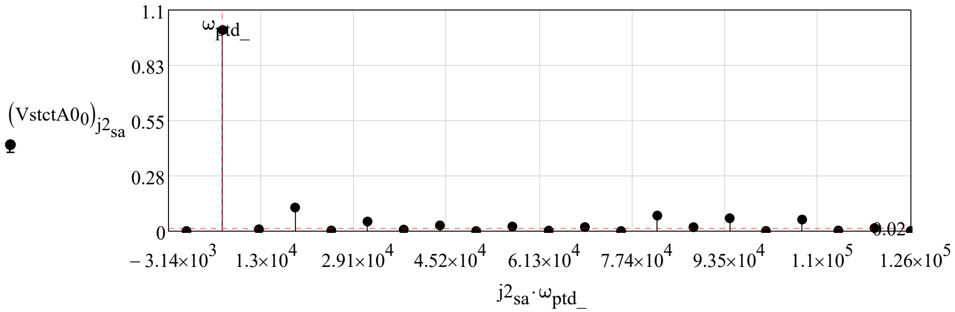
$$VstctA0 := SPCT(V_{stctA0}, rt_{gd}, N1\_, 0 \cdot s, Tstc3_{stcp\_}) \qquad N1\_ = 25$$

Signal and Fourier series

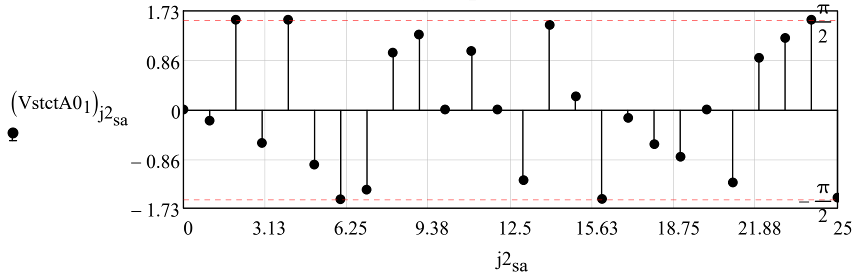


$$j2_{sa} := 0 .. rows(VstctA0_0) - 1 \qquad \omega_{ptd\_} = 6.283 \times 10^{-3} \cdot \frac{Mrads}{s}$$

Signal's Amplitude Spectrum



Phase of the N1_th order Fourier Polynomial



$$Bw_{sa} := VstctA0_3 \cdot Hz$$

$$Bw_{sa} = 0.328 \cdot MHz$$

sampling frequency: $fpt_{so} := 2 \cdot Bw_{sa}$ $\qquad fpt_{so} = 0.656 \cdot MHz$

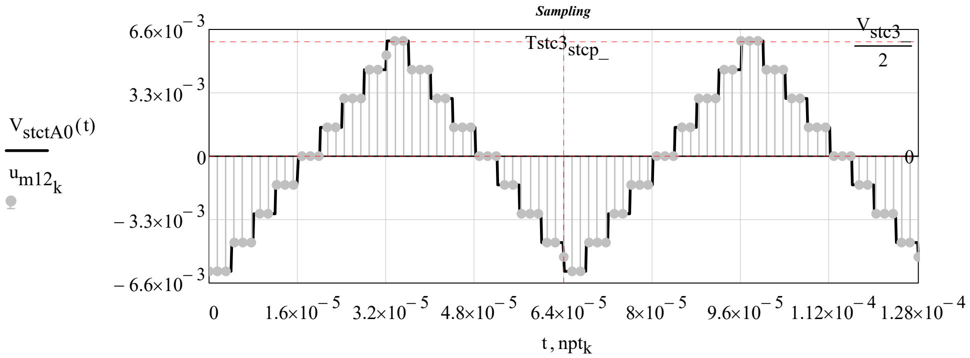
$$npt_k := \frac{k}{fpt_{so}}$$

Frequency resolution: $\frac{N0_{gd}}{fpt_{so}} \cdot \frac{1}{T2_{stp\_}} = 5.418$

$$u_{m12_k} := V_{stctA0}(npt_k)$$

$u_{m12}^T =$

	0	1	2	3	4	5
0	$-6 \cdot 10^{-3}$	$-6 \cdot 10^{-3}$	$-6 \cdot 10^{-3}$	$-4.5 \cdot 10^{-3}$	$-4.5 \cdot 10^{-3}$	...

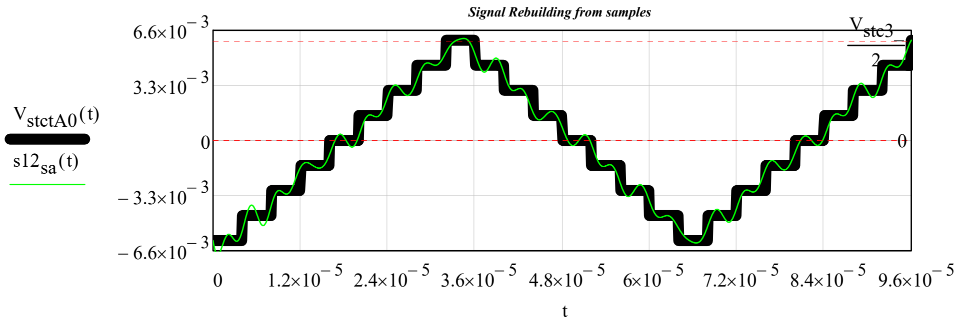


relerr = 10·% $\omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa}$ $\omega_{bwr} = 2.062 \cdot \frac{Mrads}{sec}$ $n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula: $s12_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1} \left(u_{m12_n} \cdot \mathrm{sinc}(\omega_{bwr} \cdot t - n \cdot \pi) \right) \right]$ $N0_{gd} - 1 = 255$ relerr = 10·%

$N1\_ = 25$

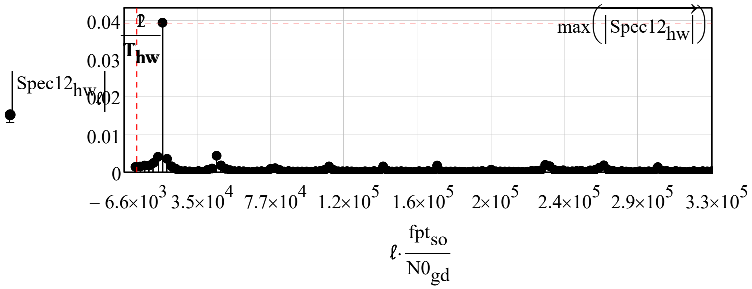


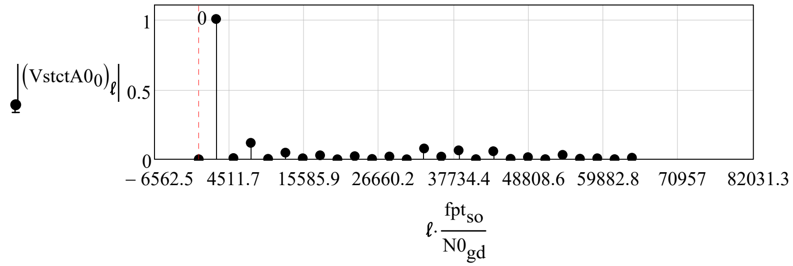
$\mathrm{length}(u_{m12}) = 256$

$fpt_{so} = 656.25 \cdot kHz$

$Spec12_{hw} := \mathrm{fft}(u_{m12})$ $\mathrm{length}(Spec12_{hw}) = 129$

$\ell := 0 .. \frac{N0_{gd}}{2}$ $\frac{N0_{gd}}{2} = 128$

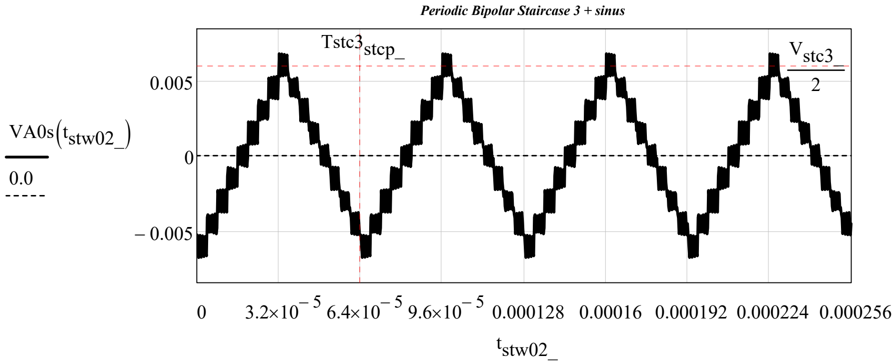




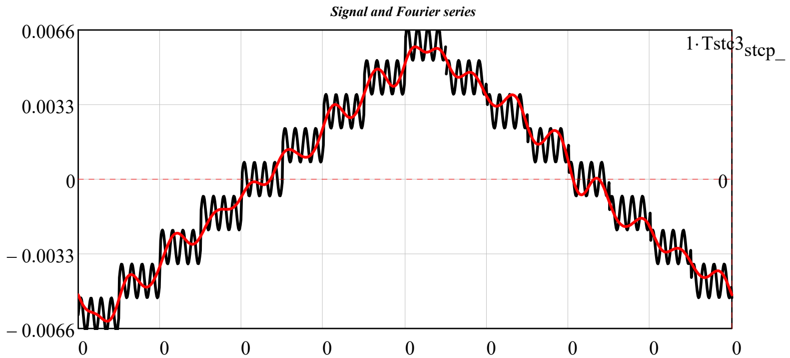
Periodic Waveforms

13 Staircase 3 Voltage Pulse Train + sinus

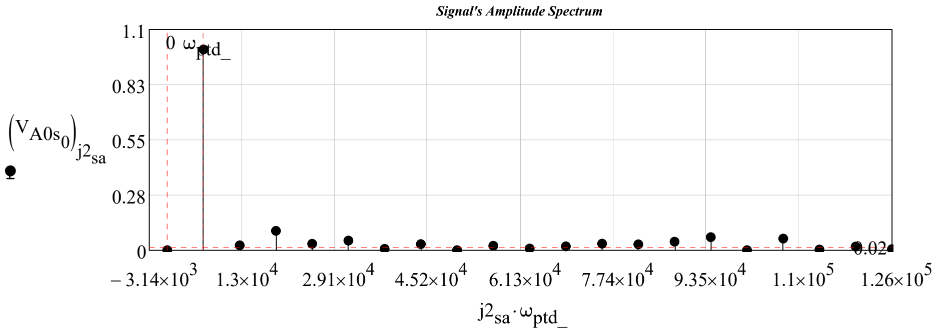
$$VA0s(t) := VistctA0sin\left(t, Tstc3_{stcp\_}, V_{stc3\_}, mstc3_{steps\_}, N0_{gd}\right)$$

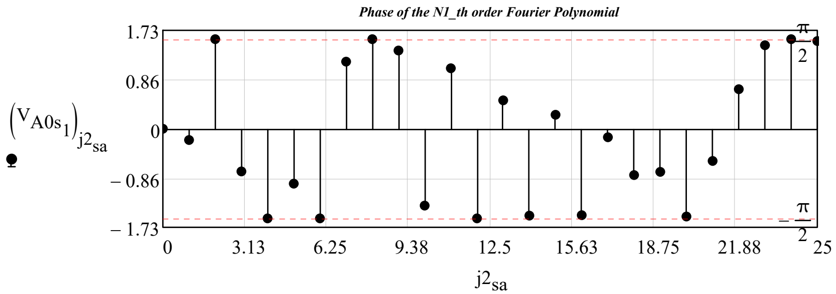


$$V_{A0s} := SPCT\left(VA0s, rt_{gd}, N1\_, 0 \cdot s, Tstc3_{stcp\_}\right) \qquad N1\_ = 25$$



$$j2_{sa} := 0 .. rows\left(V_{A0s_0}\right) - 1 \qquad \omega_{ptd\_} = 6.283 \times 10^{-3} \cdot \frac{Mrads}{s}$$





$$Bw_{sa} := V_{A0s_3} \cdot Hz$$

$$Bw_{sa} = 0.344 \cdot MHz$$

sampling frequency: $fpt_{so} := 2 \cdot Bw_{sa}$ $\qquad fpt_{so} = 0.688 \cdot MHz$

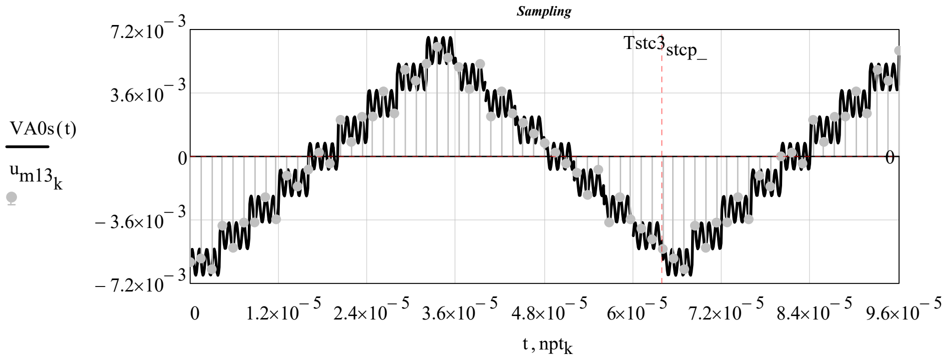
$$npt_k := \frac{k}{fpt_{so}}$$

Frequency resolution: $\frac{N0_{gd}}{fpt_{so}} \cdot \frac{1}{T2_{stp\_}} = 5.172$

$$u_{m13_k} := VA0s\left(npt_k\right)$$

$u_{m13}^{T} =$

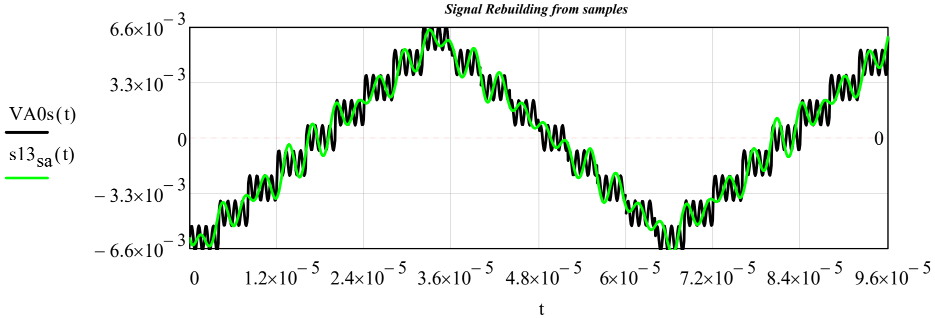
	0	1	2	3	4
0	$-6 \cdot 10^{-3}$	$-5.789 \cdot 10^{-3}$	$-6.405 \cdot 10^{-3}$	$-3.933 \cdot 10^{-3}$	...



relerr = 10·% $\omega_{bwr} := 2\cdot\pi\cdot Bw_{sa}$ $\omega_{bwr} = 2.16\cdot\frac{Mrads}{sec}$ $n\cdot\frac{\pi}{\omega_{bwr}} = n\cdot\frac{1}{2\cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula: $s13_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1}\left(u_{m13_n}\cdot sinc\left(\omega_{bwr}\cdot t - n\cdot\pi\right)\right)\right]$ $N0_{gd} - 1 = 255$ relerr = 10·%

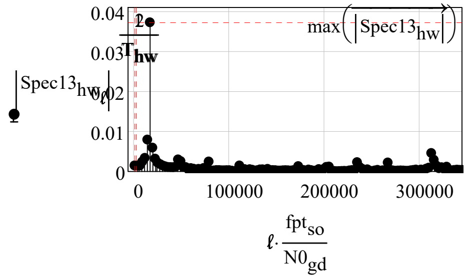


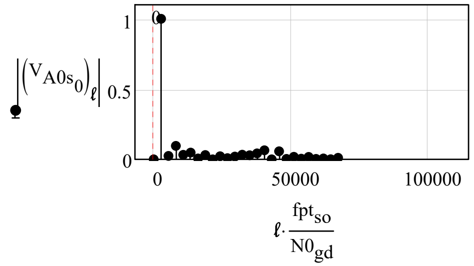
$length(u_{m13}) = 256$

$fpt_{so} = 687.5\cdot kHz$

$Spec13_{hw} := fft(u_{m13})$ $length(Spec13_{hw}) = 129$

$\ell := 0 .. \frac{N0_{gd}}{2}$ $\frac{N0_{gd}}{2} = 128$





TEST Waveforms

Periodic Waveforms

14 Staircase 4 Voltage Pulse Train

Description of the Function's parameters : vstc1p(time, step length, max amplitude, number of steps in half period, max number of periods)
To modify data, see the worksheet *"staircase 4 pulse data.xmcd"*

Step Amplitude: $V_{stc4} = 15\,V$

Step length: $T_{4stpl\_} = 1.481\cdot\mu s$

Number of steps: $2\cdot m4_{steps} + 1 = 17$

Time constant: $\tau 4\_ = 74.074\cdot ns$

Period: $T4_{stcp\_} = 0.025\cdot ms$

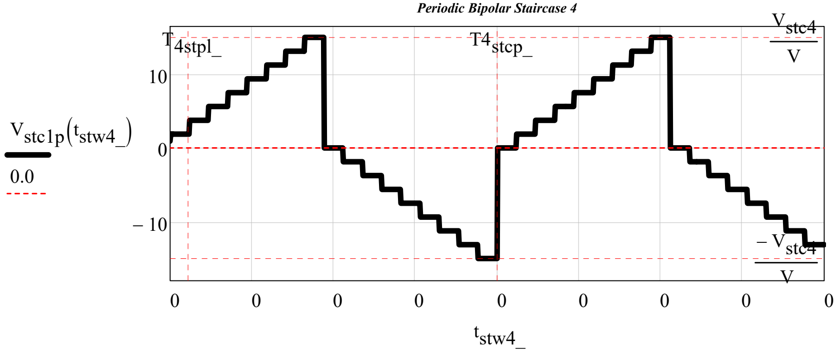
Frequency: $f_{44stcp\_} = 39.706\cdot kHz$

$\omega_{44stcp\_} = 249.479\cdot\frac{krads}{sec}$

Description of the Function's parameters : vstc1p(time, step length, max amplitude, number of steps)

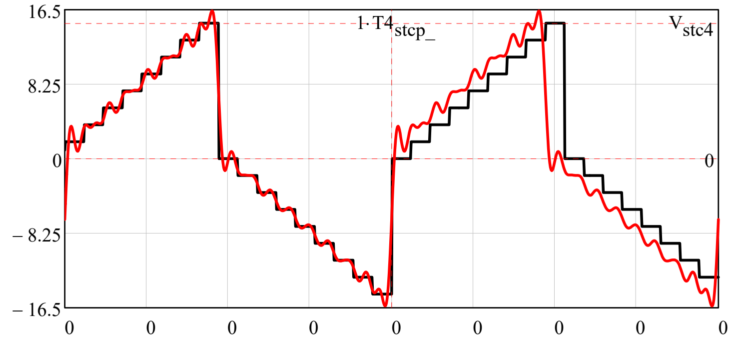
$$V_{stc1p}(t) := \frac{vstc1p\left(t, T_{4stpl\_}, V_{stc4}, m4_{steps}, N1\_\right)}{V}$$

Periodic Bipolar Staircase 4



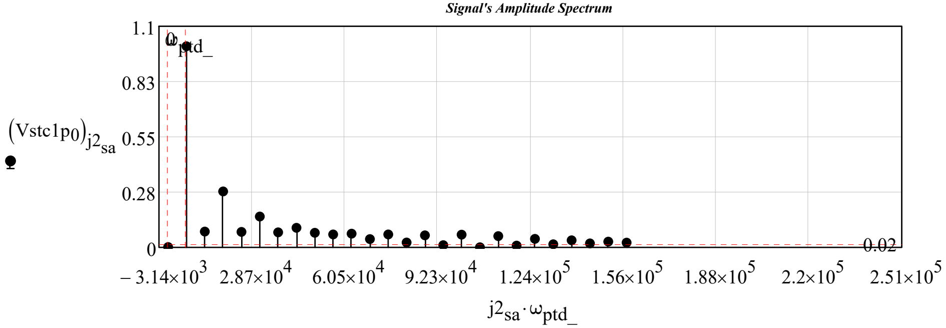
$$Vstc1p := SPCT\left(V_{stc1p}, rt_{gd}, N1\_, 0 \cdot s, T4_{stcp\_}\right) \qquad N1\_ = 25$$

Signal and Fourier series

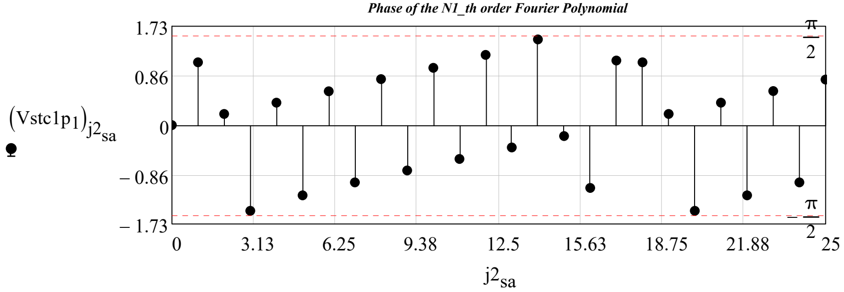


$$j2_{sa} := 0 .. rows\left(V_{A0s_0}\right) - 1 \qquad \omega_{ptd\_} = 6.283 \times 10^{-3} \cdot \frac{Mrads}{s}$$

Signal's Amplitude Spectrum



Phase of the N1_th order Fourier Polynomial



$$Bw_{sa} := Vstc1p_3 \cdot Hz$$

$$Bw_{sa} = 0.913 \cdot MHz$$

sampling frequency: $fpt_{so} := 2 \cdot Bw_{sa}$ $\qquad fpt_{so} = 1.826 \cdot MHz$

$$npt_k := \frac{k}{fpt_{so}}$$

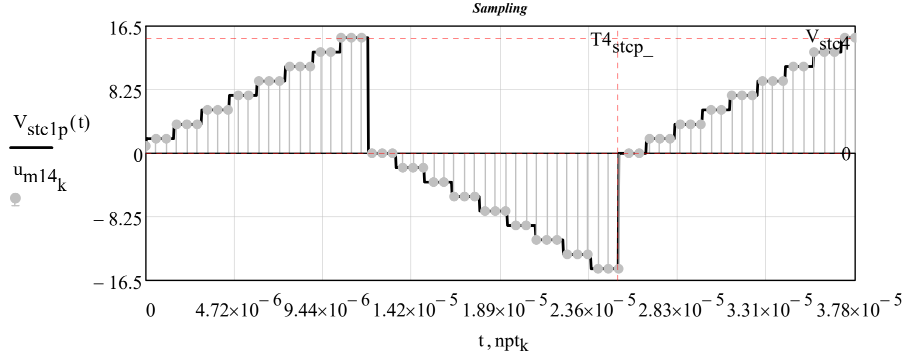
Frequency resolution: $\frac{N0_{gd}}{fpt_{so}} \cdot \frac{1}{T4_{stcp\_}} = 5.565$

$$u_{m14_k} := V_{stc1p}\left(npt_k\right)$$

$u_{m14}^T =$

	0	1	2	3	4	5	6
0	0.938	1.875	1.875	3.75	3.75	3.75	...

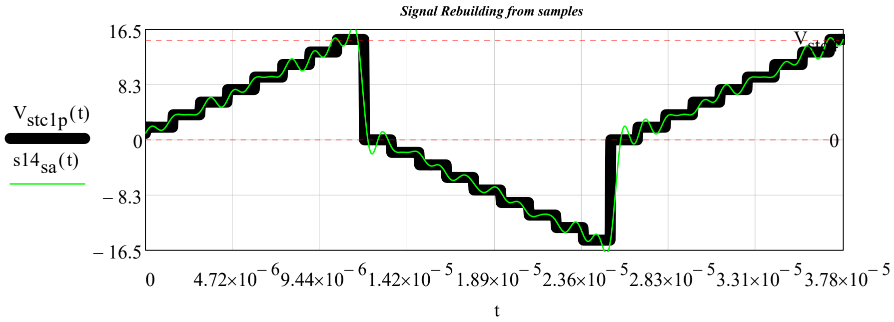
Sampling



relerr = 10·% $\qquad \omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa} \qquad \omega_{bwr} = 5.738 \cdot \frac{Mrads}{sec} \qquad n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula:: $\quad s14_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1} \left(u_{m14_n} \cdot sinc\left(\omega_{bwr} \cdot t - n \cdot \pi\right)\right)\right] \qquad N0_{gd} - 1 = 255 \qquad relerr = 10 \cdot \%$

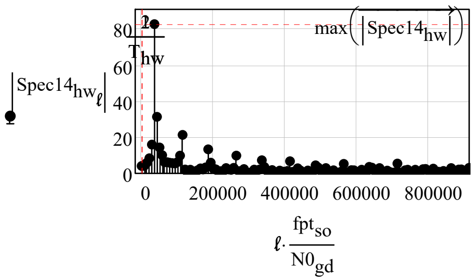


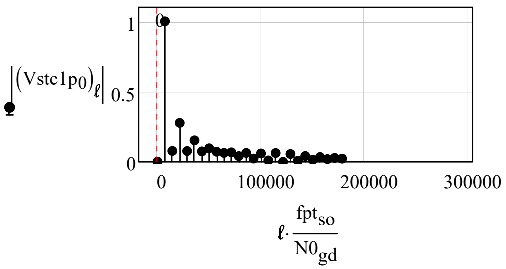
$$\text{length}\left(u_{m14}\right) = 256$$

$$fpt_{so} = 1.826 \times 10^{3} \cdot \text{kHz}$$

$$Spec14_{hw} := \text{fft}\left(u_{m14}\right) \quad \text{length}\left(Spec14_{hw}\right) = 129$$

$$\ell := 0 .. \frac{N0_{gd}}{2} \qquad \frac{N0_{gd}}{2} = 128$$





TEST Waveforms

Periodic Waveforms

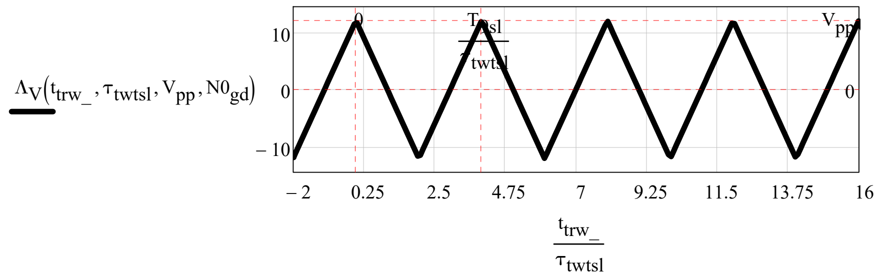
15 Bipolar Triangular Voltage Wave

Description of the Function's parameters : Λ_V(time, triangle half base, triangle amplitude, max number of periods)

Time constant: $\tau_{twtsl} := 1 \cdot \mu s$

Period: $T_{9sl} := 4 \cdot \tau_{twtsl}$ $\qquad f_{9sl} := \frac{1}{T_{9sl}}$

$$t_{trw\_} := -1 \cdot T_{9sl}, -1 \cdot T_{9sl} + \frac{20 \cdot T_{9sl} + 1 \cdot T_{9sl}}{1000} .. 20 \cdot T_{9sl}$$



Bipolar Triangular Voltage Wave Built using the Step Function

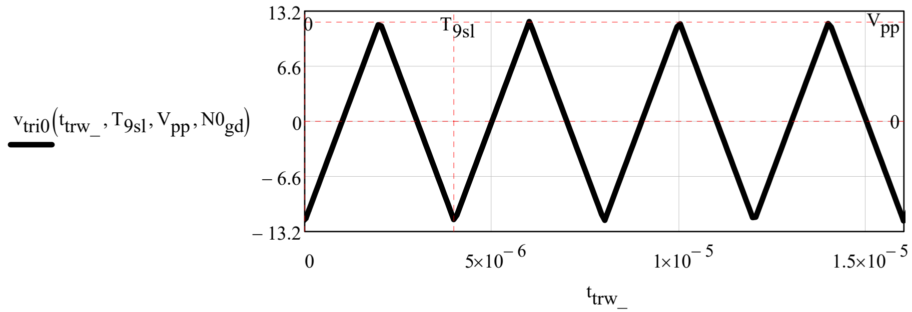
Signal amplitude: $V_{pp} = 12 \cdot V$

Time constant: $\tau_{twtsl} = 1 \cdot \mu s$

Period: $T_{9sl} = 4 \cdot \mu s$

$$\omega_{9sl} := 2 \cdot \pi \cdot f_{9sl} \qquad \omega_{9sl} = 1.571 \times 10^{6} \cdot \frac{rad}{sec}$$

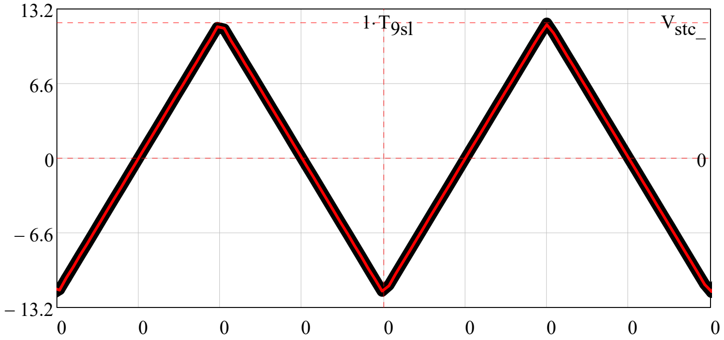
$$N0_{gd} = 256$$



$$V_{tri0}(t) := \frac{v_{tri0}\left(t, T_{9sl}, V_{pp}, N0_{gd}\right)}{V}$$

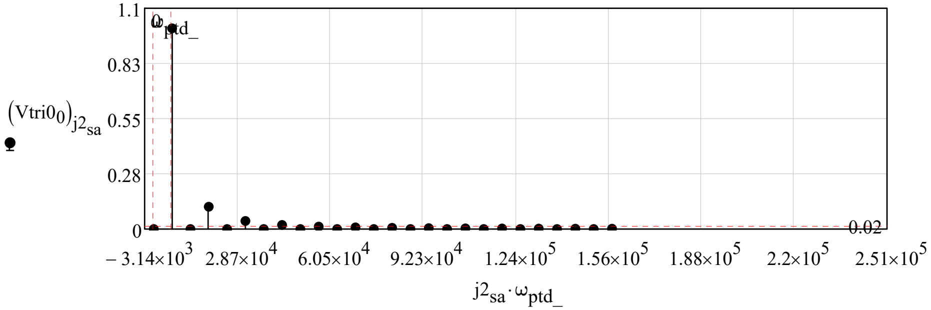
$$Vtri0 := SPCT\left(V_{tri0}, rt_{gd}, N1\_, 0\cdot s, T_{9sl}\right) \qquad N1\_ = 25$$

Signal and Fourier series

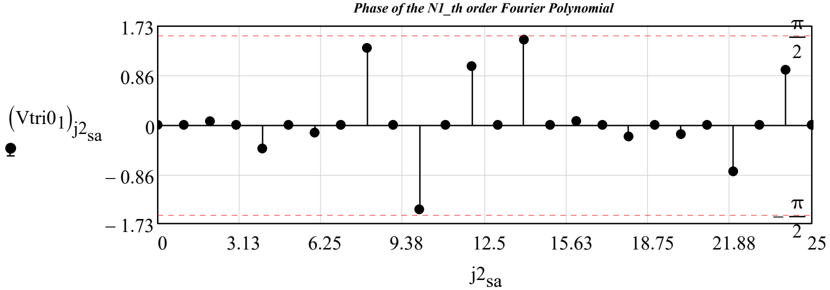


$$j2_{sa} := 0\,..\,rows\left(Vtri0_0\right) - 1 \qquad \omega_{ptd\_} = 6.283\times 10^{-3}\cdot$$

Signal's Amplitude Spectrum



Phase of the N1_th order Fourier Polynomial



$$Bw_{sa} := Vtri0_3\cdot Hz$$

$$Bw_{sa} = 3.5\cdot MHz$$

sampling frequency: $fpt_{so} := 2\cdot Bw_{sa} \qquad fpt_{so} = 7\cdot MHz$

$$npt_k := \frac{k}{fpt_{so}}$$

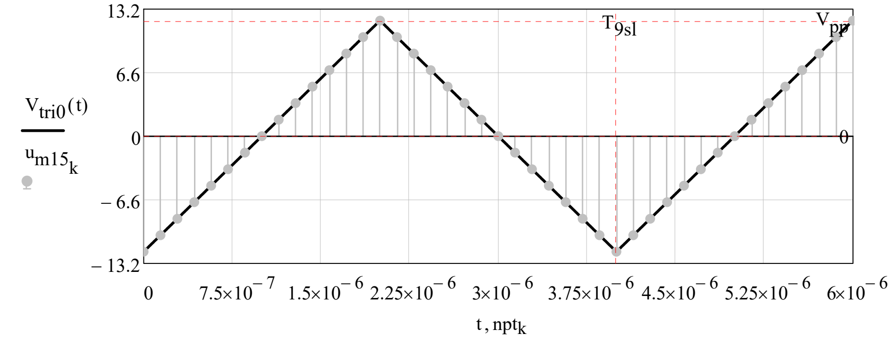
Frequency resolution: $\frac{N0_{gd}}{fpt_{so}}\cdot\frac{1}{T4_{stcp\_}} = 1.452$

$$u_{m15_k} := V_{tri0}\left(npt_k\right)$$

$u_{m15}{}^T =$

	0	1	2	3	4
0	-12	-10.286	-8.571	-6.857	...

Sampling

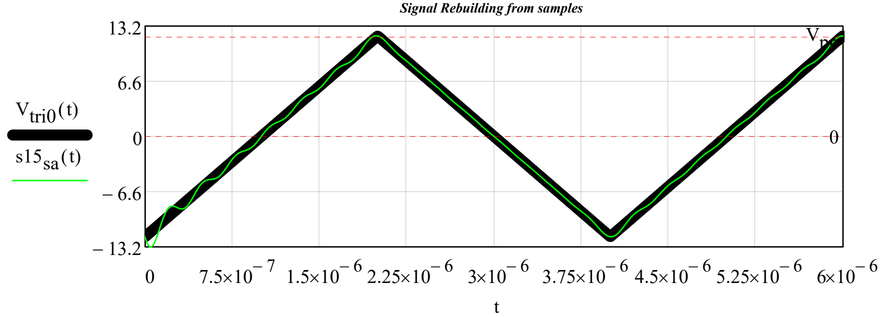


$relerr = 10\cdot\%$ $\qquad \omega_{bwr} := 2\cdot\pi\cdot Bw_{sa} \qquad \omega_{bwr} = 21.991\cdot\frac{Mrads}{sec} \qquad n\cdot\frac{\pi}{\omega_{bwr}} = n\cdot\frac{1}{2\cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula: $s15_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1}\left(u_{m15_n}\cdot sinc\left(\omega_{bwr}\cdot t - n\cdot\pi\right)\right)\right] \qquad N0_{gd} - 1 = 255 \qquad relerr = 10\cdot\%$

Signal Rebuilding from samples

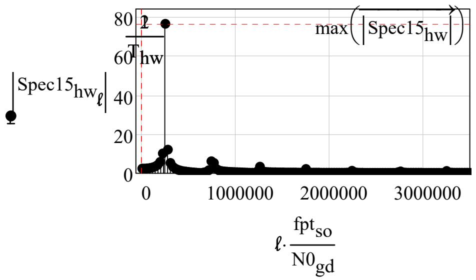


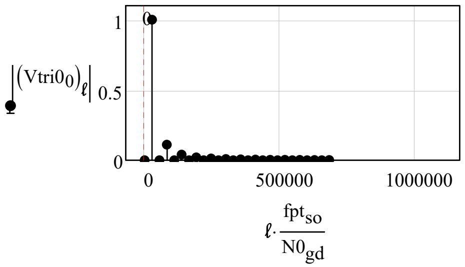
$$length\left(u_{m15}\right) = 256$$

$$fpt_{so} = 7\times 10^3\cdot kHz$$

$$Spec15_{hw} := fft\left(u_{m15}\right) \quad length\left(Spec15_{hw}\right) = 129$$

$$\ell := 0 .. \frac{N0_{gd}}{2} \qquad \frac{N0_{gd}}{2} = 128$$





Periodic Waveforms

16 Triangular Cusps Voltage Pulse Train

Signal amplitude: $V_{pp} = 12 \cdot V$

Pulse width: $p_{wsl} := \tau_{ptd\_} \qquad p_{wsl} = 250 \cdot \mu s$

Max pulse amplitude and cusp ratio: $a_{psl} := \frac{1}{4} \qquad a_{psl} < 1$

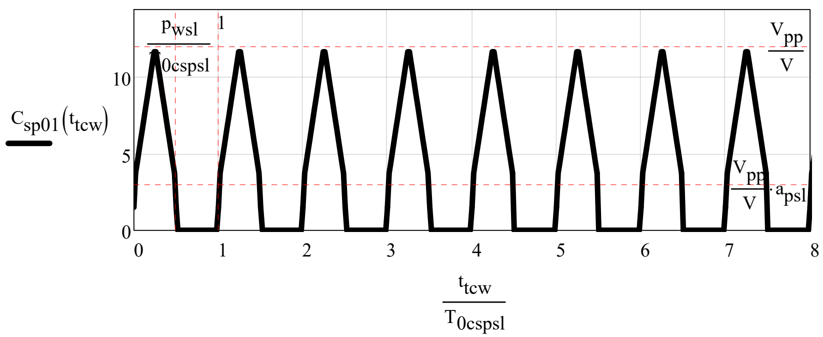
Cusp slope $c_{ssl} := V_{pp} \cdot \frac{2 \cdot (1 - a_{psl})}{p_{wsl}} \qquad c_{ssl} = 0.$

Period: $T_{0cspsl} := 2 \cdot p_{wsl}$

$$t_{tcw} := 0 \cdot T_{0cspsl}, 0 \cdot T_{0cspsl} + \frac{10 \cdot T_{0cspsl} - 0 \cdot T_{0cspsl}}{500} .. 10 \cdot T_{0cspsl}$$

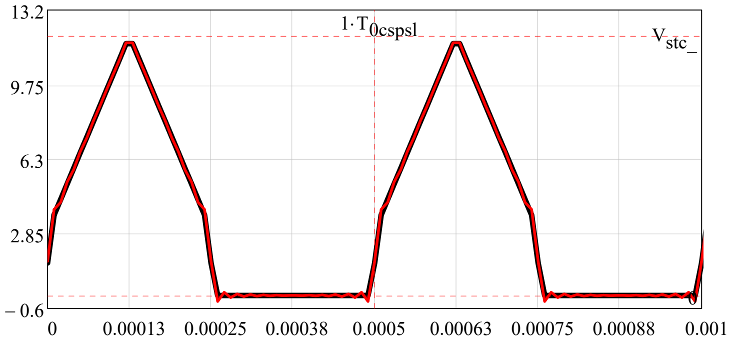
$$C_{sp01}(t) := \frac{csp01\left(t, p_{wsl}, a_{psl}, T_{0cspsl}, V_{pp}, N0_{gd}\right)}{V}$$

Cusp Pulses Train



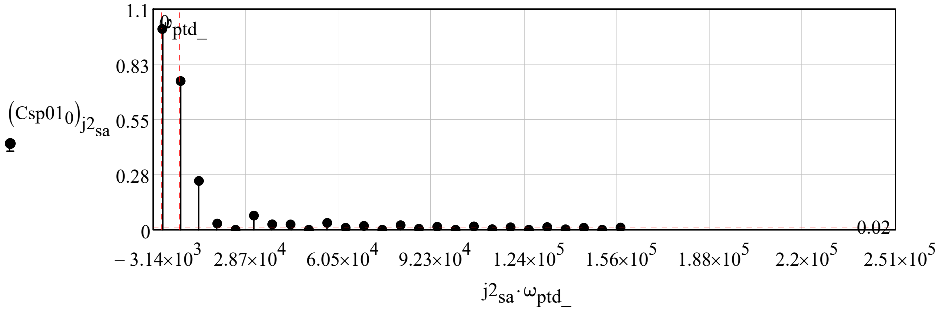
$$Csp01 := SPCT\left(C_{sp01}, rt_{gd}, N1\_, 0 \cdot s, T_{0cspsl}\right) \qquad N1\_ = 25$$

Signal and Fourier series

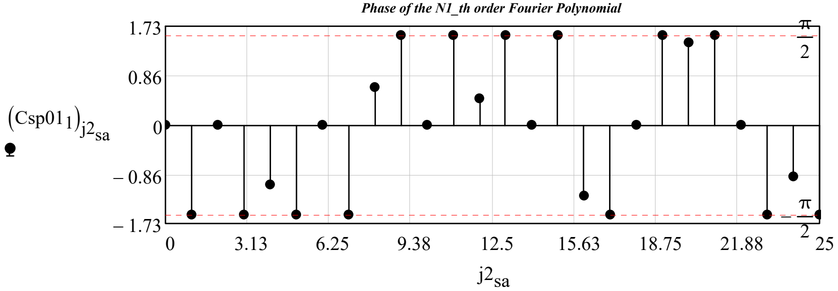


$j2_{sa} := 0 .. \mathrm{rows}(Csp01_0) - 1$ $\omega_{ptd\_} = 6.283 \times 10^{-3} \cdot \frac{Mrads}{s}$

Signal's Amplitude Spectrum



Phase of the N1_th order Fourier Polynomial



$Bw_{sa} := Csp01_3 \cdot Hz$

$Bw_{sa} = 0.046 \cdot MHz$

sampling frequency: $fpt_{so} := 2 \cdot Bw_{sa}$ $fpt_{so} = 0.092 \cdot MHz$

$npt_k := \frac{k}{fpt_{so}}$

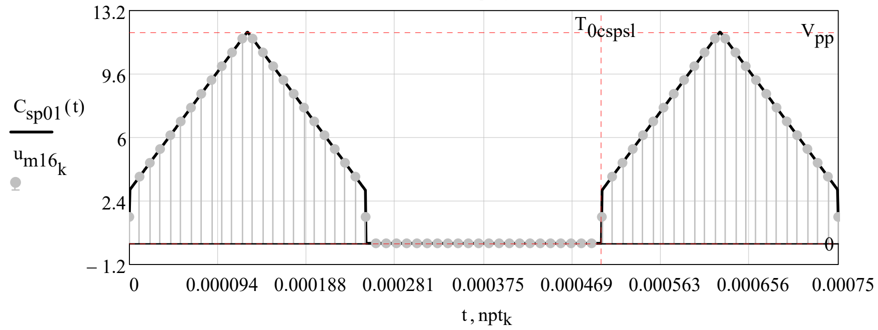
Frequency resolution: $\frac{N0_{gd}}{fpt_{so}} \cdot \frac{1}{T4_{stcp\_}} = 110.486$

$u_{m16_k} := C_{sp01}(npt_k)$

$u_{m16}^{T} =$

	0	1	2	3	4	5	6	7
0	1.5	3.783	4.565	5.348	6.13	6.913	7.696	...

Sampling

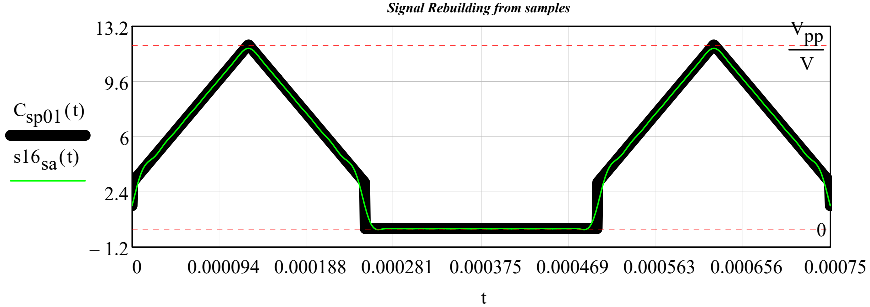


relerr $= 10 \cdot \%$ $\omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa}$ $\omega_{bwr} = 0.289 \cdot \frac{Mrads}{sec}$ $n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula: $s16_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1} \left(u_{m16_n} \cdot \mathrm{sinc}(\omega_{bwr} \cdot t - n \cdot \pi) \right) \right]$ $N0_{gd} - 1 = 255$ relerr $= 10 \cdot \%$

Signal Rebuilding from samples

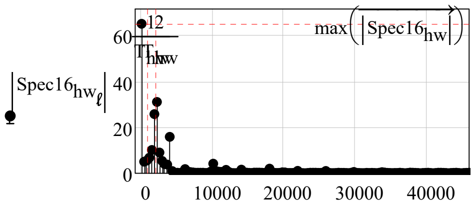


$\mathrm{length}(u_{m16}) = 256$

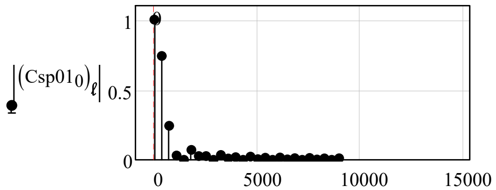
$fpt_{so} = 92 \cdot kHz$

$Spec16_{hw} := \mathrm{fft}(u_{m16})$ $\mathrm{length}(Spec16_{hw}) = 129$

$$\ell := 0 .. \frac{N0_{gd}}{2} \qquad \frac{N0_{gd}}{2} = 128$$



$$\ell \cdot \frac{fpt_{so}}{N0_{gd}}$$



$$\ell \cdot \frac{fpt_{so}}{N0_{gd}}$$

TEST Waveforms

Periodic Waveforms

17 Bipolar Sawtooth with positive slope Pulse Train

Period: $T_{sawth\_} := 1 \cdot \delta_{sawth\_}$

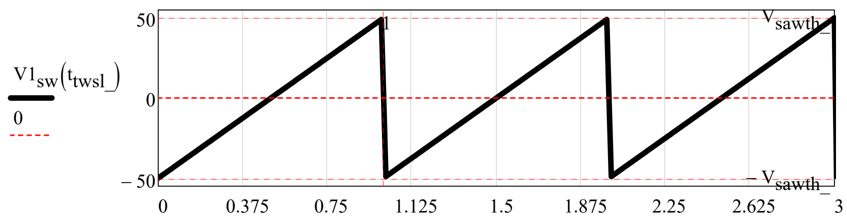
Frequency: $f_{sawth\_} := \frac{1}{T_{sawth\_}}$ $\qquad f_{sawth\_} = 1 \cdot MHz$

$T_{sawth\_} = 1 \cdot \mu s$ $\qquad \omega_{sawth\_} := 2 \cdot \pi \cdot f_{sawth\_}$ $\qquad \omega_{sawth\_} = 6.283 \cdot \frac{Mrads}{sec}$

$$t_{twsl\_} := 0, \frac{5 \cdot T_{sawth\_}}{500} .. 5 \cdot T_{sawth\_}$$

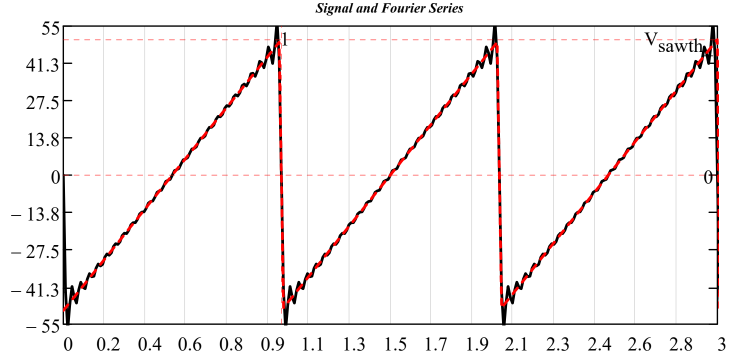
$$V1_{sw}(t) := \frac{v1_{sw}(t, T_{sawth\_}, V_{sawth\_}, N0_{gd})}{V}$$

Bipolar Sawtooth with positive slope



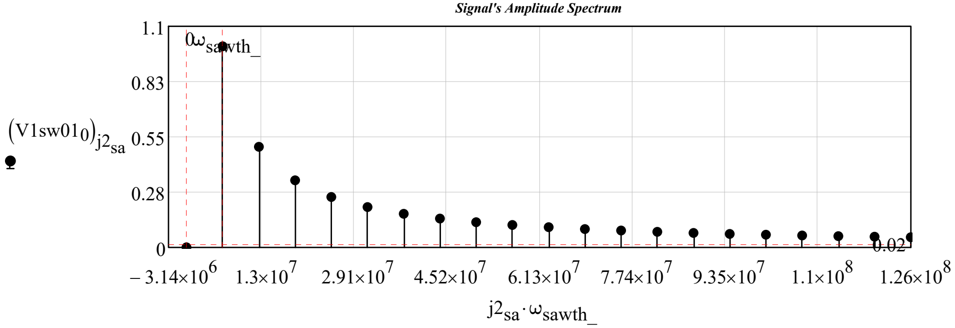
$$V1sw01 := SPCT(V1_{sw}, rt_{gd}, N1\_, 0 \cdot s, T_{sawth\_}) \qquad N1\_ = 25$$

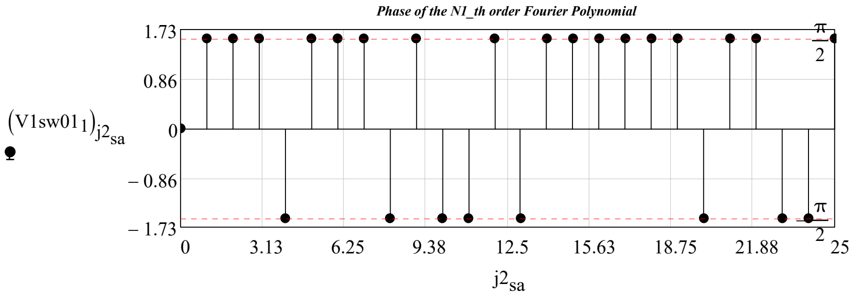
Signal and Fourier Series



$$j2_{sa} := 0 .. rows(V1sw01_0) - 1 \qquad \omega_{ptd\_} = 6.283 \times 10^{-3} \cdot \frac{Mrads}{s}$$

Signal's Amplitude Spectrum





$Bw_{sa} := V1sw01_3 \cdot Hz$

$Bw_{sa} = 23 \cdot MHz$

sampling frequency: $fpt_{so} := 2 \cdot Bw_{sa}$ $\quad fpt_{so} = 46 \cdot MHz$

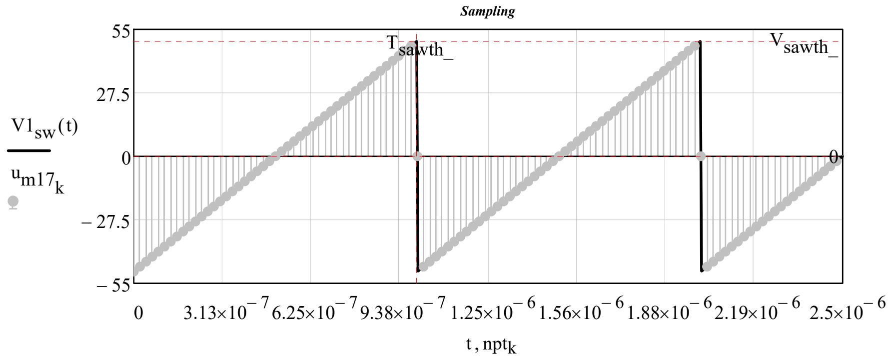
$npt_k := \frac{k}{fpt_{so}}$

Frequency resolution: $\frac{N0_{gd}}{fpt_{so}} \cdot \frac{1}{T_{sawth\_}} = 5.565$

$u_{m17_k} := V1_{sw}(npt_k)$

$u_{m17}^T =$

	0	1	2	3	4
0	-50	-47.826	-45.652	-43.478	...

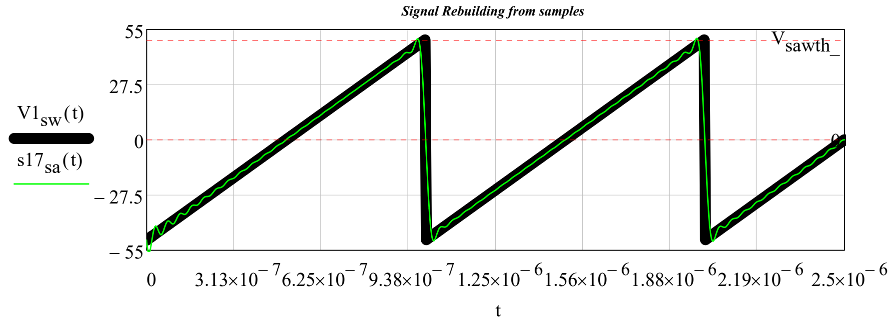


relerr = 10·%

$\omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa}$ $\quad \omega_{bwr} = 144.513 \cdot \frac{Mrads}{sec}$ $\quad n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula: $s17_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1} \left(u_{m17_n} \cdot \text{sinc}(\omega_{bwr} \cdot t - n \cdot \pi)\right)\right]$ $\quad N0_{gd} - 1 = 255$ $\quad relerr = 10 \cdot \%$

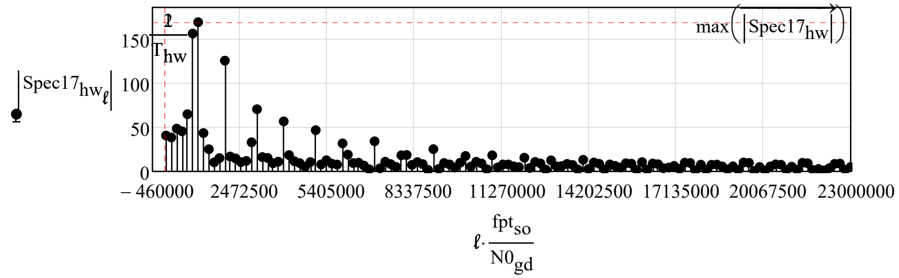


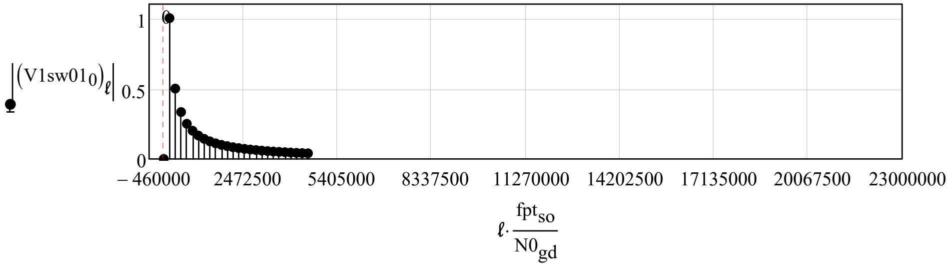
$\text{length}(u_{m17}) = 256$

$fpt_{so} = 46 \cdot MHz$

$Spec17_{hw} := \text{fft}(u_{m17})$ $\quad \text{length}(Spec17_{hw}) = 129$

$\ell := 0 .. \frac{N0_{gd}}{2}$ $\quad \frac{N0_{gd}}{2} = 128$





Periodic Waveforms

18 Bipolar Sawtooth with negative slope Pulse Train

Amplitude: $V_{sawth\_} = 50 \cdot V$

Sawtooth length: $\delta_{sawth\_} = 1 \cdot \mu s$

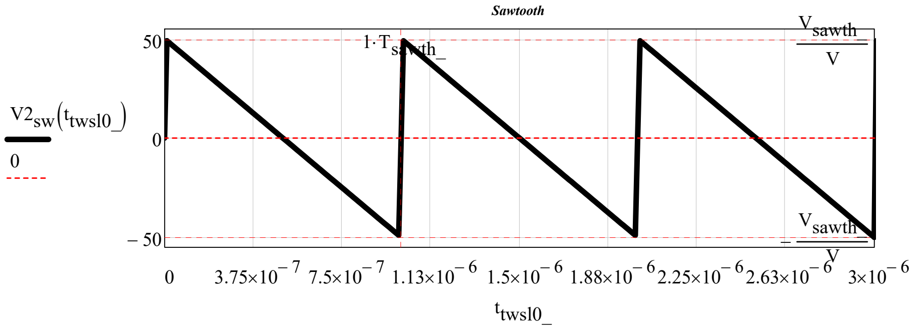
Slope: $sp_{sawth\_} = 50 \cdot \frac{V}{\mu s}$

Period: $T_{sawth\_} = 1 \cdot \mu s$

Frequency: $\frac{1}{T_{sawth\_}} = 1 \cdot MHz$

$$t_{twsl0\_} := -T_{sawth\_} \cdot 0, T_{sawth\_} \cdot 0 + \frac{5 \cdot T_{sawth\_} + T_{sawth\_} \cdot 0}{500} .. 5 \cdot T_{sawth\_}$$

$$V2_{sw}(t) := \frac{v2_{sw}(t, T_{sawth\_}, V_{sawth\_}, N0_{gd})}{V}$$



Dirichlet conditions

A periodic function s(t)=s(t+T), can be expressed by the Fourier series provided that (Dirichlet conditions):
(1) it is discontinuous and presents a finite number of discontinuities in the period T;
(2) has a limited average value in the period T;
(3) it has a finite number of maximum positive or negative.
If these conditions are met, the considered function can be developed in Fourier series in trigonometric form.

The Dirichlet conditions apply to:

1) signals of energy for which holds: $\int_{-\infty}^{\infty} \left(\left|s_{fs}(t)\right|\right)^2 dt < \infty$,

2) power signals for which holds: $\lim_{T \to \infty} \left[\frac{1}{T} \cdot \int_{-T}^{T} \left(\left|s_{fs}(t)\right|\right)^2 dt\right] < \infty$

Fourier series definition $s_{fs}(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} \left(a_k \cos(\omega \cdot k \cdot t) + b_k \cdot \sin(\omega \cdot k \cdot t)\right)$

The coefficients are defined as follows:

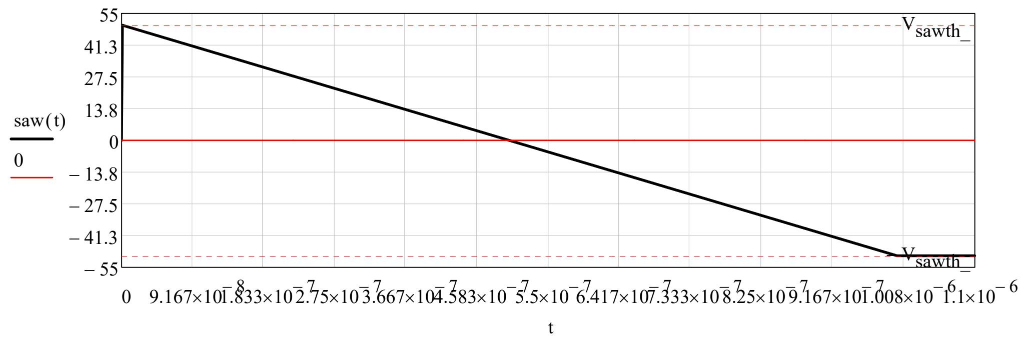
$$\frac{a_0}{2} = A_{fs} = \frac{1}{T} \cdot \int_{t_0}^{t_0+T} s_{fs}(t)\, dt$$

$$a_k = \frac{2}{T} \cdot \int_{t_0}^{t_0+T} s_{fs}(t) \cdot \cos(\omega \cdot k \cdot t)\, dt$$

$$b_k = \frac{2}{T} \cdot \int_{t_0}^{t_0+T} s_{fs}(t) \cdot \sin(\omega \cdot k \cdot t)\, dt$$

$V_{sawth\_} = 50\,V$

$$saw(t) := 2 \cdot V_{sawth\_} \cdot \left[\left(\frac{-t}{T_{sawth\_}} + 1\right) \cdot \left(\Phi(t) - \Phi\left(t - T_{sawth\_}\right)\right) - \frac{1}{2}\right]$$



$$\frac{a_0}{2} = A_{fs} = \frac{2 \cdot V_{sawth\_}}{T_{sawth\_}} \cdot \int_{t,0}^{t_0+T_{so}} \left(\frac{-t}{T_{sawth\_}} + 1\right) \cdot \left(\Phi(t) - \Phi\left(t - T_{sawth\_}\right)\right) - \frac{1}{2}\, dt = \frac{2 \cdot V_{sawth\_}}{T_{sawth\_}} \cdot \int_{0}^{T_{sawth\_}} \left(\frac{-t}{T_{sawth}}\right.$$

$$\frac{2 \cdot V_{sawth\_}}{T_{sawth\_}} \cdot \int_{0}^{T_{sawth\_}} \left(\frac{-t}{T_{sawth\_}} + \frac{1}{2}\right) dt = 0$$

$$a_k = \frac{2}{T} \cdot \int_{t_0}^{t_0+T} s_{fs}(t) \cdot \cos(\omega \cdot k \cdot t)\, dt = 2 \cdot \frac{2 \cdot V_{sawth\_}}{T_{sawth\_}} \cdot \int_{0}^{T_{sawth\_}} \left(\frac{-t}{T_{sawth\_}} + \frac{1}{2}\right) \cdot \cos(\omega \cdot k \cdot t)\, dt$$

$$2\cdot\frac{2\cdot V_{sawth\_}}{T_{sawth\_}}\cdot\int_0^{T_{sawth\_}}\left(\frac{-t}{T_{sawth\_}}+\frac{1}{2}\right)\cdot\cos(\omega\cdot k\cdot t)\,dt = \frac{2\cdot V_{sawth\_}\cdot\left(4\cdot\sin\left(\frac{T_{sawth\_}\cdot\omega\cdot k}{2}\right)^2 - T_{sawth\_}\cdot\omega\cdot k\cdot\sin\left(T_{sav\right.}{T_{sawth\_}{}^2\cdot\omega^2\cdot k^2}$$

$$a_k = \frac{2\cdot V_{sawth\_}\cdot\left(4\cdot\sin\left(\frac{T_{sawth\_}\cdot\omega\cdot k}{2}\right)^2 - T_{sawth\_}\cdot\omega\cdot k\cdot\sin\left(T_{sawth\_}\cdot\omega\cdot k\right)\right)}{T_{sawth\_}{}^2\cdot\omega^2\cdot k^2}$$

$$b_k = \frac{2}{T}\cdot\int_{t_0}^{t_0+T}s_{fs}(t)\cdot\sin(\omega\cdot k\cdot t)\,dt = 2\cdot\frac{2\cdot V_{sawth\_}}{T_{sawth\_}}\cdot\int_{t_0}^{t_0+T}\left(\frac{-t}{T_{sawth\_}}+\frac{1}{2}\right)\cdot\sin(\omega\cdot k\cdot t)\,dt$$

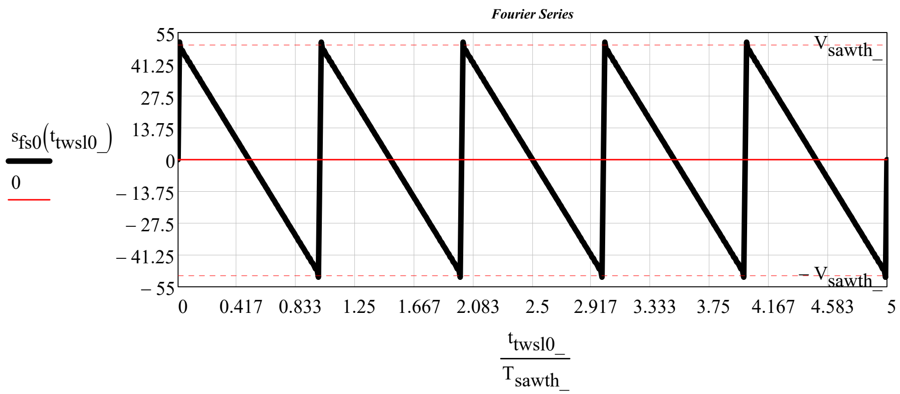
$$2\cdot\frac{2\cdot V_{sawth\_}}{T_{sawth\_}}\cdot\int_{t_0}^{t_0+T}\left(\frac{-t}{T_{sawth\_}}+\frac{1}{2}\right)\cdot\sin(\omega\cdot k\cdot t)\,dt = \left(\cos\left(\frac{T_{sawth\_}\cdot\omega\cdot k}{2}\right)^2 - \frac{\sin\left(T_{sawth\_}\cdot\omega\cdot k\right)}{T_{sawth\_}\cdot\omega\cdot k}\right)\cdot\frac{4\cdot V_{saw}}{\left(T_{sawth\_}\right.}$$

$$b_k = \left(\cos\left(\frac{T_{sawth\_}\cdot\omega\cdot k}{2}\right)^2 - \frac{\sin\left(T_{sawth\_}\cdot\omega\cdot k\right)}{T_{sawth\_}\cdot\omega\cdot k}\right)\cdot\frac{4\cdot V_{sawth\_}}{\left(T_{sawth\_}\cdot\omega\cdot k\right)}$$

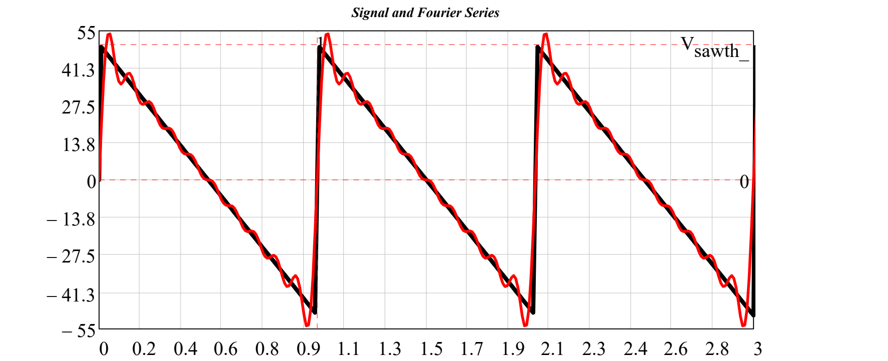
$\omega_{sf} := \omega_{sawth\_}$

$$s_{fs0}(t) := \frac{2\cdot V_{sawth\_}}{\left(T_{sawth\_}\cdot\omega_{sf}\right)}\cdot\sum_{k=1}^{N0_{gd}}\left[\frac{\left(4\cdot\sin\left(\frac{T_{sawth\_}\cdot\omega_{sf}\cdot k}{2}\right)^2 - T_{sawth\_}\cdot\omega_{sf}\cdot k\cdot\sin\left(T_{sawth\_}\cdot\omega_{sf}\cdot k\right)\right)}{T_{sawth\_}\cdot\omega_{sf}\cdot k}\cdot\cos\left(\omega_{sf}\cdot k\right.\right. \\ \left.+\left(\cos\left(\frac{T_{sawth\_}\cdot\omega_{sf}\cdot k}{2}\right)^2 - \frac{\sin\left(T_{sawth\_}\cdot\omega_{sf}\cdot k\right)}{T_{sawth\_}\cdot\omega_{sf}\cdot k}\right)\cdot\frac{2}{k}\cdot\sin\left(\omega_{sf}\cdot k\cdot t\right)\right]$$

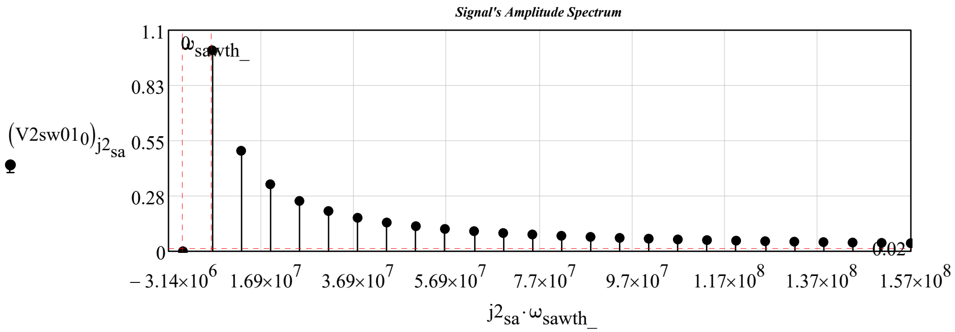
$N0_{gd} = 256 \qquad V_{sawth\_} = 50\,V$

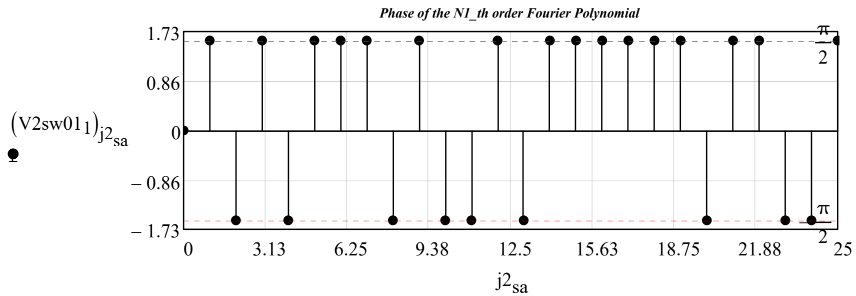


$V2sw01 := SPCT\left(V2_{sw}, rt_{gd}, N1\_, 0\cdot s, T_{sawth\_}\right) \qquad N1\_ = 25$



$j2_{sa} := 0\,..\,rows\left(V2sw01_0\right) - 1 \qquad \omega_{ptd\_} = 6.283\times 10^{-3}\cdot\frac{Mrads}{s}$





$Bw_{sa} := V2sw01_3\cdot Hz$

$Bw_{sa} = 23\cdot MHz$

sampling frequency: $fpt_{so} := 2\cdot Bw_{sa} \qquad fpt_{so} = 46\cdot MHz$

$npt_k := \frac{k}{fpt_{so}}$

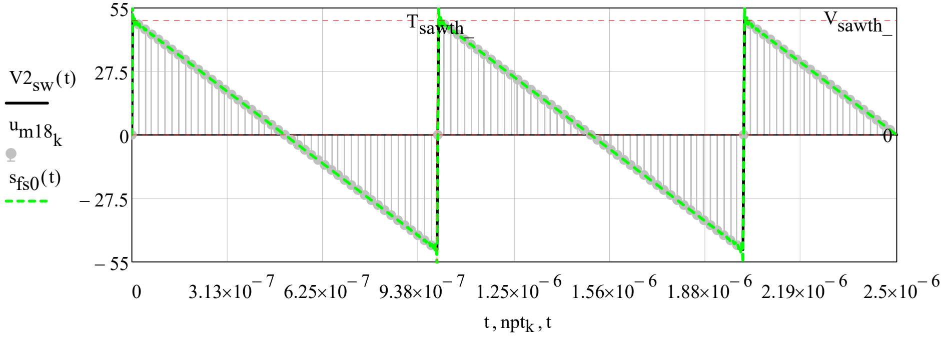
Frequency resolution: $\frac{N0_{gd}}{fpt_{so}} \cdot \frac{1}{T_{sawth\_}} = 5.565$

$u_{m18_k} := V2_{sw}(npt_k)$

$u_{m18}^T =$

	0	1	2	3	4
0	0	47.826	45.652	43.478	...

Comparison among the given signal, the sampled and the calculated series

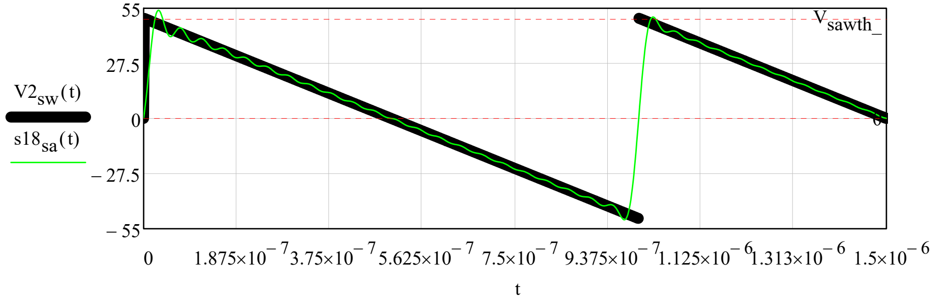


relerr = 10·% $\omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa}$ $\omega_{bwr} = 144.513 \cdot \frac{Mrads}{sec}$ $n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula: $s18_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1} \left(u_{m18_n} \cdot sinc(\omega_{bwr} \cdot t - n \cdot \pi)\right)\right]$ $N0_{gd} - 1 = 255$ relerr = 10·%

Signal Rebuilding from samples

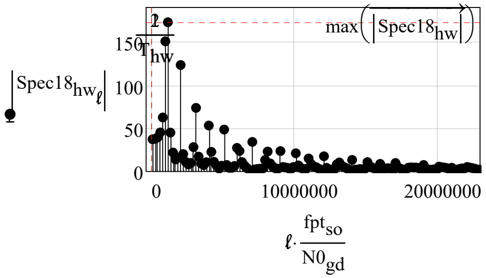


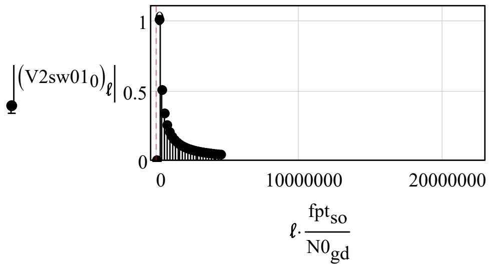
$length(u_{m18}) = 256$

$fpt_{so} = 46 \cdot MHz$

$Spec18_{hw} := fft(u_{m18})$ $length(Spec18_{hw}) = 129$

$\ell := 0 .. \frac{N0_{gd}}{2}$ $\frac{N0_{gd}}{2} = 128$





TEST Waveforms

Periodic Waveforms

19 Bipolar Sawtooth with adjustable rising and falling edges Pulse Train

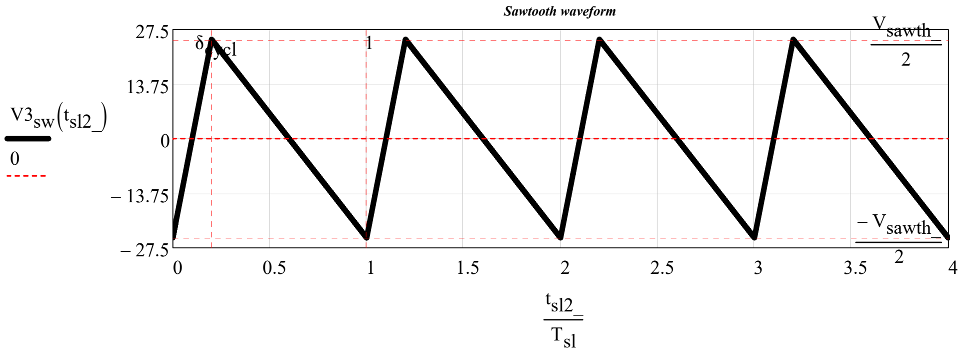
$\delta_{cycl} \cdot T_{sl} = 220 \cdot ns$

$T_{sl} = 1.1 \cdot \mu s$ $f_{3sw} := \frac{1}{T_{sl}}$ $\omega_{3sw} := 2 \cdot \pi \cdot f_{3sw}$ $\omega_{3sw} = 5.712 \cdot \frac{Mrads}{sec}$

$\delta_{cycl} = 20 \cdot \%$

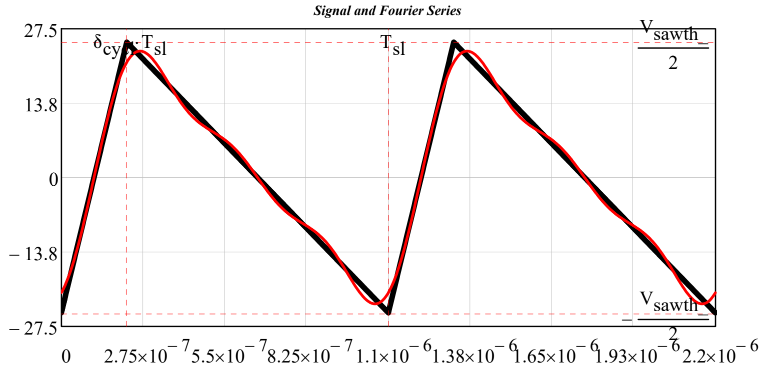
$t_{sl2\_} := 0 \cdot T_{sl}, 0 \cdot T_{sl} + \frac{4 \cdot T_{sl}}{10000} .. 4 \cdot T_{sl}$

$V3_{sw}(t) := \frac{V_s\left(t \cdot sec^{-1}, T_{sl} \cdot sec^{-1}, \delta_{cycl}, V_{sawth\_}, N_{gd}\right)}{V}$

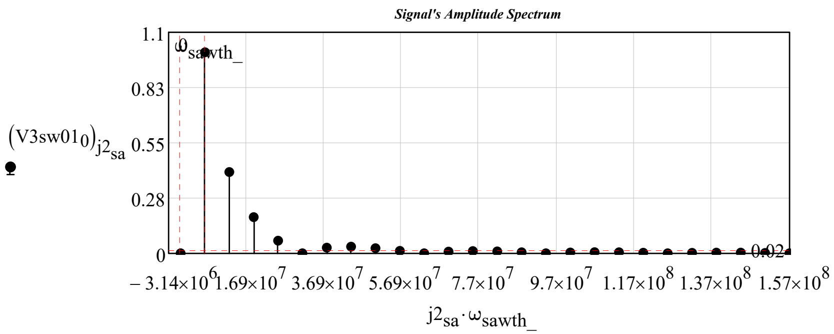


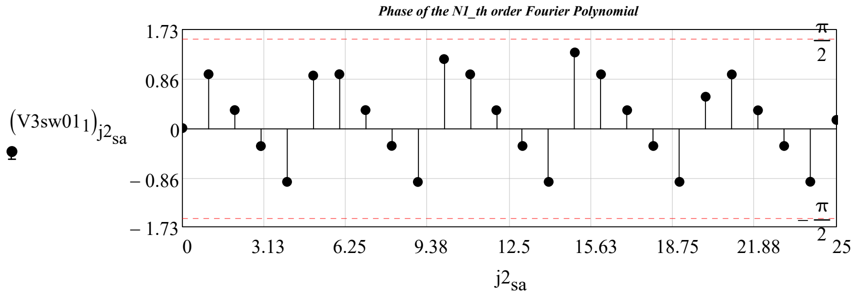
$$V3sw01 := SPCT\left(V3_{sw}, rt_{gd}, N1\_, 0 \cdot s, T_{sl}\right)$$

$N1\_ = 25$



$$j2_{sa} := 0 .. rows\left(V3sw01_0\right) - 1 \qquad \omega_{ptd\_} = 6.283 \times 10^{-3} \cdot \frac{Mrads}{s}$$





$$Bw_{sa} := V3sw01_3 \cdot Hz$$

$$Bw_{sa} = 11.818 \cdot MHz$$

sampling frequency: $fpt_{so} := 2 \cdot Bw_{sa}$ $\qquad fpt_{so} = 23.636 \cdot MHz$

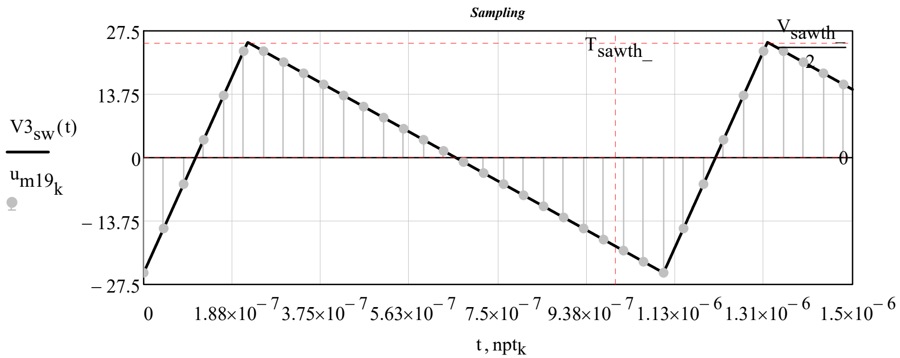
$$npt_k := \frac{k}{fpt_{so}}$$

Frequency resolution: $\dfrac{N0_{gd}}{fpt_{so}} \cdot \dfrac{1}{T_{sawth\_}} = 10.831$

$$u_{m19_k} := V3_{sw}\left(npt_k\right)$$

$u_{m19}{}^T =$

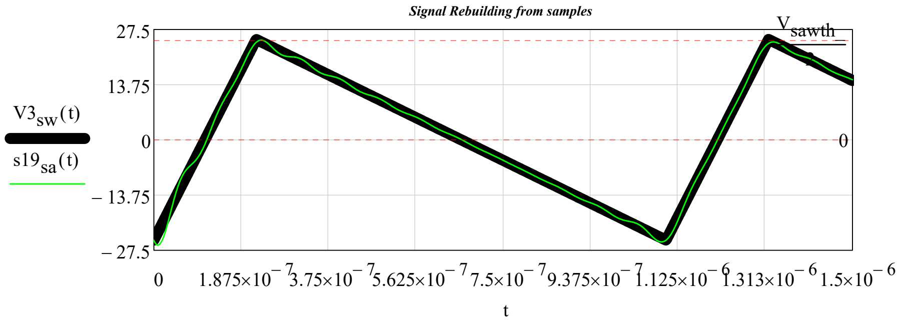
	0	1	2	3	4	5	6
0	-25	-15.385	-5.769	3.846	13.462	23.077	...



$relerr = 10 \cdot \%$ $\qquad \omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa}$ $\qquad \omega_{bwr} = 74.256 \cdot \dfrac{Mrads}{sec}$ $\qquad n \cdot \dfrac{\pi}{\omega_{bwr}} = n \cdot \dfrac{1}{2 \cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula: $s19_{sa}(t) := \left[\displaystyle\sum_{n=0}^{N0_{gd}-1} \left(u_{m19_n} \cdot sinc\left(\omega_{bwr} \cdot t - n \cdot \pi\right)\right)\right]$ $\qquad N0_{gd} - 1 = 255$ $\qquad relerr = 10 \cdot \%$



TEST Waveforms

Periodic Waveforms

20 AM test signal (single tone)

Carrier Amplitude: $A1_{sl} := 20\cdot volt$

Modulating signal's amplitude: $B1_{sl} := 12\cdot volt$

Carrier pulsation: $\omega1_{csl} := 15\cdot\omega_{0gd}$ Carrier period: $T1_{csl} := \frac{2\cdot\pi}{\omega1_{csl}}$

Carrier frequency: $f_{1csl} := \frac{\omega1_{csl}}{2\cdot\pi}$

Modulating signal's pulsation: $\omega1_{msl} := \frac{\omega1_{csl}}{20}$ Modulating signal's period: $T1_{msl} := \frac{2\cdot\pi}{\omega1_{msl}}$

Modulating signal's frequency: $f1_{msl} := \frac{\omega1_{msl}}{2\cdot\pi}$

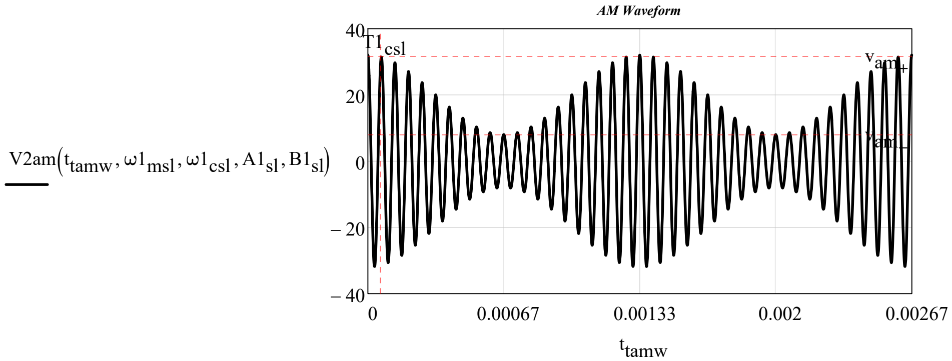
$\omega1_{csl} = 94.248\cdot\frac{krads}{sec}$ $\frac{\omega1_{csl}}{\omega1_{msl}} = 20$ $\omega_{0gd} = 6.283\cdot\frac{krads}{s}$

$v_{am_+} := A1_{sl} + B1_{sl}$ $v_{am\_} := A1_{sl} - B1_{sl}$ $A1_{sl} \equiv v_{am_+} + v_{am\_}$ $B1_{sl} \equiv v_{am_+} - v_{am\_}$

$v_{am_+} = 32\cdot volt$

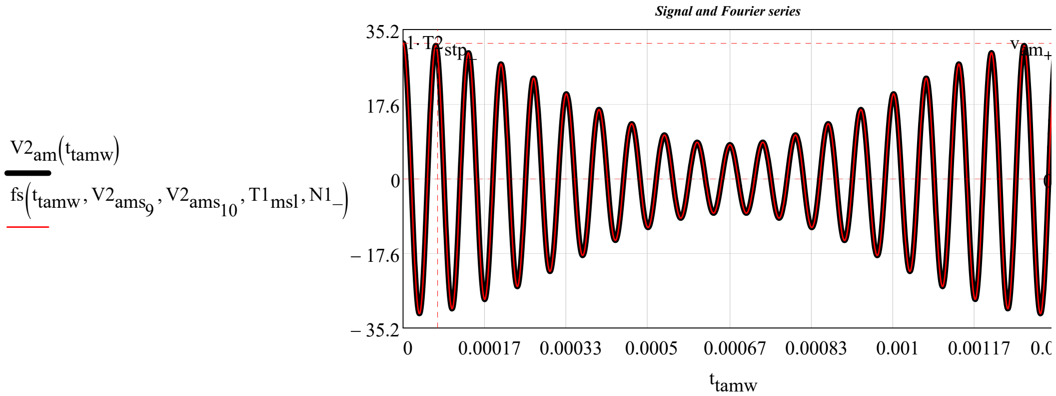
$v_{am\_} = 8\cdot volt$ AM modulation index: $m_{amsl} := \frac{v_{am_+} - v_{am\_}}{v_{am_+} + v_{am\_}}$ $m_{amsl} = 60\cdot\%$ $\frac{B1_{sl}}{A1_{sl}} = 60\cdot\%$

$$t_{tamw} := -T_{0gd}\cdot 3, -T_{0gd}\cdot 3 + \frac{40\cdot T1_{csl} + T_{0gd}\cdot 3}{5000} .. 40\cdot T1_{csl}$$

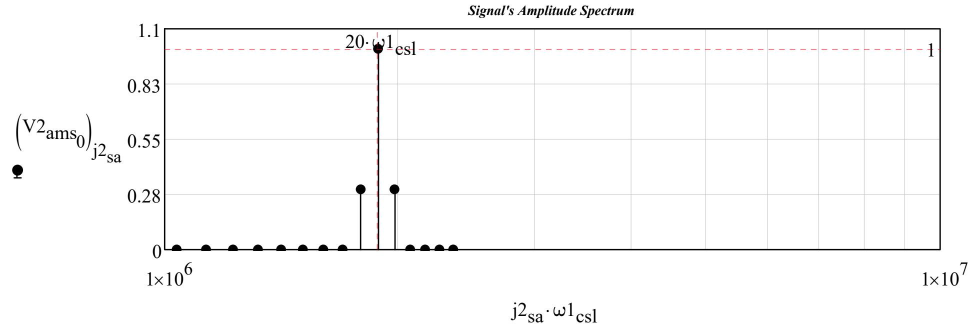


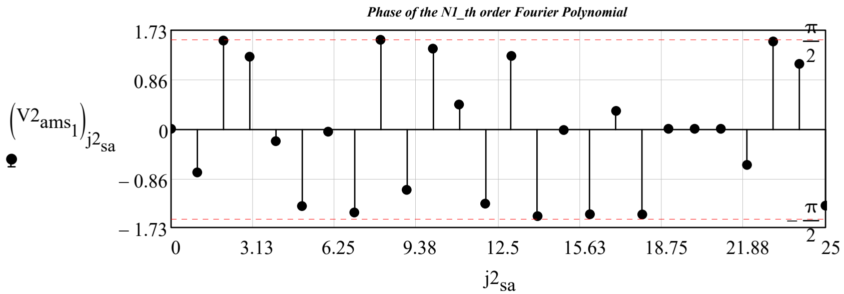
$$V2_{am}(t) := V2am(t, \omega1_{msl}, \omega1_{csl}, A1_{sl}, B1_{sl})$$

$$V2_{ams} := SPCT(V2_{am}, rt_{gd}, N1\_, 0\cdot s, T1_{msl}) \quad N1\_ = 25$$



$j2_{sa} := 0 .. rows(V2ams_0) - 1$ $\omega_{ptd\_} = 6.283 \times 10^{-3} \cdot \frac{Mrads}{s}$





$Bw_{sa} := V2ams_3 \cdot Hz$

$Bw_{sa} = 16.5 \cdot kHz$

sampling frequency: $fpt_{so} := 2 \cdot Bw_{sa}$ $fpt_{so} = 33 \cdot kHz$

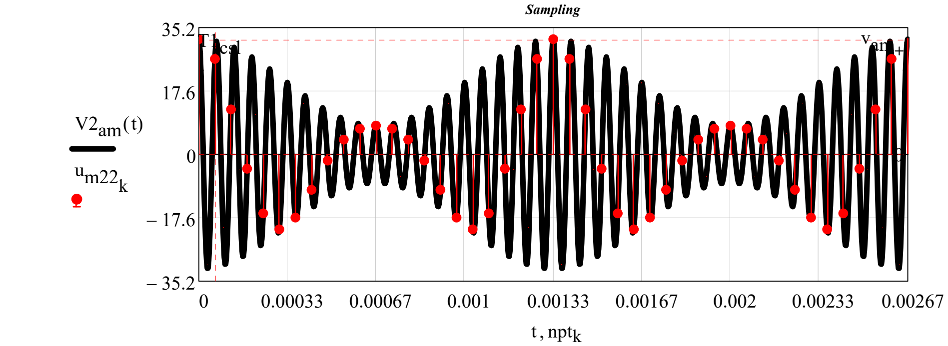
$npt_k := \frac{k}{fpt_{so}}$

Frequency resolution: $\frac{N0_{gd}}{fpt_{so}} \cdot \frac{1}{T1_{csl}} = 116.364$

$u_{m22_k} := V2_{am}(npt_k)$

$u_{m22}^T =$

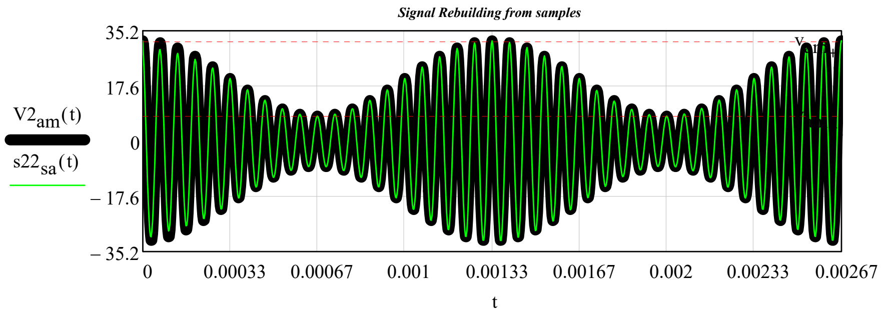
	0	1	2	3	4	5	6
0	32	-30.587	26.511	-20.245	12.502	-4.137	...



relerr = 10·% $\omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa}$ $\omega_{bwr} = 0.104 \cdot \frac{Mrads}{sec}$ $n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot B}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula: $s22_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1} \left(u_{m22_n} \cdot sinc(\omega_{bwr} \cdot t - n \cdot \pi) \right) \right]$ $N0_{gd} - 1 = 255$ rel



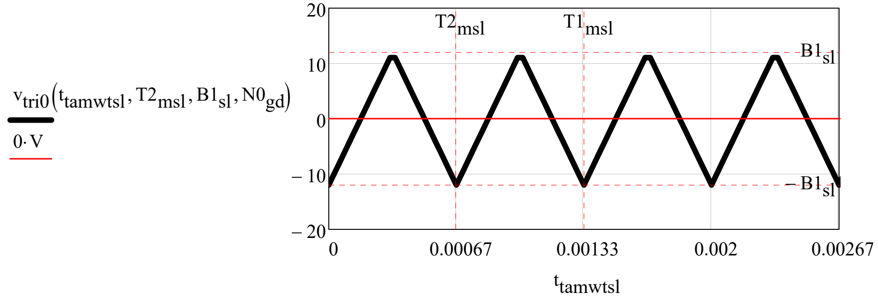
Periodic Waveforms

21 AM test signal (triangular wave)

$$\omega_{2msl} := \frac{\omega 1_{csl}}{10} \qquad T2_{msl} := \frac{2\cdot\pi}{\omega_{2msl}}$$

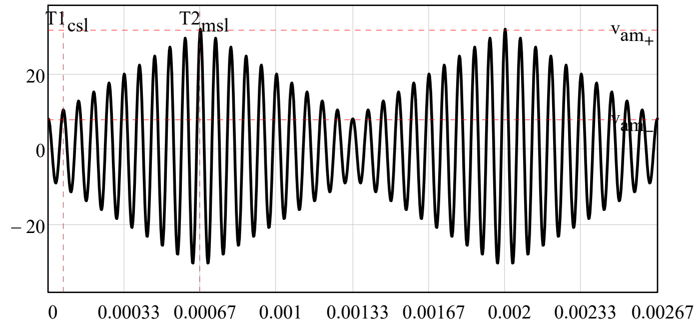
$$t_{tamwtsl} := 0\cdot sec, 40\cdot\frac{T2_{msl}}{1000} .. 40\cdot T2_{msl}$$

Madulating Signal



$$t_{twsl\_} := -T_{0gd}\cdot 3, -T_{0gd}\cdot 3 + \frac{8\cdot T2_{msl} + T_{0gd}\cdot 3}{500} .. 8\cdot T2_{msl}$$

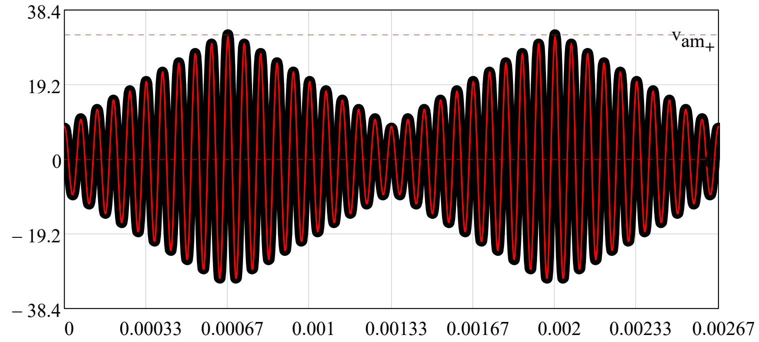
AM Waveform



$$V3_{am}(t) := V3am\left(t, \omega 1_{msl}, \omega 1_{csl}, m_{amsl}, A1_{sl}, B1_{sl}, N0_{gd}\right)$$

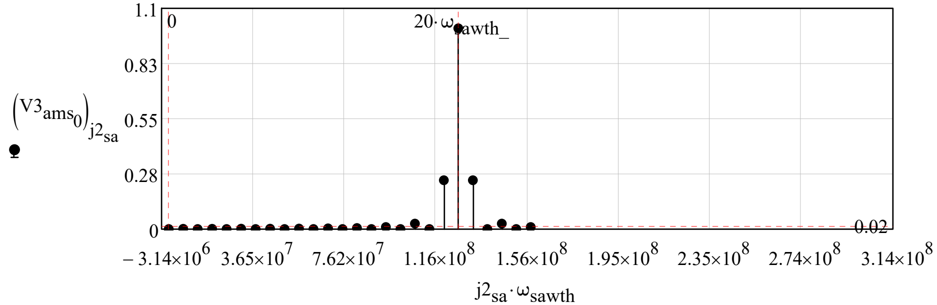
$$V3_{ams} := SPCT\left(V3_{am}, rt_{gd}, N1\_, 0\cdot s, 2\cdot T2_{msl}\right) \qquad N1\_ = 25$$

Signal and Fourier Series

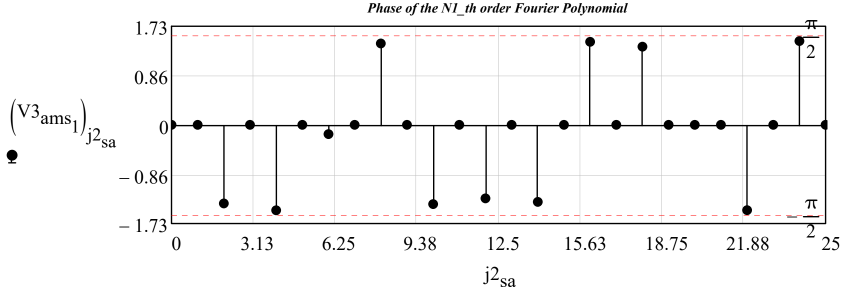


$$j2_{sa} := 0 .. rows\left(V3_{ams_0}\right) - 1 \qquad \omega_{ptd\_} = 6.283\times 10^{-3}\cdot\frac{Mrads}{s}$$

Signal's Amplitude Spectrum



Phase of the N1_th order Fourier Polynomial



$$Bw_{sa} := V3_{ams_3}\cdot Hz$$

$$Bw_{sa} = 0.017\cdot MHz$$

sampling frequency: $fpt_{so} := 2\cdot Bw_{sa}$ $\qquad fpt_{so} = 0.035\cdot MHz$

$$npt_k := \frac{k}{fpt_{so}}$$

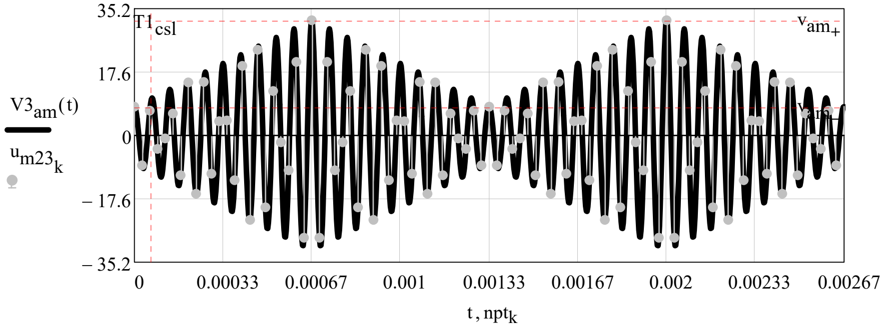
Frequency resolution: $\frac{N0_{gd}}{fpt_{so}}\cdot\frac{1}{T2_{msl}} = 11.13$

$$u_{m23_k} := V3_{am}(npt_k)$$

$u_{m23}^T =$

	0	1	2	3	4	5	6
0	8	-8.295	6.885	-3.727	-0.831	6.081	...

Sampling



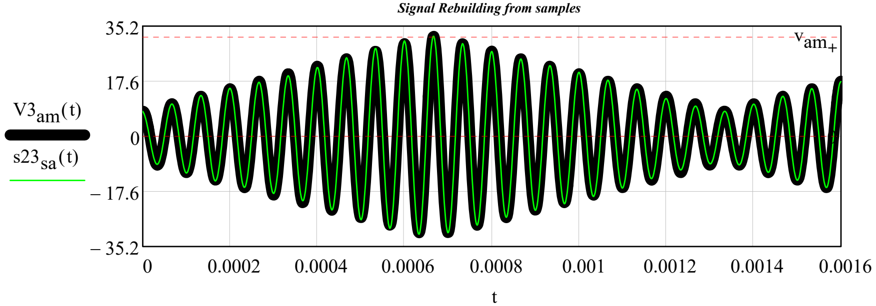
$relerr = 10 \cdot \%$ $\qquad \omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa}$ $\qquad \omega_{bwr} = 0.108 \cdot \frac{Mrads}{sec}$ $\qquad n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula: $$s23_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1} \left(u_{m23_n} \cdot \mathrm{sinc}(\omega_{bwr} \cdot t - n \cdot \pi) \right) \right] \qquad N0_{gd} - 1 = 255$$

$relerr = 10 \cdot \%$

Signal Rebuilding from samples



Periodic Waveforms

22 AM DSBSC test signal (single tone)

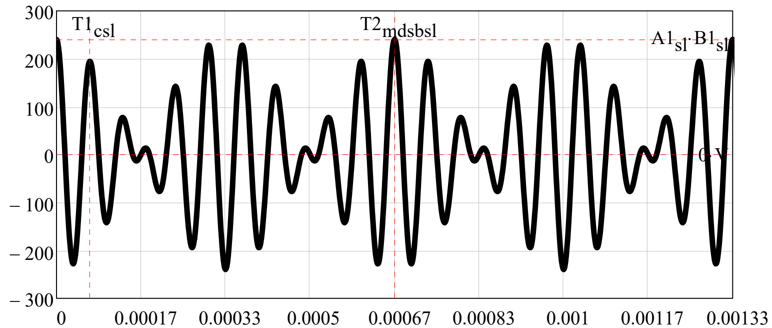
$\omega2_{msl} := \frac{\omega1_{csl}}{10}$ $\qquad T2_{mdsbsl} := \frac{2 \cdot \pi}{\omega2_{msl}}$ $\qquad \omega2_{msl} = \frac{2 \cdot \pi}{T2_{mdsbsl}}$ $\qquad \frac{A1_{sl} \cdot B1_{sl}}{2} = 120 \cdot volt^2$

$\omega1_{csl} = 94.248 \cdot \frac{krads}{sec}$ $\qquad \omega2_{msl} = 9.425 \cdot \frac{krads}{sec}$ $\qquad f2_{msl} := \frac{1}{T2_{msl}}$ $\qquad f1_{csl} := \frac{\omega1_{csl}}{2 \cdot \pi}$

$T_{1csl} := \frac{1}{f1_{csl}}$ $\qquad \nu_{sl} := 40$

$$t_{tdsbw} := 0 \cdot sec, \nu_{sl} \cdot \frac{T2_{mdsbsl}}{20000} .. \nu_{sl} \cdot T2_{mdsbsl}$$

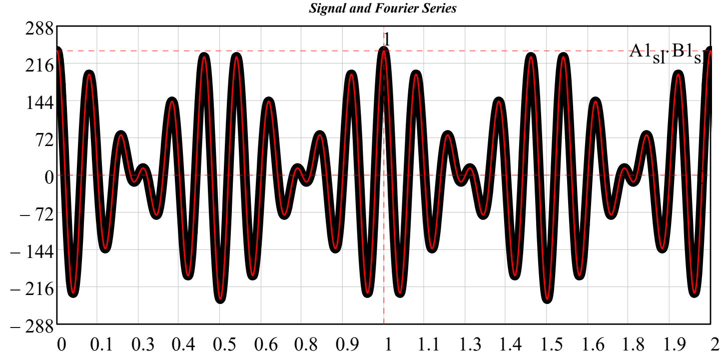
DSBSC single tone



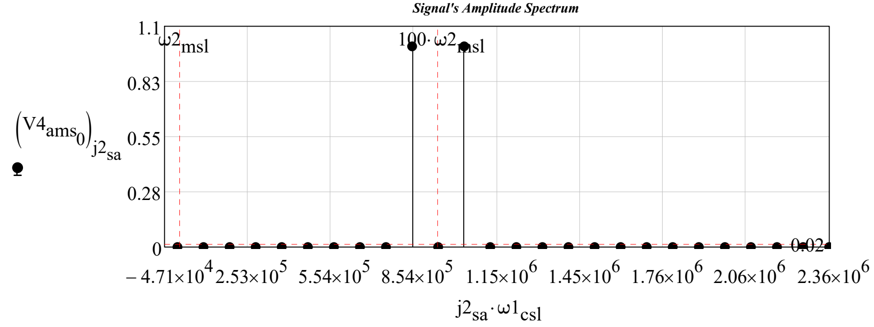
$$V4_{am}(t) := V4dsbsc(t, f1_{csl}, f2_{msl}, A1_{sl}, B1_{sl})$$

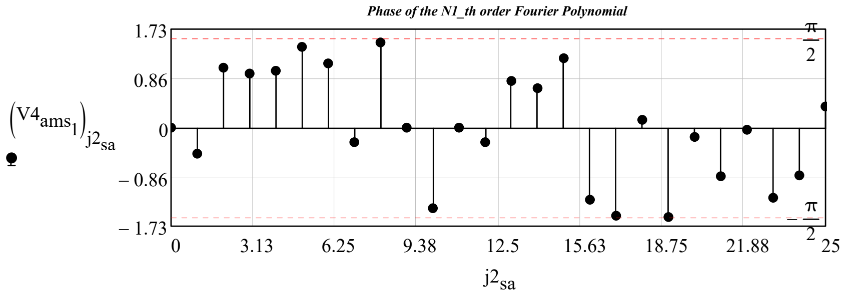
$$V4_{ams} := SPCT(V4_{am}, rt_{gd}, N1\_, 0 \cdot s, T2_{mdsbsl}) \qquad N1\_ = 25$$

Signal and Fourier Series



$j2_{sa} := 0 .. \text{rows}\left(V4_{ams_0}\right) - 1 \qquad \omega_{ptd\_} = 6.283 \times 10^{-3} \cdot \frac{Mrads}{s}$





$Bw_{sa} := V4_{ams_3} \cdot Hz$

$Bw_{sa} = 0.024 \cdot MHz$

sampling frequency: $fpt_{so} := 2 \cdot Bw_{sa} \qquad fpt_{so} = 0.048 \cdot MHz$

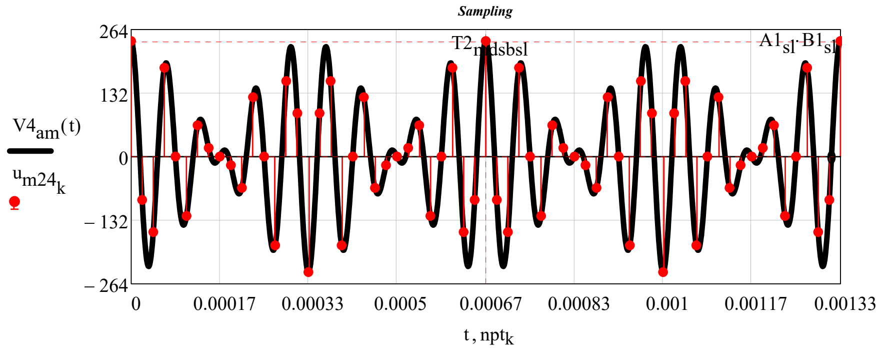
$npt_k := \frac{k}{fpt_{so}}$

Frequency resolution: $\frac{N0_{gd}}{fpt_{so}} \cdot \frac{1}{T2_{mdsbsl}} = 8$

$u_{m24_k} := V4_{am}\left(npt_k\right)$

$u_{m24}^T =$

	0	1	2	3	4
0	240	-90.079	-156.788	184.363	...

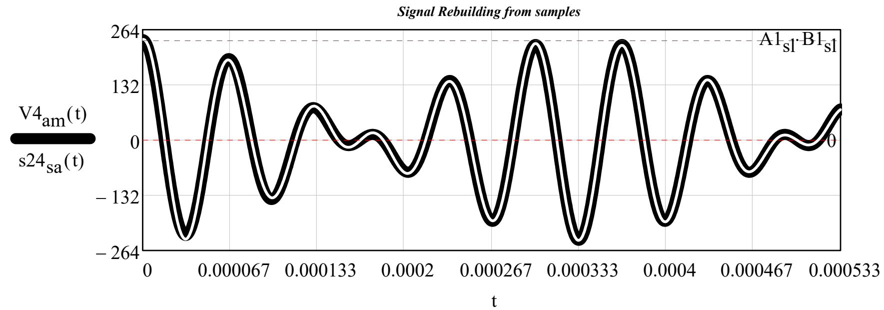


$relerr = 10 \cdot \%$ $\qquad \omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa} \qquad \omega_{bwr} = 0.151 \cdot \frac{Mrads}{sec} \qquad n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula: $s24_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1} \left(u_{m24_n} \cdot \text{sinc}\left(\omega_{bwr} \cdot t - n \cdot \pi\right)\right) \right] \qquad N0_{gd} - 1 = 255$

$relerr = 10 \cdot \%$



TEST Waveforms

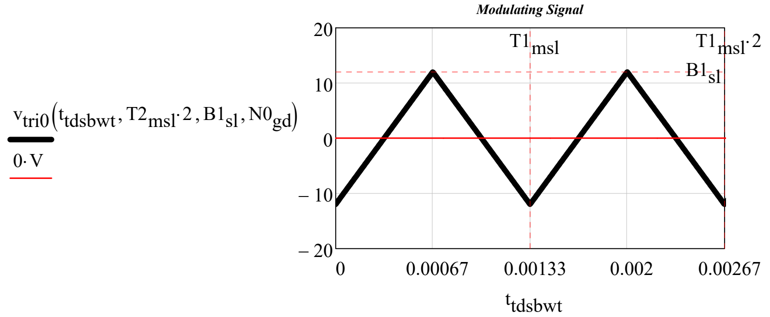
Periodic Waveforms

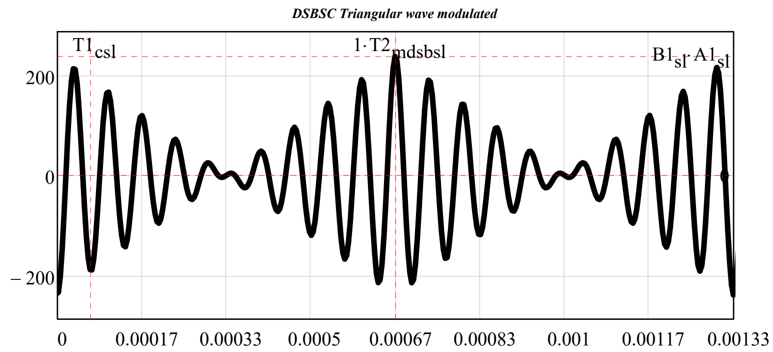
23 AM DSBSC test signal (triangular wave)

$T_{18} := T2_{mdsbsl}$

$f_{18} := \frac{1}{T_{18}}$

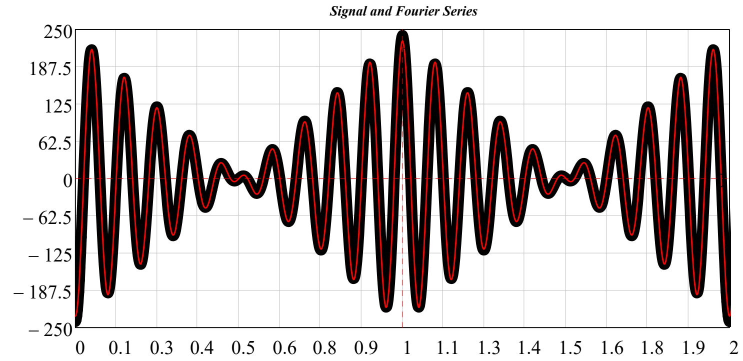
$t_{tdsbwt} := -T_{18}\cdot 3, -T_{18}\cdot 3 + \frac{8\cdot T_{18} + T_{18}\cdot 3}{2000} .. 8\cdot T_{18}$



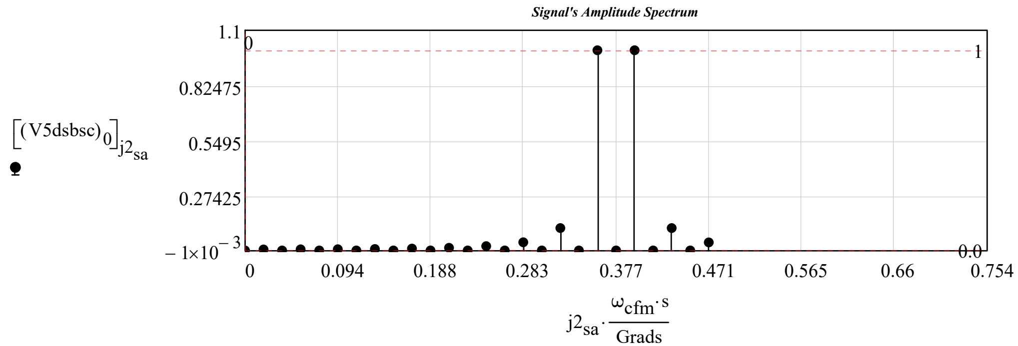


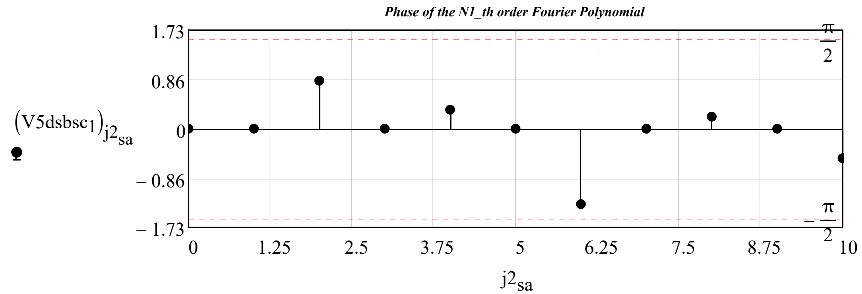
$N1\_ := 25$ $\quad v5_{dsbsc}(t) := V5_{dsbsc}(t, T2_{mdsbsl}, f1_{csl}, f2_{msl}, A1_{sl}, B1_{sl}, N0_{gd})$

$f_{cfm} = 3\cdot MHz$ $\quad V5dsbsc := SPCT(v5_{dsbsc}, rt_{gd}, N1\_, 0\cdot s, 2\cdot T2_{mdsbsl})$ $\quad N1\_ = 25$



$\omega_{cfm} = 0.019\cdot\frac{Grads}{s}$ $\quad j2_{sa} := 0 .. rows(V5dsbsc_0) - 1$ $\quad \omega_{fmm} = 0.754\cdot\frac{Mrads}{s}$





Bandwidth: $\quad Bw_{sa} := V5dsbsc_3\cdot Hz$

$Bw_{sa} = 0.017\cdot MHz$

sampling frequency: $\quad fpt_{so} := 2\cdot Bw_{sa}$ $\quad fpt_{so} = 0.035\cdot MHz$

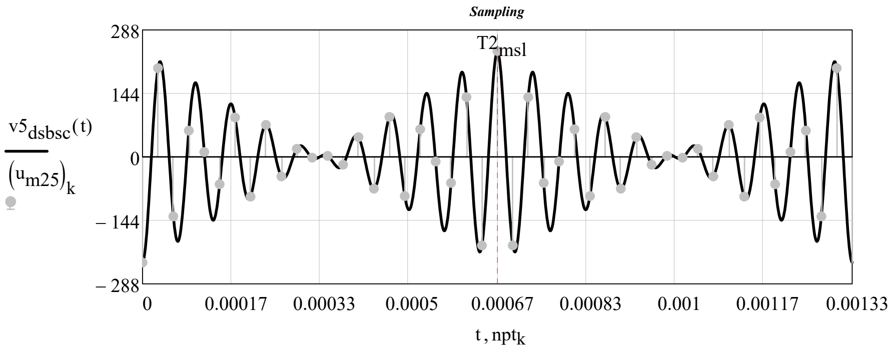
$npt_k := \frac{k}{fpt_{so}}$

Frequency resolution: $\frac{N0_{gd}}{fpt_{so}} \cdot \frac{1}{T2_{mdsbsl}} = 11.13$

$(u_{m25})_k := v5_{dsbsc}(npt_k)$

$u_{m25}^T =$

	0	1	2	3	4	5
0	-240	200.989	-135.324	59.405	10.681	...



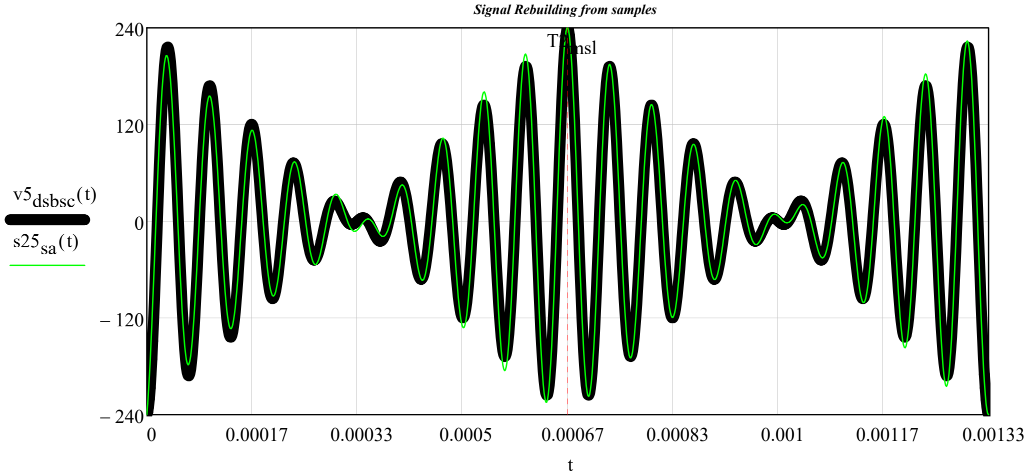
relerr = 10·%

$\omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa}$ $\omega_{bwr} = 0.108 \cdot \frac{Mrads}{sec}$ $n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula: $s25_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1} \left(u_{m25_n} \cdot sinc(\omega_{bwr} \cdot t - n \cdot \pi) \right) \right]$ $N0_{gd} - 1 = 255$

relerr = 10·%



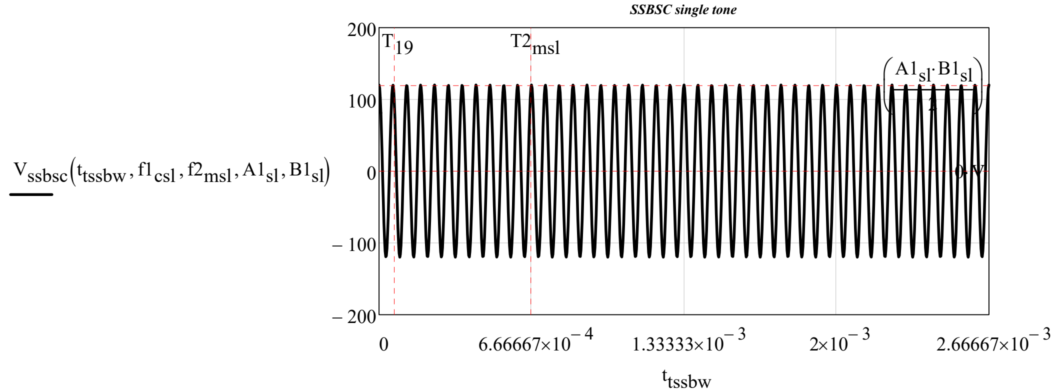
TEST Waveforms

Periodic Waveforms

24 AM SSBSC test signal (single tone)

$f_{19} := \frac{\omega 1_{csl}}{2 \cdot \pi}$ $T_{19} := \frac{1}{f_{19}}$

$t_{tssbw} := 0 \cdot sec, \frac{4 \cdot T2_{msl}}{1000} .. 4 \cdot T2_{msl}$



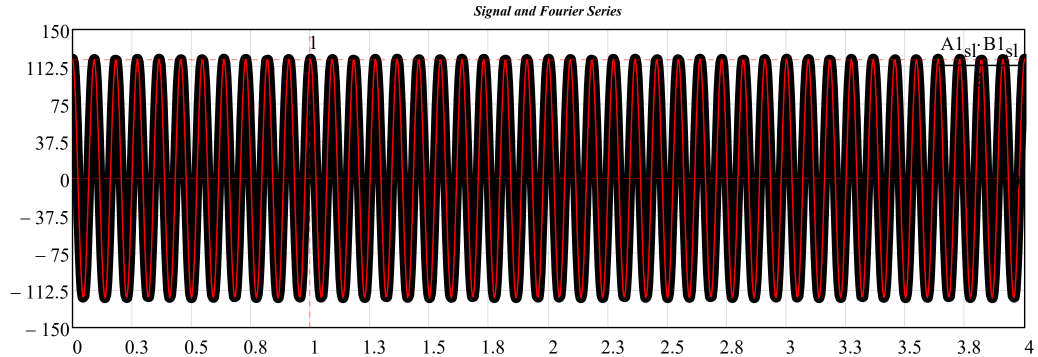
$\frac{A1_{sl} \cdot B1_{sl}}{2} = 120\,V^2$

$v_{ssbsc}(t) := \frac{V_{ssbsc}(t, f1_{csl}, f2_{msl}, A1_{sl}, B1_{sl})}{V^2}$ $v_{ssbsc}(T_{19}) = 97.082$

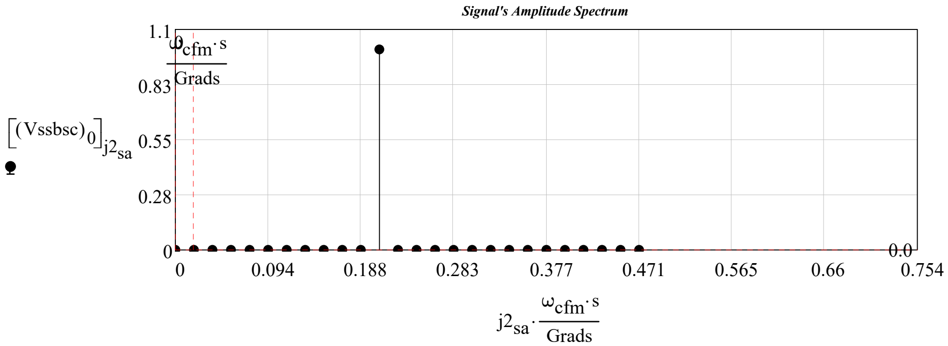
$N1\_ := 25$

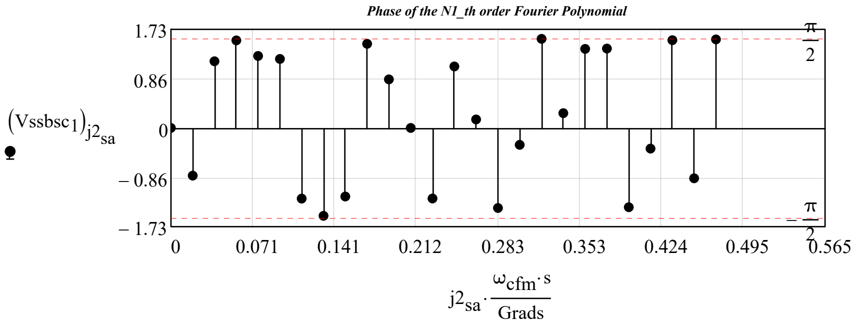
$f_{cfm} = 3 \cdot MHz$

$Vssbsc := SPCT(v_{ssbsc}, rt_{gd}, N1\_, 0 \cdot s, T2_{mdsbsl})$ $N1\_ = 25$



$\omega_{cfm} = 0.019 \cdot \frac{Grads}{s}$ $j2_{sa} := 0 .. rows(Vssbsc_0) - 1$ $\omega_{fmm} = 0.754 \cdot \frac{Mrads}{s}$





$Bw_{sa} := Vssbsc_3 \cdot Hz$

$Bw_{sa} = 0.023 \cdot MHz$

sampling frequency: $fpt_{so} := 2 \cdot Bw_{sa}$ $\qquad fpt_{so} = 0.045 \cdot MHz$

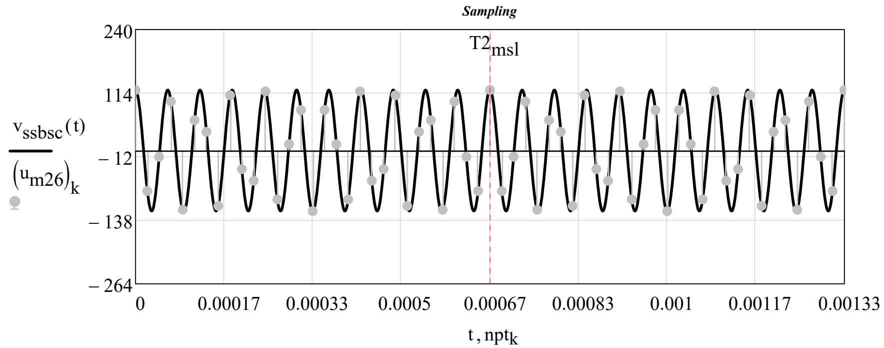
$npt_k := \frac{k}{fpt_{so}}$

Frequency resolution: $\frac{N0_{gd}}{fpt_{so}} \cdot \frac{1}{T2_{msl}} = 8.533$

$(u_{m26})_k := v_{ssbsc}(npt_k)$

$u_{m26}^{T} =$

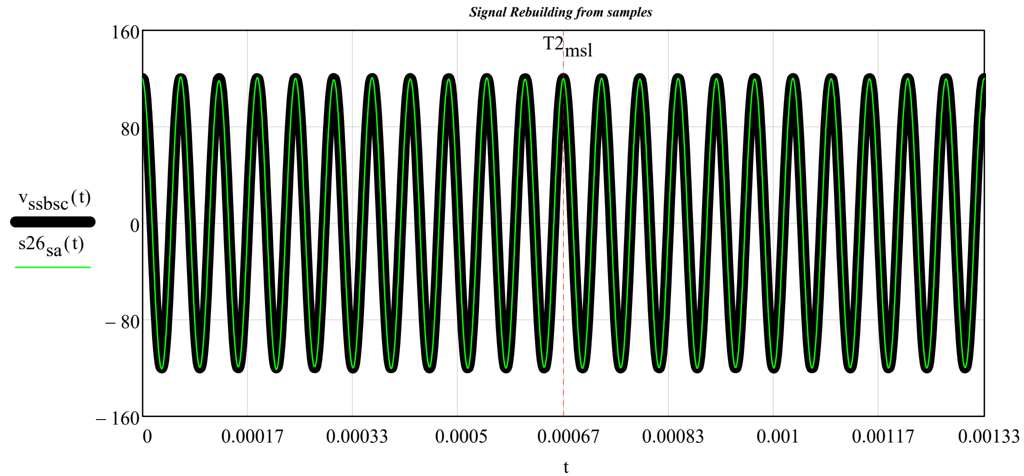
	0	1	2	3	4	5
0	120	-80.296	-12.543	97.082	-117.378	...



relerr = 10·% $\qquad \omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa}$ $\qquad \omega_{bwr} = 0.141 \cdot \frac{Mrads}{sec}$ $\qquad n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula: $s26_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1} \left(u_{m26_n} \cdot sinc(\omega_{bwr} \cdot t - n \cdot \pi) \right) \right]$ $\qquad N0_{gd} - 1 = 255$ $\qquad$ relerr = 10·%

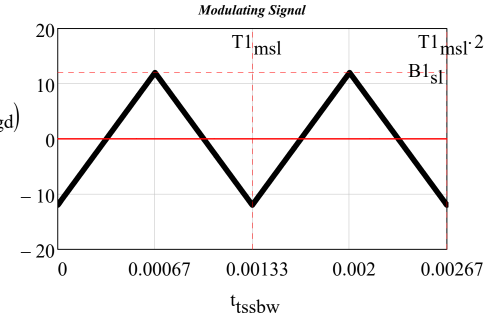


Periodic Waveforms

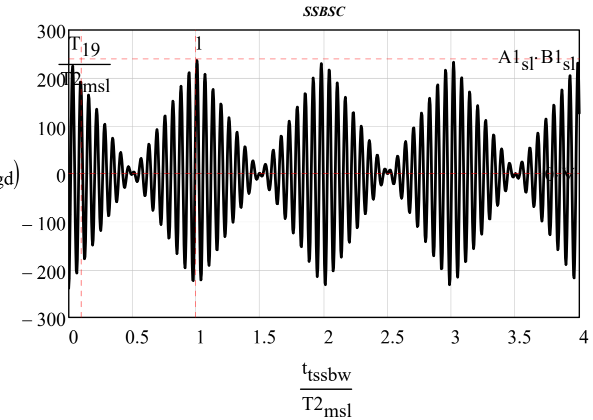
25 AM SSBSC test signal (triangular wave)

$f_{20} := \frac{10}{T1_{csl}}$ $\frac{A1_{sl} \cdot A1_{sl}}{2} = 200\,V^2$

$v_{tri0}(t_{tssbw}, T2_{msl} \cdot 2, B1_{sl}, N0_{gd})$
$0 \cdot V$



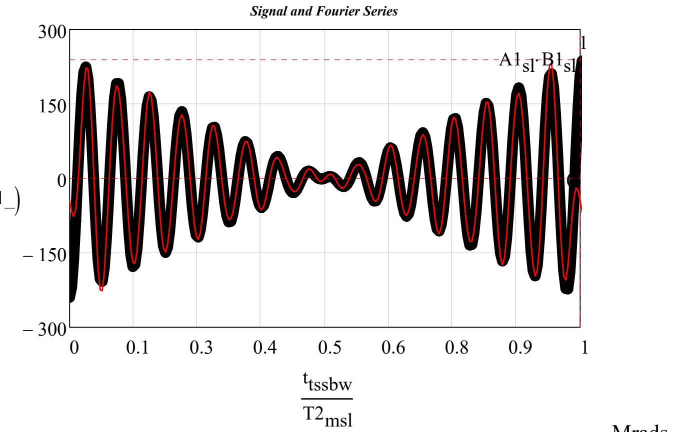
$V7_{ssbsc}(t_{tssbw}, f_{20}, f2_{msl}, A1_{sl}, B1_{sl}, N0_{gd})$



$N1\_ := 25$ $v7_{ssbsc}(t) := V7_{ssbsc}(t, f_{20}, f2_{msl}, A1_{sl}, B1_{sl}, N0_{gd})$

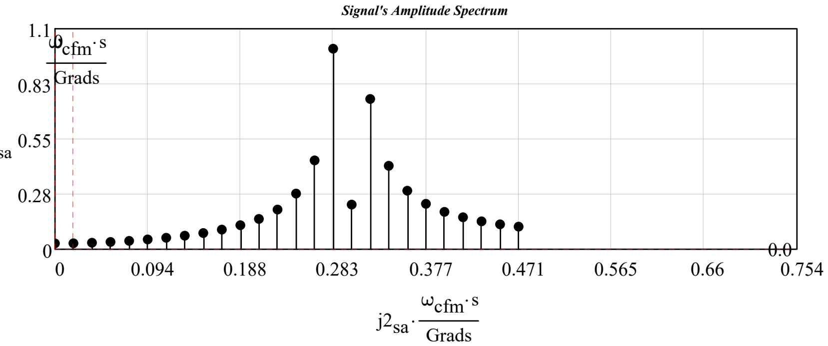
$f_{cfm} = 3 \cdot MHz$ $V7ssbsc := SPCT(v7_{ssbsc}, rt_{gd}, N1\_, 0 \cdot s, T2_{msl})$ $N1\_ = 25$

$v7_{ssbsc}(t_{tssbw})$
$fs(t_{tssbw}, V7ssbsc_9, V7ssbsc_{10}, T2_{msl}, N1\_)$

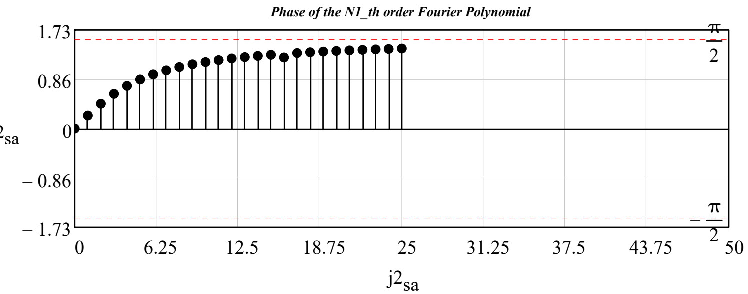


$\omega_{cfm} = 0.019 \cdot \frac{Grads}{s}$ $j2_{sa} := 0 .. rows(V7ssbsc_0) - 1$ $\omega_{fmm} = 0.754 \cdot \frac{Mrads}{s}$

$\left[(V7ssbsc)_0\right]_{j2_{sa}}$



$(V7ssbsc_1)_{j2_{sa}}$



$Bw_{sa} := V7ssbsc_3 \cdot Hz$

$Bw_{sa} = 0.035 \cdot MHz$

sampling frequency: $fpt_{so} := 2 \cdot Bw_{sa}$ $fpt_{so} = 0.069 \cdot MHz$

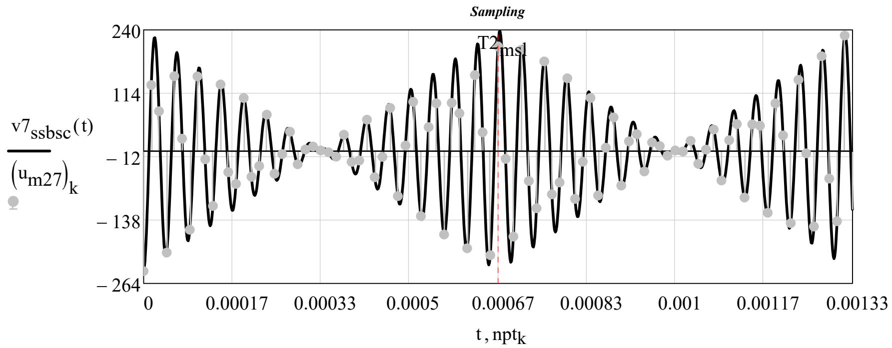
$npt_k := \frac{k}{fpt_{so}}$

Frequency resolution: $\frac{N0_{gd}}{fpt_{so}} \cdot \frac{1}{T2_{msl}} = 5.565$

$(u_{m27})_k := v7_{ssbsc}(npt_k)$

$u_{m27}^T =$

	0	1	2	3	4
0	-240	130.212	78.129	-202.786	...

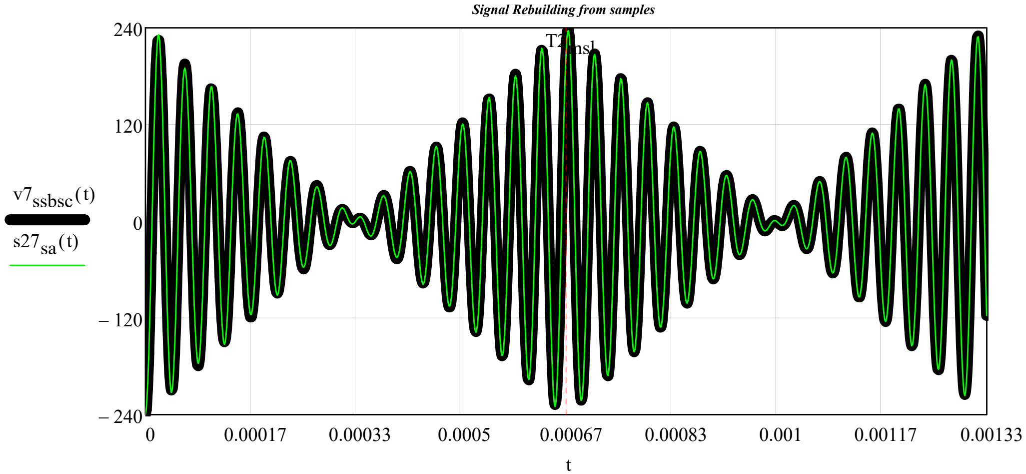


relerr = 10·%

$\omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa}$ $\omega_{bwr} = 0.217 \cdot \frac{Mrads}{sec}$ $n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula: $s27_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1} \left(u_{m27_n} \cdot \mathrm{sinc}(\omega_{bwr} \cdot t - n \cdot \pi) \right) \right]$ $N0_{gd} - 1 = 255$ relerr = 10·%



TEST Waveforms

Periodic Waveforms

26 FM test signal (single tone) (change data in FM data.xmcd)

Carrier Amplitude:..: $A_{fm} = 20 \cdot V$

Carrier Frequency...: $f_{cfm} = 3 \cdot MHz$

Carrier period...: $T_{cfm} = 333.333 \cdot ns$

Angular frequency of the carrier.....................................: $\omega_{cfm} = 18.85 \cdot \frac{Mrads}{sec}$

Amplitude of the single tone modulating signal............: $B_{fmm} = 15 \cdot V$

Period of the modulating signal..: $T_{fmm} = 8.333 \cdot \mu s$

Frequency of the single tone modulating signal........ .....: $f_{fmm} := \frac{1}{T_{fmm}}$ $f_{fmm} = 0.12 \cdot MHz$

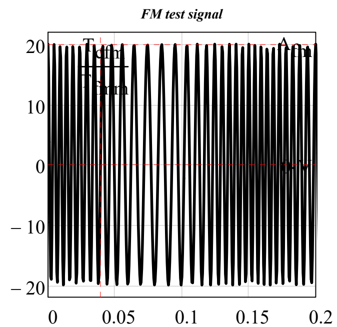
Angular frequency of the single tone modulating signal: $\omega_{fmm} = 0.754 \cdot \frac{Mrads}{sec}$

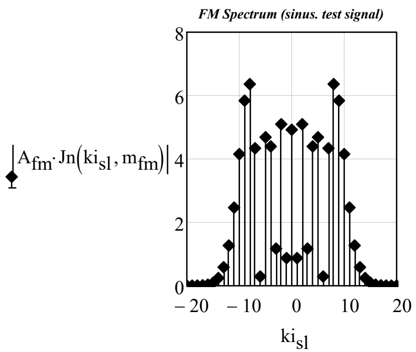
Frequency modulation index: $m_{fm} = 10$

$\frac{T_{fmm}}{T_{cfm}} = 25$ $\frac{\omega_{cfm}}{\omega_{fmm}} = 25$ $k_{fm} = 8 \times 10^4 \frac{1}{Wb}$

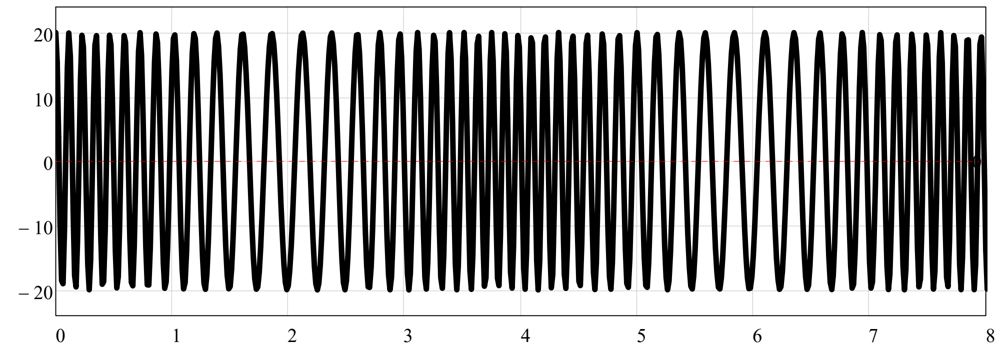
$m_{fm} = 10$ $ki_{sl} := -30 .. 30$

$t_{fmsl} := T_{fmm} \cdot 0, T_{fmm} \cdot 0 + \frac{10 \cdot T_{fmm} - T_{fmm} \cdot 0}{20000} .. 10 \cdot T_{fmm}$





Dimensionless FM Waveform (Single Tone)

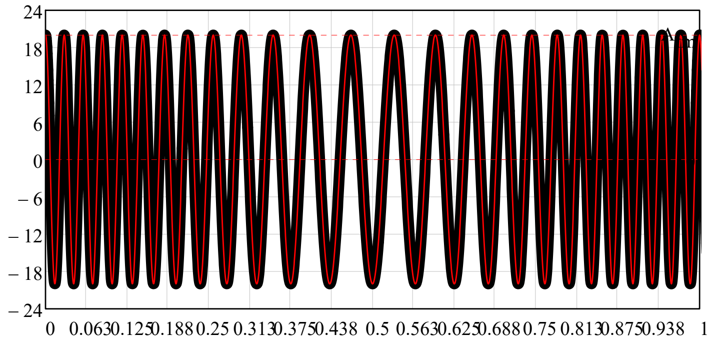


$m_{fm} = 10 \quad A_{fm} = 20\,V \quad B_{fmm} = 15\,V \quad f_{cfm} = 3 \times 10^{6}\,\frac{1}{s}$

$N1\_ := 50$ Dimensionless: $v7_{fm}(t) := V7_{fm}(t, f_{cfm}, f_{fmm}, A_{fm}, m_{fm}, N_{gd})$ $T_{fmm} = 8.333\cdot\mu s$

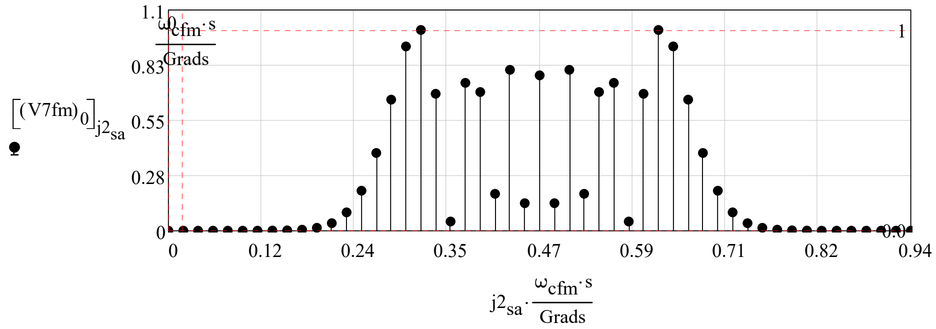
$f_{cfm} = 3\cdot MHz$ $V7fm := SPCT(v7_{fm}, rt_{gd}, N1\_, 0\cdot s, T_{fmm})$ $N1\_ = 50$

Signal and Fourier Series

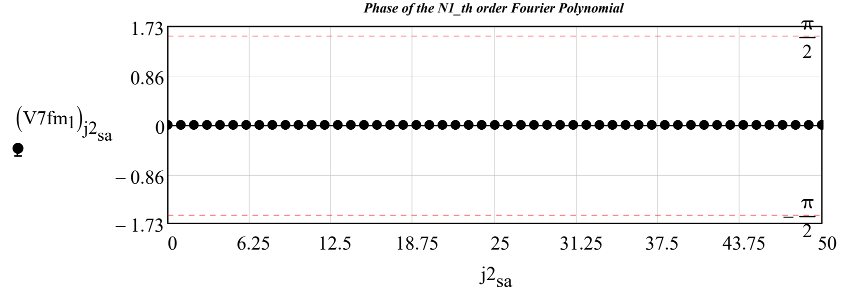


$\omega_{cfm} = 0.019\cdot\frac{Grads}{s}$ $j2_{sa} := 0..rows(V7fm_0) - 1$ $\omega_{fmm} = 0.754\cdot\frac{Mrads}{s}$

Signal's Amplitude Spectrum



Phase of the N1_th order Fourier Polynomial



$Bw_{sa} := V7fm_3\cdot Hz$

$Bw_{sa} = 4.68\cdot MHz$

sampling frequency: $fpt_{so} := 2\cdot Bw_{sa}$ $fpt_{so} = 9.36\cdot MHz$

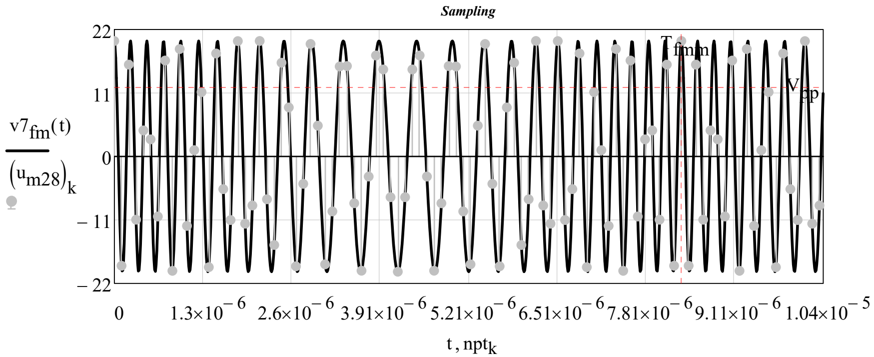
$npt_k := \frac{k}{fpt_{so}}$

Frequency resolution: $\frac{N0_{gd}}{fpt_{so}}\cdot\frac{1}{T_{fmm}} = 3.282$

$(u_{m28})_k := v7_{fm}(npt_k)$

$u_{m28}^{T} =$

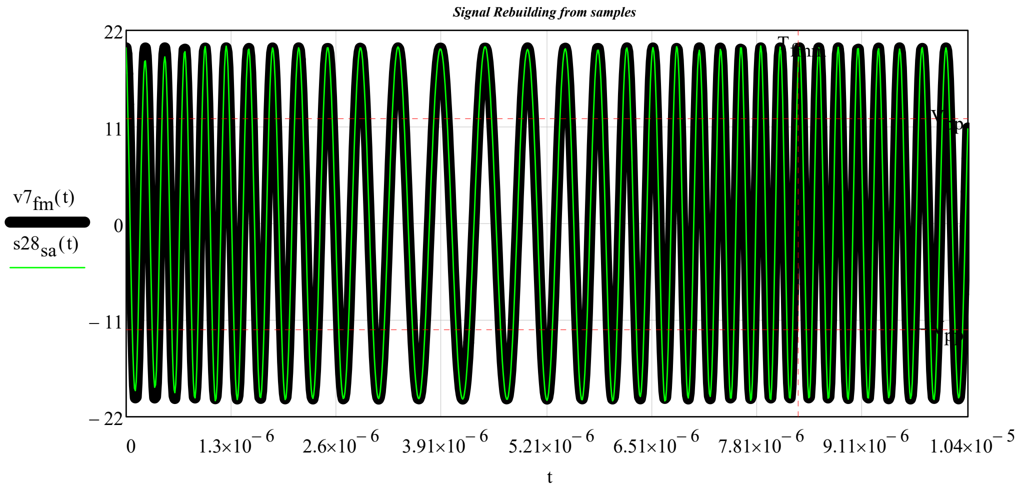
	0	1	2	3	4	5	6
0	20	-18.965	15.905	-10.972	4.491	2.955	...



relerr = 10·% $\omega_{bwr} := 2\cdot\pi\cdot Bw_{sa}$ $\omega_{bwr} = 29.405\cdot\frac{Mrads}{sec}$ $n\cdot\frac{\pi}{\omega_{bwr}} = n\cdot\frac{1}{2\cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

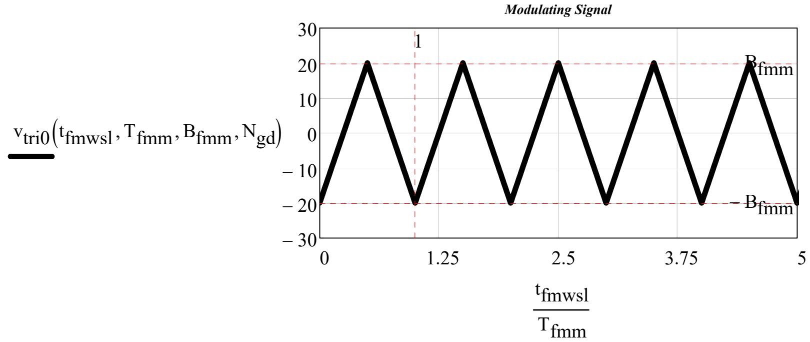
interpolation formula: $s28_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1}\left(u_{m28_n}\cdot \mathrm{sinc}\left(\omega_{bwr}\cdot t - n\cdot\pi\right)\right)\right]$ $N0_{gd} - 1 = 255$ relerr = 10·%

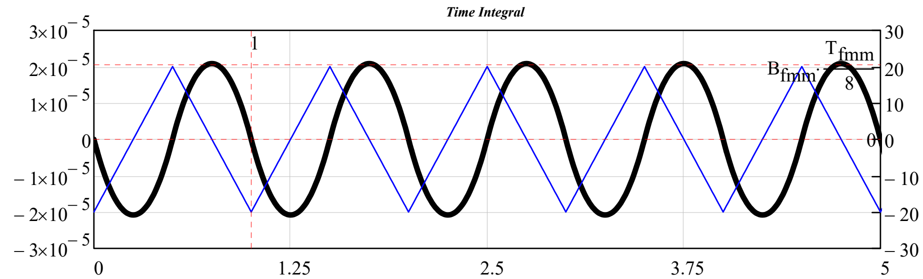


Periodic Waveforms

27 FM test signal (triangular wave)

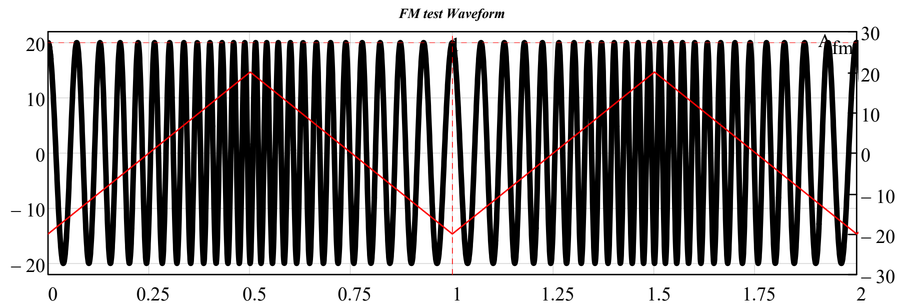
$B_{fmm} := 20\cdot V$ $m_{fm} := 80$ $t_{fmwsl} := 0\cdot T_{fmm}, \frac{10\cdot T_{fmm} - 0\cdot T_{fmm}}{1000} .. 10\cdot T_{fmm}$





$k_{fm} := \frac{m_{fm}\cdot\omega_{fmm}}{2\cdot\pi\cdot B_{fmm}}$ $f_{fmm} := \frac{1}{T_{fmm}}$ $k_{fm} = 0.48\cdot(\mu V\cdot s)^{-1}$ $f_{fmm} = 120\cdot kHz$ $f_{cfm} = 3\cdot MHz$

$m_{fm} = 80$

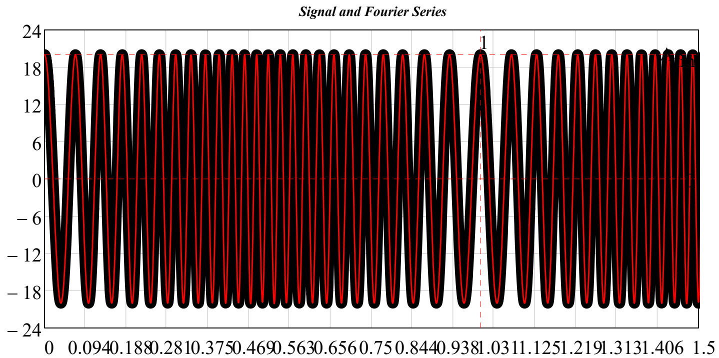


$N1\_ := 50$

$v8_{fm}(t) := V8_{fm}(t, f_{cfm}, f_{fmm}, A_{fm}, B_{fmm}, m_{fm}, k_{fm}, N1\_)$

$f_{cfm} = 3\cdot MHz$

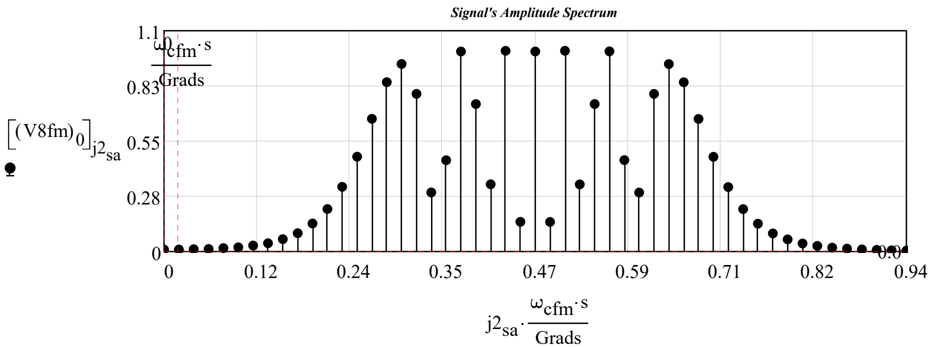
$V8fm := SPCT(v8_{fm}, rt_{gd}, N1\_, 0\cdot s, T_{fmm})$

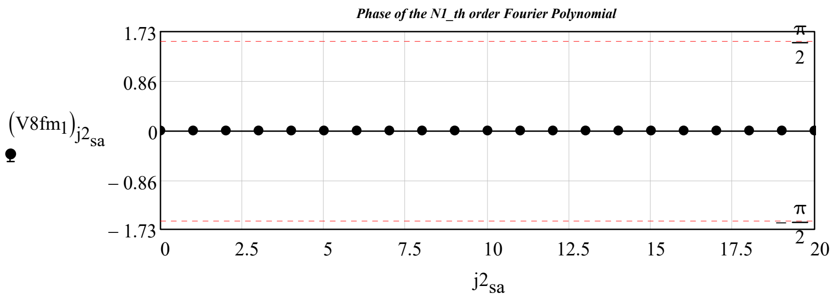


$\omega_{cpm} = 3.77\cdot\frac{Grads}{s}$

$j2_{sa} := 0 .. rows(V8fm_0) - 1$

$\omega_{pmm} = 94.248\cdot\frac{Mrads}{s}$





$Bw_{sa} := V8fm_3\cdot Hz$

$Bw_{sa} = 5.16\cdot MHz$

sampling frequency: $fpt_{so} := 2\cdot Bw_{sa}$ $\qquad fpt_{so} = 10.32\cdot MHz$

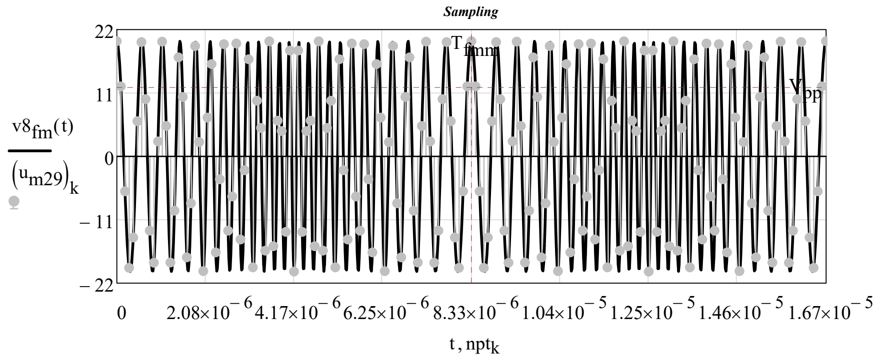
$npt_k := \frac{k}{fpt_{so}}$

Frequency resolution: $\frac{N0_{gd}}{fpt_{so}}\cdot\frac{1}{T_{fmm}} = 2.977$

$(u_{m29})_k := v8_{fm}(npt_k)$

$u_{m29}^T =$

	0	1	2	3	4	5	6
0	20	12.15	-6.069	-19.338	-14.082	6.098	...

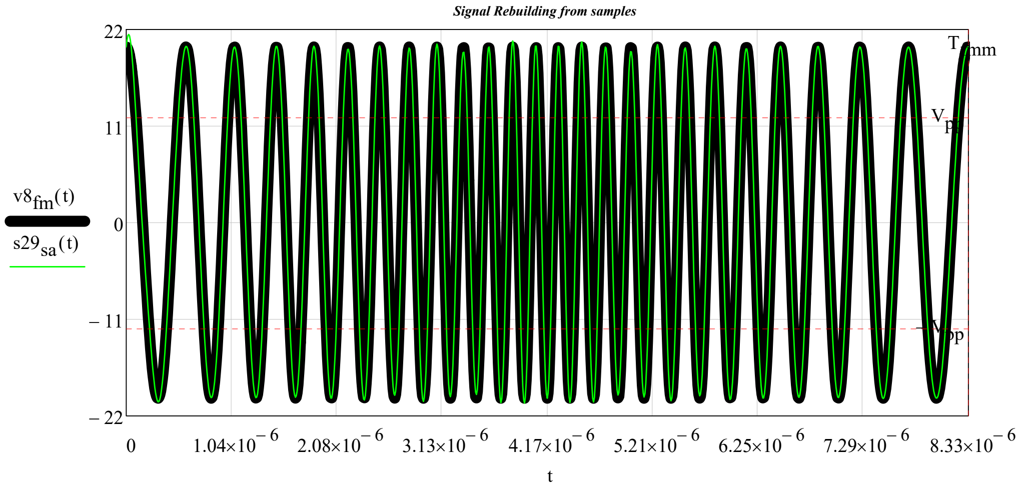


$relerr = 10\cdot\%$ $\qquad \omega_{bwr} := 2\cdot\pi\cdot Bw_{sa}$ $\qquad \omega_{bwr} = 32.421\cdot\frac{Mrads}{sec}$ $\qquad n\cdot\frac{\pi}{\omega_{bwr}} = n\cdot\frac{1}{2\cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula: $s29_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1}\left(u_{m29_n}\cdot sinc(\omega_{bwr}\cdot t - n\cdot\pi)\right)\right]$ $\qquad N0_{gd} - 1 = 255$

$relerr = 10\cdot\%$



TEST Waveforms

Periodic Waveforms

28 PM test signal (single tone)

Carrier Amplitude..: $A_{pm} := 20\cdot V$, $\quad A_{pm} = 20\cdot V$

Carrier Frequency..: $f_{cpm} = 600\cdot MHz$,

Carrier period...: $T_{cpm} = 1.667\cdot ns$,

Angular frequency of the carrier................................: $\omega_{cpm} = 3.77\cdot\frac{Grads}{sec}$,

Amplitude of the modulating signal.................... ....: $B_{pm} = 30\,V$,

Modulating signal period..: $T_{pmm} = 0.067\cdot\mu s$,

Frequency of the harmonic modulating signal.........: $f_{pmm} = 15\cdot MHz$, $\quad \frac{T_{pmm}}{T_{cpm}} = 40$,

Angular frequency of the modulating signal:...........: $\omega_{pmm} = 94.248\cdot\frac{Mrads}{sec}$.

Phase modulation index...: $m_{pm} = 30\cdot rad$

Phase-sensitivity factor..: $k_{pm} = 1\cdot\frac{rad}{V}$

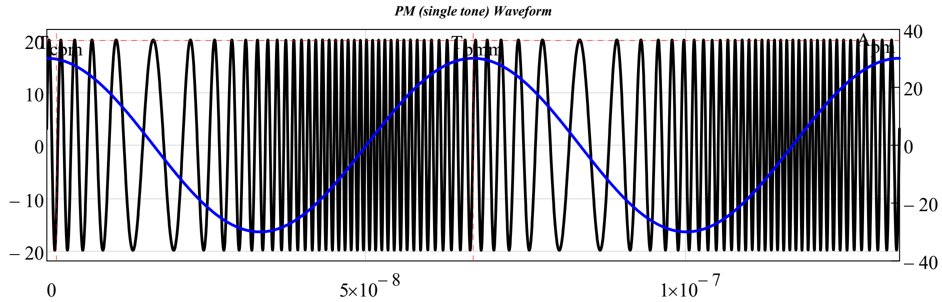
$$k_{pm} = \frac{m_{pm}}{B_{pm}}$$

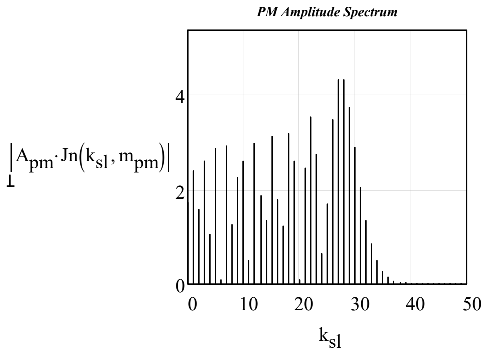
$$v_{pm}(t, f_{cpm}, f_{pmm}, A_{pm}, m_{pm}, N_{gd}) = \mathrm{Re}\left[A_{pm}\cdot e^{j\cdot 2\cdot\pi\cdot f_{cpm}\cdot t}\cdot\sum_{k=-N_{gd}}^{N_{gd}}\left(e^{j\cdot\frac{k\cdot\pi}{2}}\cdot Jn(k, m_{pm})\cdot\cos(k\cdot 2\cdot\pi\cdot f_{pmm}\cdot t)\right)\right]$$

Dimensionless function: $\quad V9_{pm}(t, f_{cpm}, f_{pmm}, A_{pm}, m_{pm}, N_{gd}) = \frac{v_{pm}(t, f_{cpm}, f_{pmm}, A_{pm}, m_{pm}, N_{gd})}{V}$

$f_{cpm} = 600\cdot MHz \qquad t_{pm} := T_{cpm}\cdot 0, T_{cpm}\cdot 0 + \frac{80\cdot T_{cpm} - 0\cdot T_{cpm}}{4000} .. 80\cdot T_{cpm}$

$f_{pmm} = 15\cdot MHz \quad m_{pm} = 30 \quad A_{pm} = 20\,V$





$m_{pm} = 30$

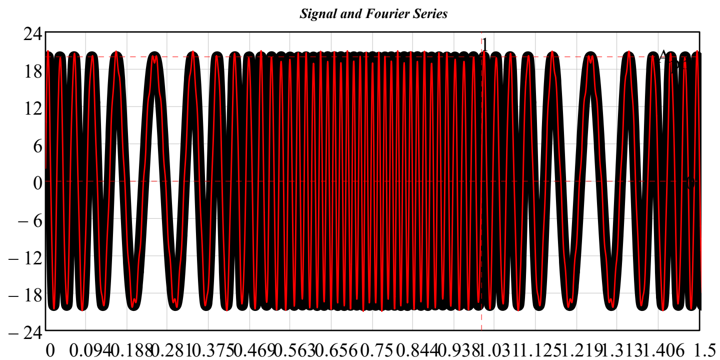
$N1\_ := 100$

$f_{cpm} = 600 \cdot MHz$

$v9_{pm}(t) := V9_{pm}(t, f_{cpm}, f_{pmm}, A_{pm}, m_{pm}, N1\_)$

$V9pm := SPCT(v9_{pm}, rt_{gd}, N1\_, 0 \cdot s, T_{pmm})$

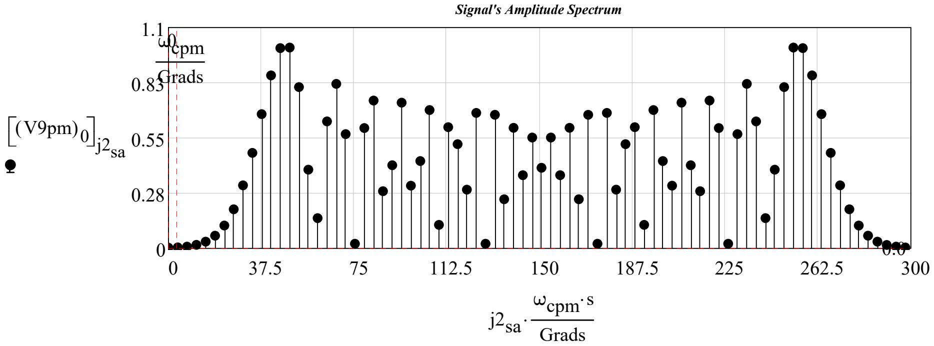
$N1\_ = 100$

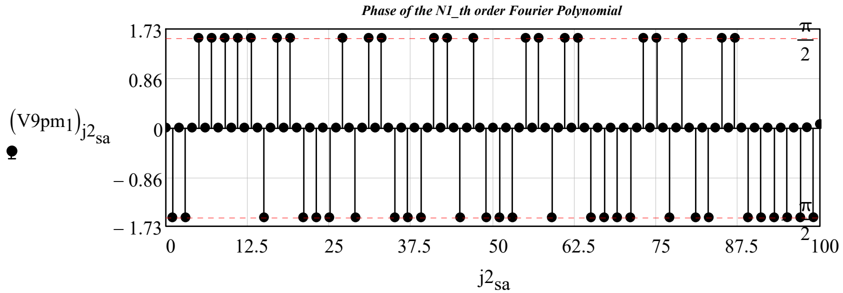


$\omega_{cpm} = 3.77 \cdot \frac{Grads}{s}$

$j2_{sa} := 0 .. rows(V9pm_0) - 1$

$\omega_{pmm} = 94.248 \cdot \frac{Mrads}{s}$





$Bw_{sa} := V9pm_3 \cdot Hz$

$Bw_{sa} = 1.125 \times 10^3 \cdot MHz$

sampling frequency: $fpt_{so} := 2 \cdot Bw_{sa}$ $\quad fpt_{so} = 2.25 \times 10^3 \cdot MHz$

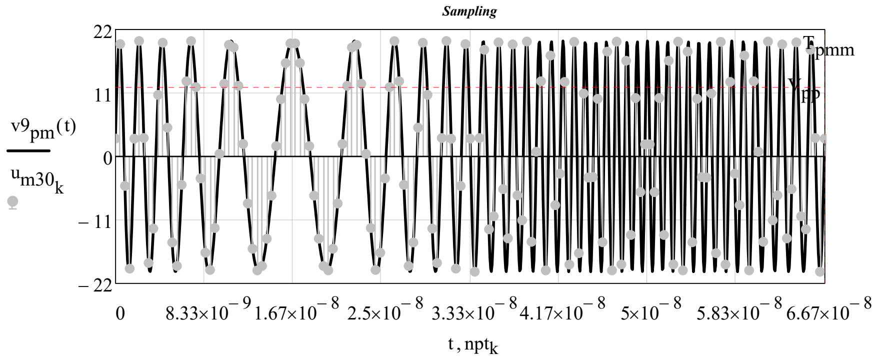
$npt_k := \frac{k}{fpt_{so}}$

Frequency resolution: $\frac{N0_{gd}}{fpt_{so}} \cdot \frac{1}{T_{pmm}} = 1.707$

$(u_{m30})_k := v9_{pm}(npt_k)$

$u_{m30}^T =$

	0	1	2	3	4	5	6
0	3.085	19.458	-5.124	-19.462	3.061	19.995	...

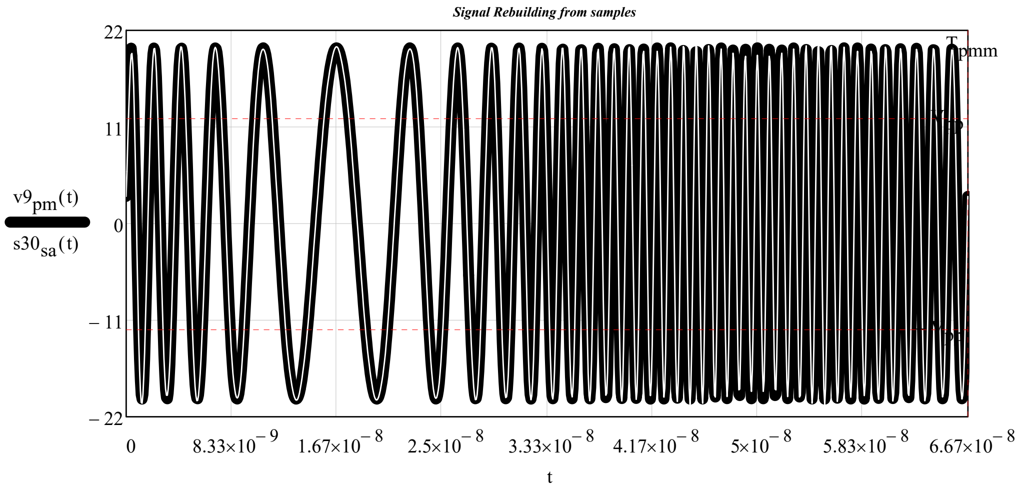


relerr = 10·%

$\omega_{bwr} := 2\cdot\pi\cdot Bw_{sa}$ $\quad \omega_{bwr} = 7.069\times 10^{3}\cdot\frac{Mrads}{sec}$ $\quad n\cdot\frac{\pi}{\omega_{bwr}} = n\cdot\frac{1}{2\cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula: $s30_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1}\left(u_{m30_n}\cdot \mathrm{sinc}(\omega_{bwr}\cdot t - n\cdot\pi)\right)\right]$ $\quad N0_{gd} - 1 = 255$ $\quad$ relerr



TEST Waveforms

Periodic Waveforms

29 PM test signal (triangular wave)

$T_{tri} := \frac{T_{pmm}}{2}$

$v_{mtri}(t_{sl}) := v_{tri0}(t_{sl}, T_{tri}, A_{pm}, N0_{gd})$

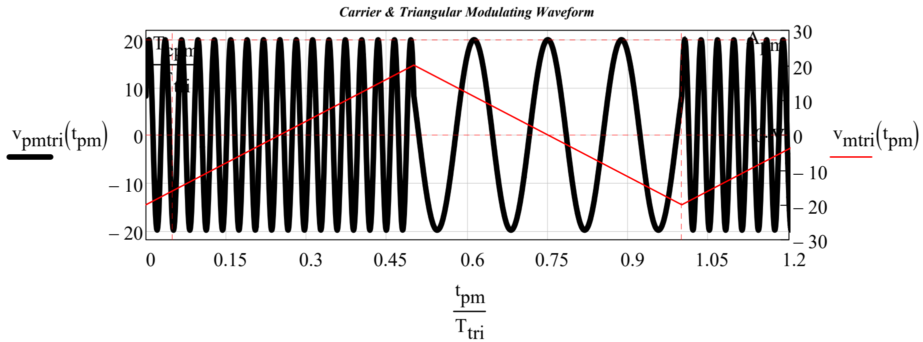
$v_{pmtri}(t_{sl}) := A_{pm}\cdot\cos(\omega_{cpm}\cdot t_{sl} + k_{pm}\cdot v_{mtri}(t_{sl}))$

$k_{pm} = \frac{m_{pm}}{B_{pm}}$ $\quad k_{pm} = 1\cdot V^{-1}$

$v_{pmtri}(t, T_{pmm}, f_{cpm}, k_{pm}, A_{pm}, B_{pm}, N0_{gd}) = A_{pm}\cdot\cos(2\cdot\pi\cdot f_{cpm}\cdot t + k_{pm}\cdot v_{tri0}(t, T_{tri}, B_{pm}, N0_{gd}))$

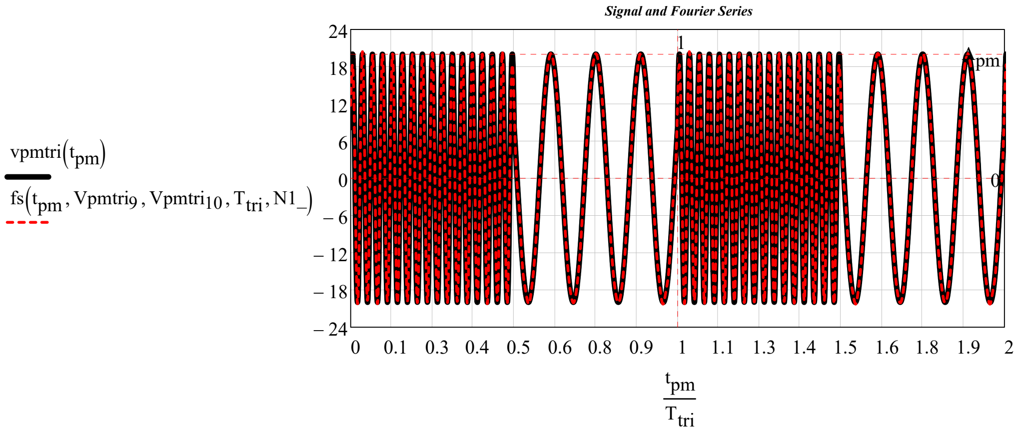
$V10_{pm}(t, T_{pmm}, f_{cpm}, m_{pm}, A_{pm}, B_{pm}, N0_{gd}) = \frac{v_{pmtri}(t, T_{tri}, f_{cpm}, m_{pm}, A_{pm}, B_{pm}, N0_{gd})}{V}$

$t_{pm} := T_{tri}\cdot 0, T_{tri}\cdot 0 + \frac{5\cdot T_{tri} - 0\cdot T_{tri}}{10000} .. 5\cdot T_{tri}$

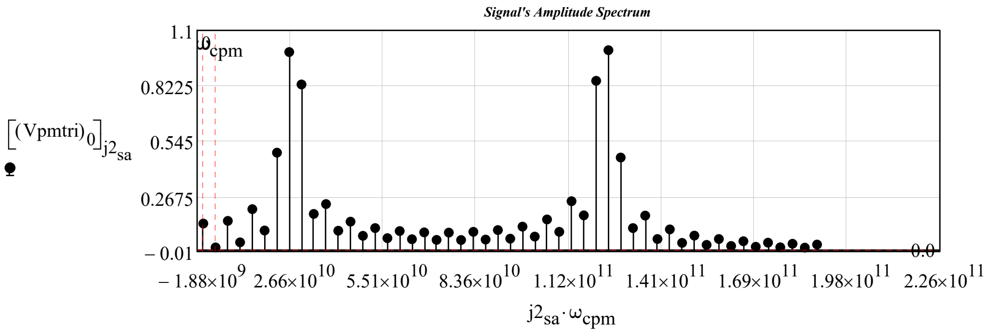


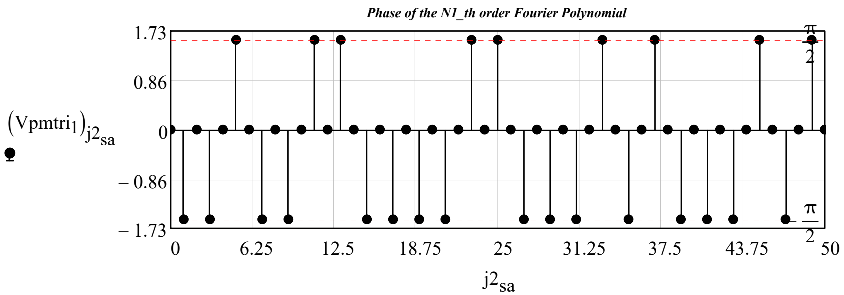
$N1\_ := 50$ $\quad vpmtri(t) := \frac{v_{pmtri}(t)}{V}$

$Vpmtri := SPCT(vpmtri, rt_{gd}, N1\_, 0\cdot s, T_{tri})$ $\quad N1\_ = 50$



$relerr := Vpmtri_7$ $\quad j2_{sa} := 0..rows(Vpmtri_0) - 1$ $\quad \omega_{pmm} = 94.248 \cdot \frac{Mrads}{s}$ $\quad relerr = 10 \cdot \%$





$Bw_{sa} := Vpmtri_3 \cdot Hz$

$Bw_{sa} = 1.44 \times 10^3 \cdot MHz$

sampling frequency: $fpt_{so} := 2 \cdot Bw_{sa}$ $\quad fpt_{so} = 2.88 \cdot GHz$

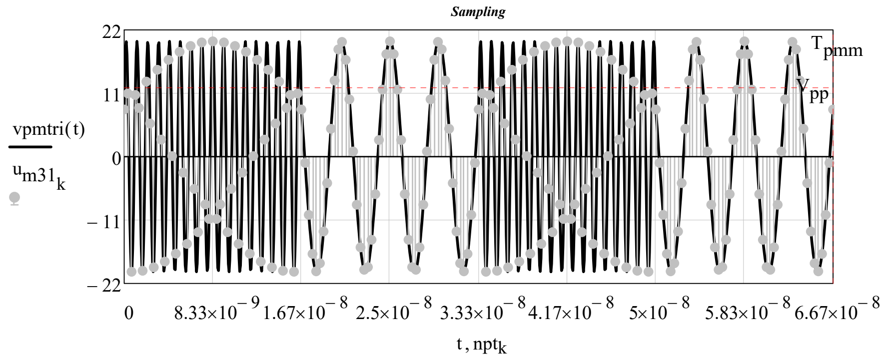
$k := 0..2^8 - 1$ $\quad npt_k := \frac{k}{fpt_{so}}$

Frequency resolution: $\frac{N0_{gd}}{fpt_{so}} \cdot \frac{1}{T_{pmm}} = 1.333$

$(u_{m31})_k := vpmtri(npt_k)$

$u_{m31}^T =$

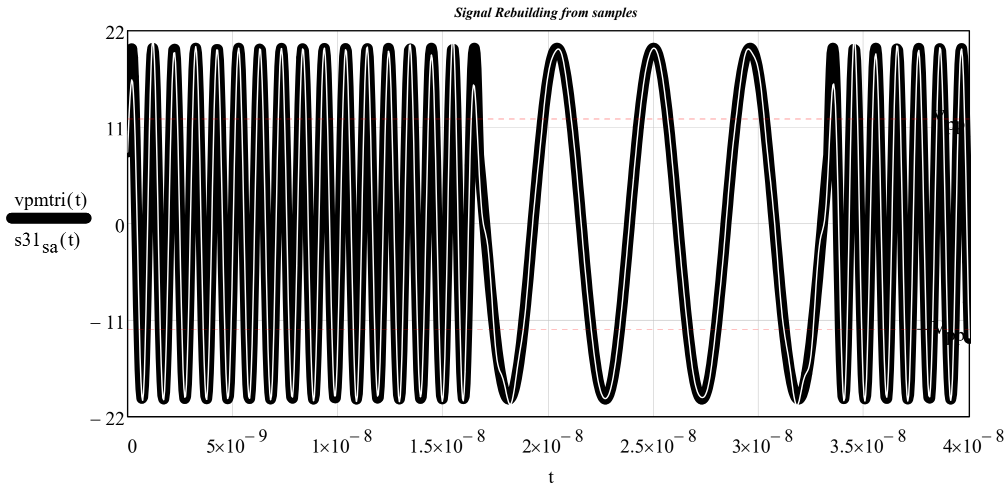
	0	1	2	3	4	5	6
0	8.162	10.942	-19.999	10.694	8.43	-19.814	...



$relerr = 10 \cdot \%$ $\quad \omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa}$ $\quad \omega_{bwr} = 9.048 \times 10^3 \cdot \frac{Mrads}{sec}$ $\quad n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula: $s31_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1} \left(u_{m31_n} \cdot sinc(\omega_{bwr} \cdot t - n \cdot \pi) \right) \right]$ $\quad N0_{gd} - 1 = 255$



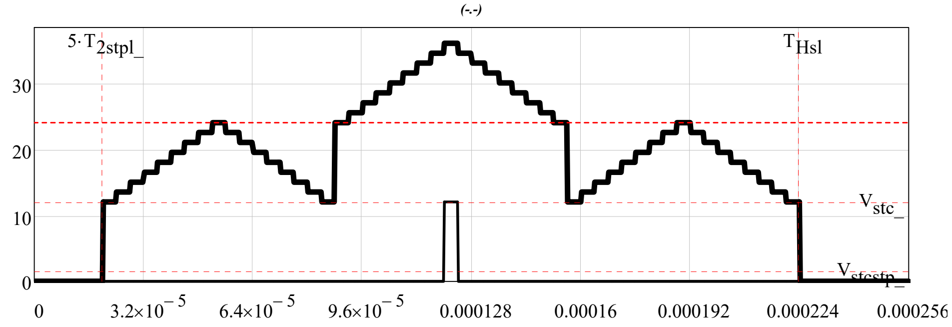
TEST Waveforms

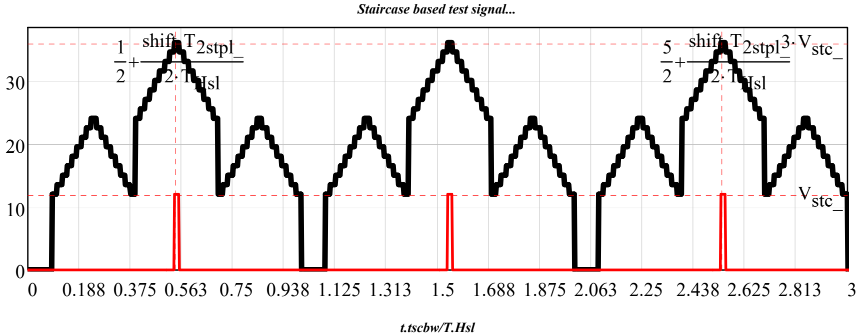
Periodic Waveforms

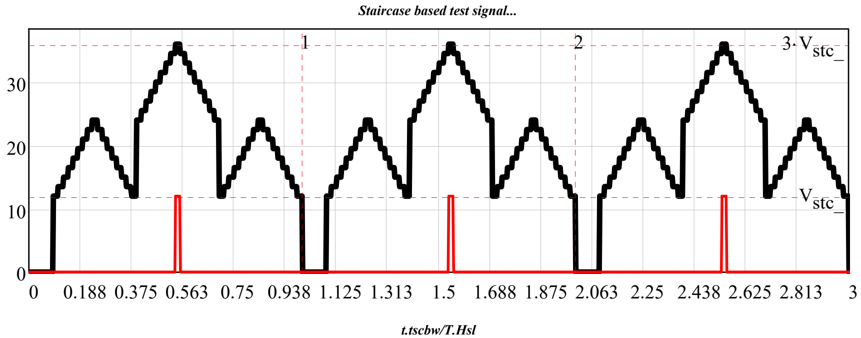
30 Staircase based test signal

$shift := 5$

$T_{Hsl} := (6 \cdot m2_{steps\_} + shift + 3) \cdot T_{2stpl\_} \qquad t_{tscbw} := 0 \cdot T_{Hsl}, 0 \cdot T_{Hsl} + \frac{5 \cdot T_{Hsl}}{2000} .. 5 \cdot T_{Hsl}$

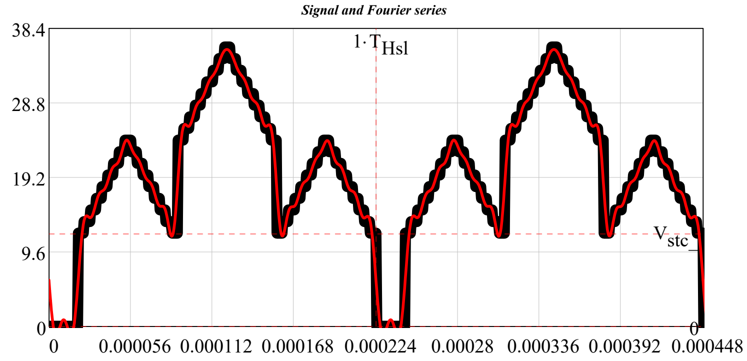




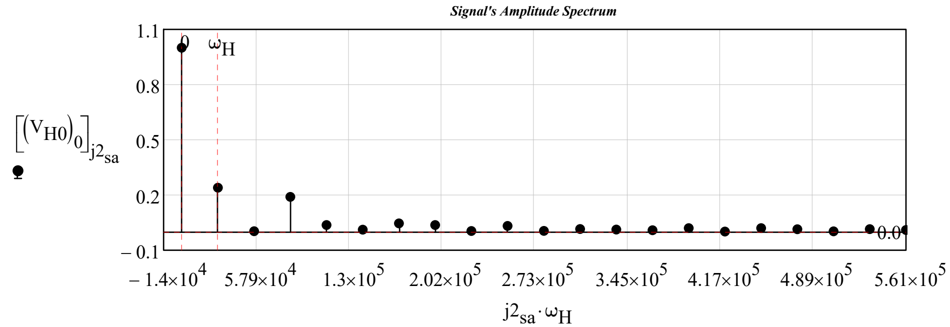


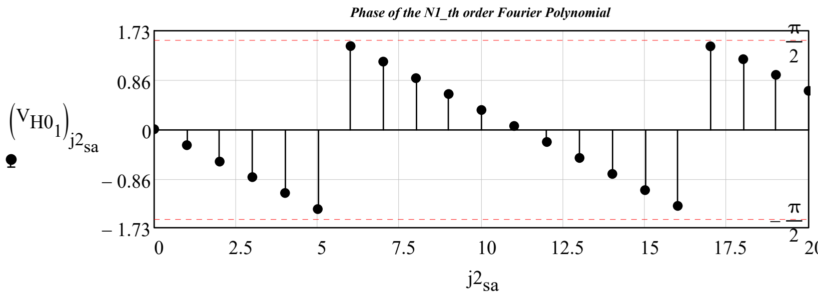
$N1\_ := 25 \qquad \omega_H := \frac{2 \cdot \pi}{T_{Hsl}} \qquad VH(t) := V_H(t, T_{Hsl}, T_{2stpl\_}, V_{stc\_}, mstc3_{steps\_}, shift, N_{gd})$

$5 \cdot T_{2stpl\_} = 20 \cdot \mu s \qquad V_{H0} := SPCT(VH, rt_{gd}, N1\_, 5 \cdot T_{2stpl\_}, T_{Hsl}) \qquad N1\_ = 25$



$j2_{sa} := 0 .. rows\left(V_{H0_0}\right) - 1 \qquad \omega_H = 28.05 \cdot \frac{krads}{s}$





$Bw_{sa} := VH0_3 \cdot Hz$

$Bw_{sa} = 0.103 \cdot MHz$

sampling frequency: $fpt_{so} := 2 \cdot Bw_{sa}$ $\qquad fpt_{so} = 0.205 \cdot MHz$

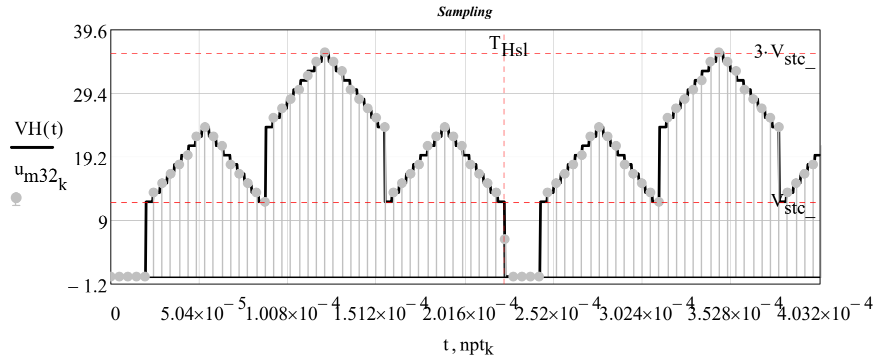
$npt_k := \frac{k}{fpt_{so}}$

Frequency resolution: $\frac{N0_{gd}}{fpt_{so}} \cdot \frac{1}{T_{Hsl}} = 5.565$

$(u_{m32})_k := VH(npt_k)$

$u_{m32}^T =$

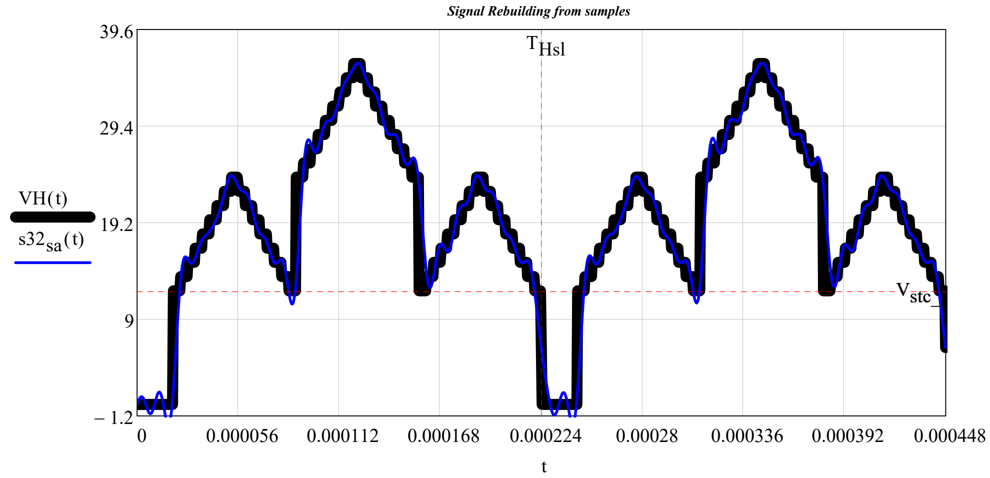
	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	13.5	15	16.5	...



$relerr = 10 \cdot \%$ $\qquad \omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa}$ $\qquad \omega_{bwr} = 0.645 \cdot \frac{Mrads}{sec}$ $\qquad n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula: $s32_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1} \left(u_{m32_n} \cdot sinc(\omega_{bwr} \cdot t - n \cdot \pi)\right)\right]$ $\qquad N0_{gd} - 1 = 255$ $\qquad relerr = 10 \cdot \%$



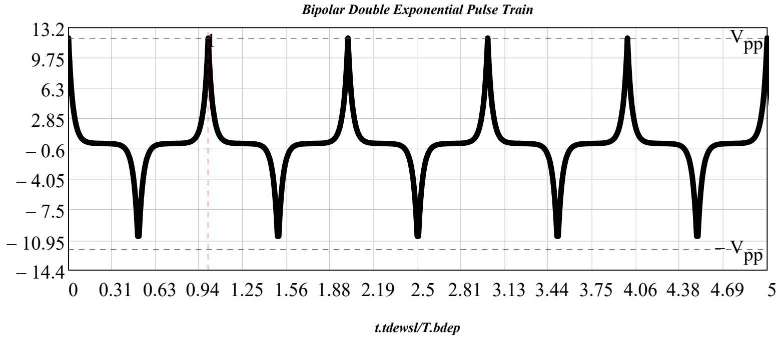
TEST Waveforms

Periodic Waveforms

31 Bipolar Double Exponential Pulse Train

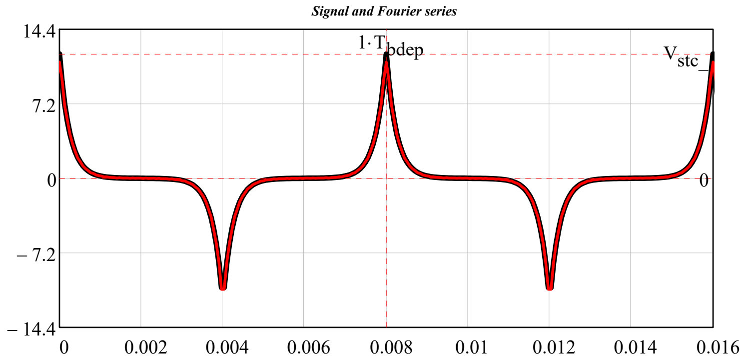
$$T_{bdep} := 32 \cdot \tau_{ptd\_}$$

$$t_{tdewsl} := -20 \cdot T_{bdep}, -20 \cdot T_{bdep} + \frac{20 \cdot T_{bdep} + 20 \cdot T_{bdep}}{5000} .. 20 \cdot T_{bdep}$$

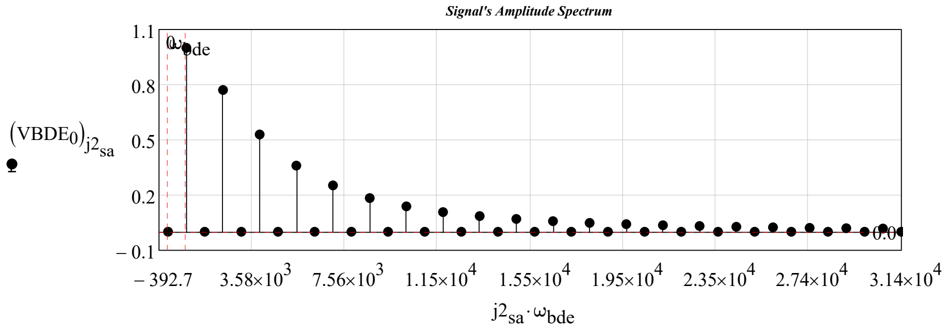


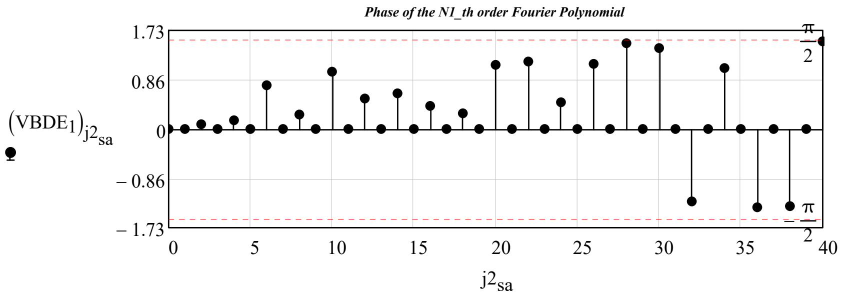
$N1\_ := 50$ $\qquad \omega_{bde} := \frac{2 \cdot \pi}{T_{bdep}}$ $\qquad Vbdept(t) := \frac{V_{bdept}(t, \tau_{ptd\_}, T_{bdep}, V_{pp}, N0_{gd})}{V}$

$$VBDE := SPCT(Vbdept, rt_{gd}, N1\_, 0 \cdot s, T_{bdep}) \qquad N1\_ = 50$$



$$j2_{sa} := 0 .. rows(VBDE_0) - 1 \qquad \omega_{bde} = 0.785 \cdot \frac{krads}{s}$$





$$Bw_{sa} := VBDE_3 \cdot Hz$$

$$Bw_{sa} = 6 \times 10^{-3} \cdot MHz$$

sampling frequency: $\quad fpt_{so} := 2 \cdot Bw_{sa} \qquad fpt_{so} = 0.012 \cdot MHz$

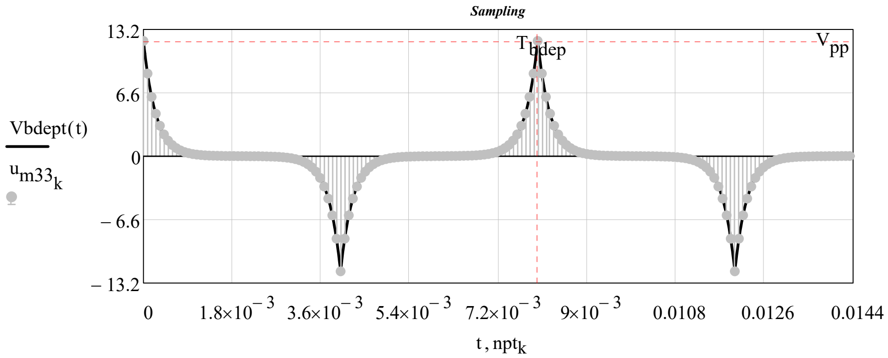
$$npt_k := \frac{k}{fpt_{so}}$$

Frequency resolution: $\quad \frac{N0_{gd}}{fpt_{so}} \cdot \frac{1}{T_{bdep}} = 2.667$

$$(u_{m33})_k := Vbdept(npt_k)$$

$u_{m33}{}^T =$

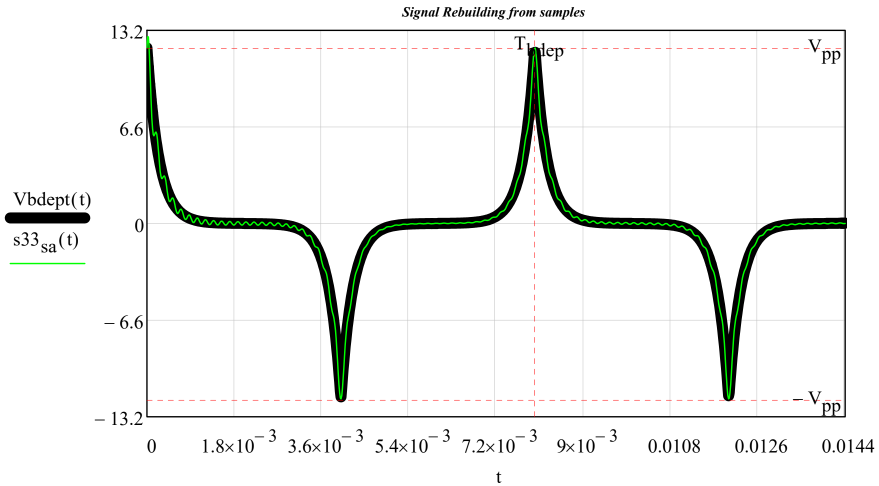
	0	1	2	3	4
0	12	8.598	6.161	4.415	...



$relerr = 10\cdot\%$ $\qquad \omega_{bwr} := 2\cdot\pi\cdot Bw_{sa}$ $\qquad \omega_{bwr} = 0.038\cdot\frac{Mrads}{sec}$ $\qquad n\cdot\frac{\pi}{\omega_{bwr}} = n\cdot\frac{1}{2\cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

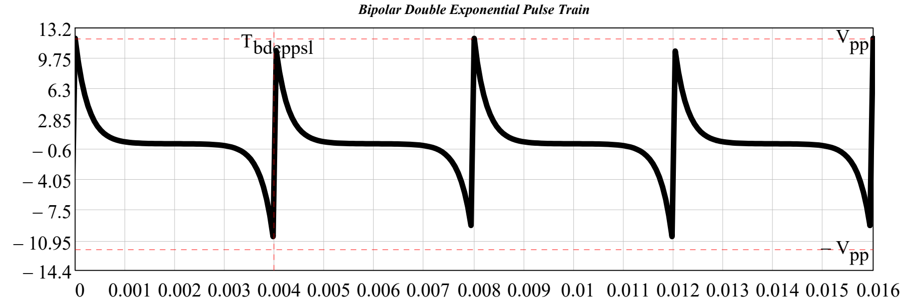
interpolation formula: $\quad s33_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1}\left(u_{m33_n}\cdot \mathrm{sinc}(\omega_{bwr}\cdot t - n\cdot\pi)\right)\right]$ $\qquad N0_{gd} - 1 = 255$



Periodic Waveforms

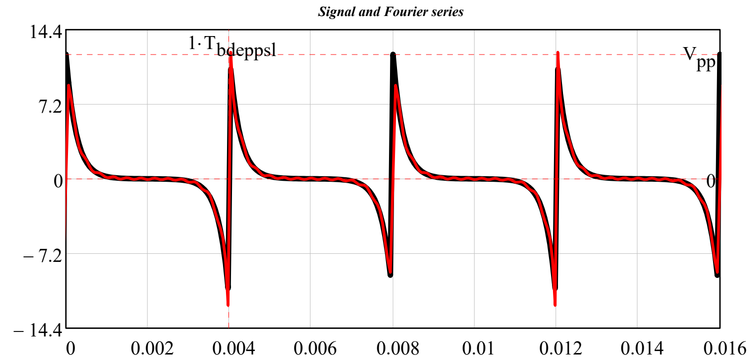
32 Bipolar Double Exponential Odd symmetric Pulse Train

$$T_{bdeppsl} := 16\cdot\tau_{ptd\_}$$

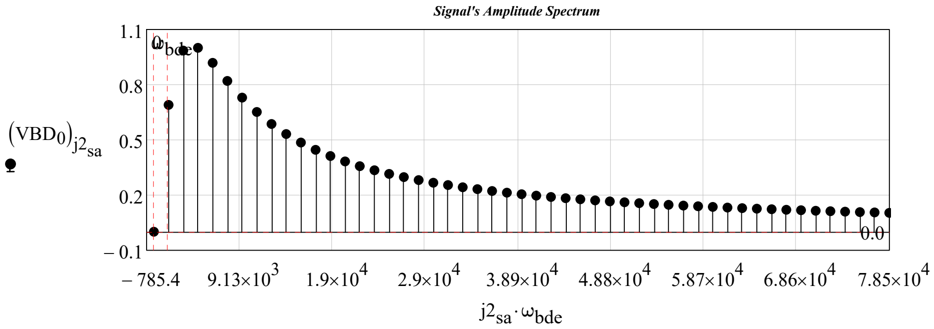


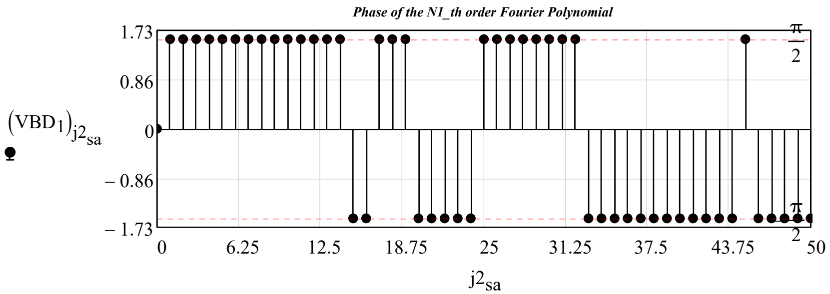
$N1\_ := 50$ $\qquad \omega_{bde} := \frac{2\cdot\pi}{T_{bdeppsl}}$ $\qquad Vbdeospp(t) := \frac{V_{bdeospp}(t, \tau_{ptd\_}, T_{bdeppsl}, V_{pp}, N0_{gd})}{V}$

$VBD := SPCT(Vbdeospp, rt_{gd}, N1\_, 0\cdot s, T_{bdeppsl})$ $\qquad N1\_ = 50$



$j2_{sa} := 0 .. rows(VBD_0) - 1$ $\qquad \omega_{bde} = 1.571\cdot\frac{krads}{s}$





$Bw_{sa} := VBD_3\cdot Hz$

$Bw_{sa} = 0.012\cdot MHz$

sampling frequency: $fpt_{so} := 2\cdot Bw_{sa}$ $\qquad fpt_{so} = 0.024\cdot MHz$

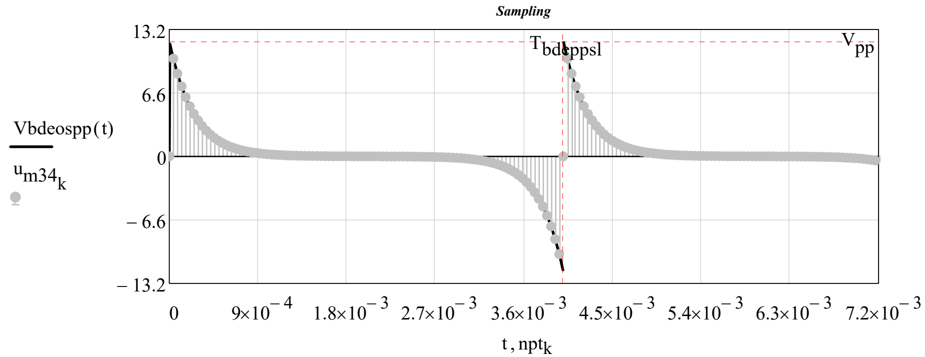
$npt_k := \dfrac{k}{fpt_{so}}$

Frequency resolution: $\dfrac{N0_{gd}}{fpt_{so}}\cdot\dfrac{1}{T_{bdeppsl}} = 2.667$

$\left(u_{m34}\right)_k := Vbdeospp\left(npt_k\right)$

$u_{m34}^{\ T} =$

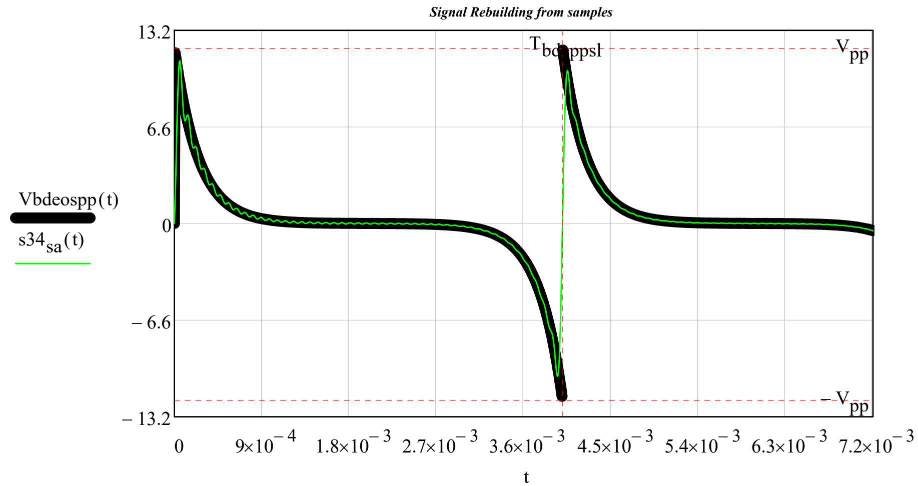
	0	1	2	3	4
0	-1.35·10^-6	10.158	8.598	7.278	...



$relerr = 10\cdot\%$ $\qquad \omega_{bwr} := 2\cdot\pi\cdot Bw_{sa}$ $\qquad \omega_{bwr} = 0.075\cdot\dfrac{Mrads}{sec}$ $\qquad n\cdot\dfrac{\pi}{\omega_{bwr}} = n\cdot\dfrac{1}{2\cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula: $s34_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1}\left(u_{m34_n}\cdot sinc\left(\omega_{bwr}\cdot t - n\cdot\pi\right)\right)\right]$ $\qquad N0_{gd} - 1 = 255$

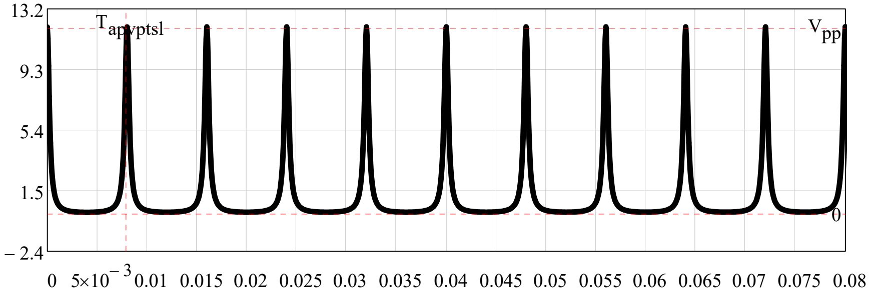


Periodic Waveforms

33 Agnesi Profile Voltage Pulse Train

$$T_{apvptsl} := 32\cdot\tau_{ptd\_}$$

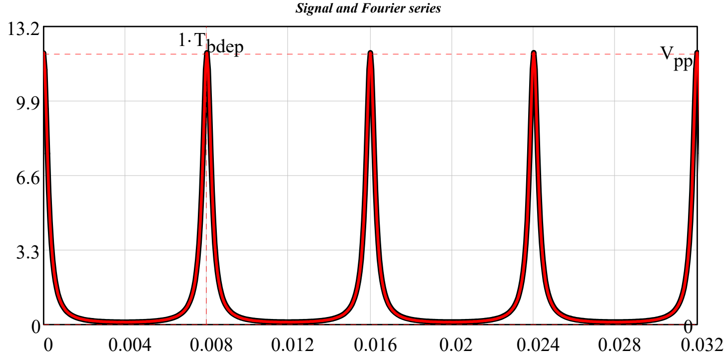
Agnesi Voltage Pulse Train



$N1\_ := 50$ $\qquad \omega_{agn} := \frac{2\cdot\pi}{T_{apvptsl}}$ $\qquad Vagnp(t) := \frac{V_{agnp}(t, \tau_{ptd\_}, T_{apvptsl}, V_{pp}, N0_{gd})}{V}$

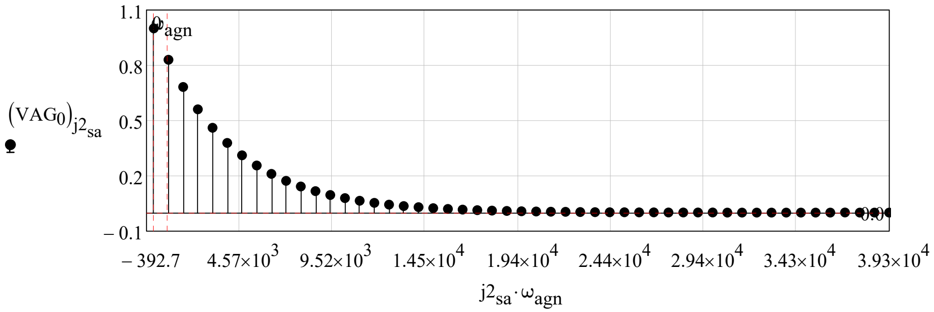
$$VAG := SPCT(Vagnp, rt_{gd}, N1\_, 0\cdot s, T_{apvptsl}) \qquad N1\_ = 50$$

Signal and Fourier series

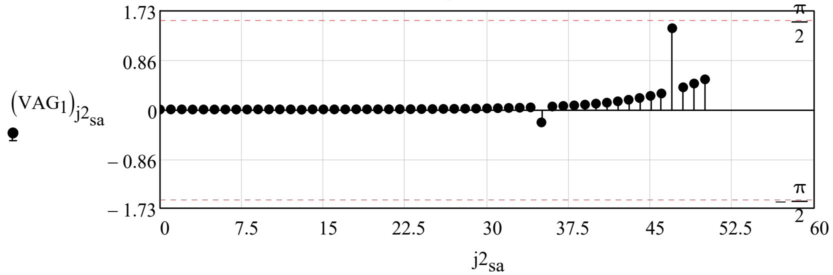


$$j2_{sa} := 0 .. rows(VAG_0) - 1 \qquad \omega_{agn} = 0.785\cdot\frac{krads}{s}$$

Signal's Amplitude Spectrum



Phase of the N1_th order Fourier Polynomial



$$Bw_{sa} := VAG_3\cdot Hz$$

$$Bw_{sa} = 2.875\times10^{-3}\cdot MHz$$

sampling frequency: $\quad fpt_{so} := 2\cdot Bw_{sa} \qquad fpt_{so} = 5.75\times10^{-3}\cdot MHz$

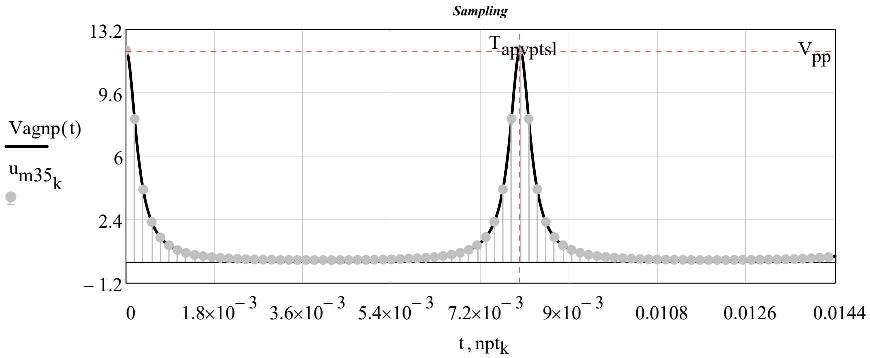
$$npt_k := \frac{k}{fpt_{so}}$$

Frequency resolution: $\quad \frac{N0_{gd}}{fpt_{so}}\cdot\frac{1}{T_{bdeppsl}} = 11.13$

$$(u_{m35})_k := Vagnp(npt_k)$$

$u_{m35}^T =$

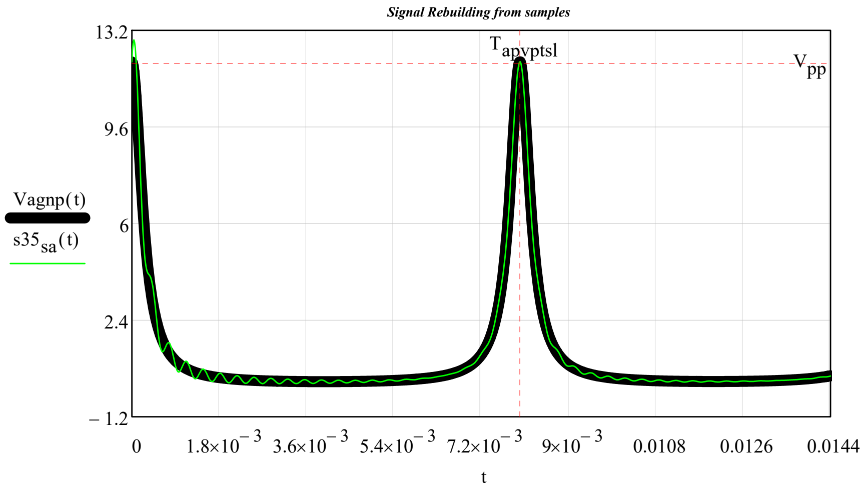
	0	1	2	3	4	5	6	7
0	12.019	8.106	4.108	2.262	1.395	0.939	0.675	...



$relerr = 10\cdot\%$ $\qquad \omega_{bwr} := 2\cdot\pi\cdot Bw_{sa}$ $\qquad \omega_{bwr} = 0.018\cdot\frac{Mrads}{sec}$ $\qquad n\cdot\frac{\pi}{\omega_{bwr}} = n\cdot\frac{1}{2\cdot Bw_{sa}}$

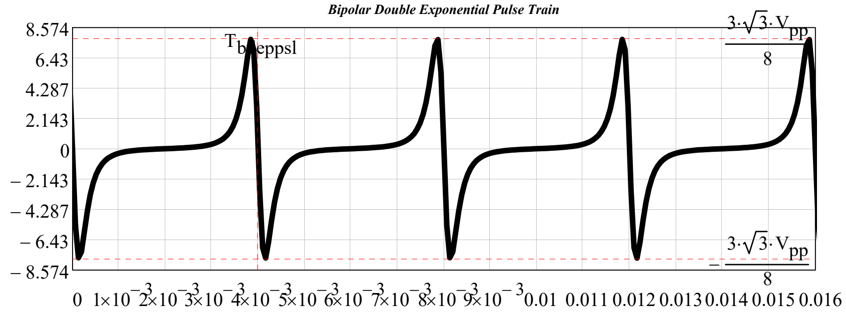
Signal reconstruction according to the Shannon sampling theorem:

interpolation formula: $$s35_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1}\left(u_{m35_n}\cdot \mathrm{sinc}\left(\omega_{bwr}\cdot t - n\cdot\pi\right)\right)\right] \qquad N0_{gd} - 1 = 255$$



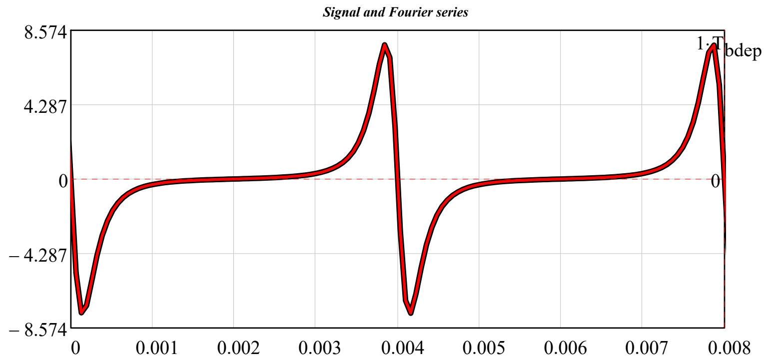
Periodic Waveforms

34 Agnesi Derivative Profile Voltage Pulse Train

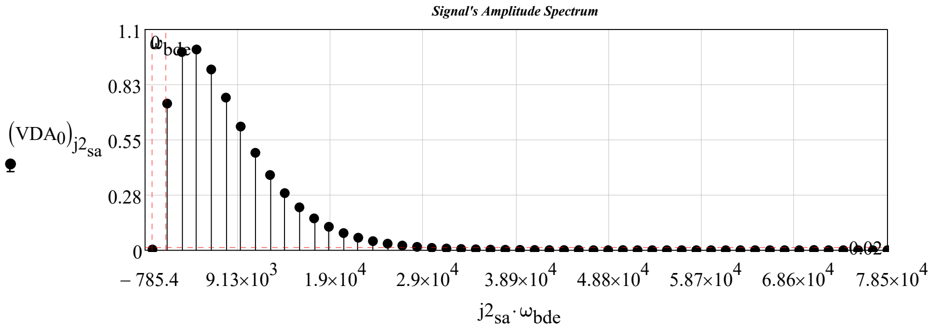


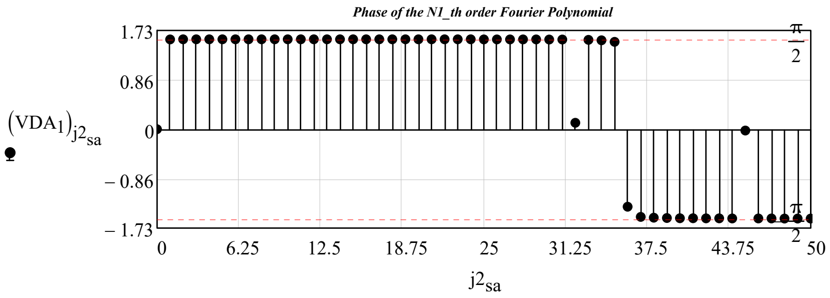
$N1\_ := 50$ $\qquad \omega_{bde} := \frac{2\cdot\pi}{T_{bdeppsl}}$ $\qquad VDagnp(t) := \frac{V_{Dagnp}\left(t, \tau_{ptd\_}, T_{bdeppsl}, V_{pp}, N0_{gd}\right)}{V}$

$VDA := SPCT\left(VDagnp, rt_{gd}, N1\_, 0\cdot s, T_{bdeppsl}\right)$ $\qquad N1\_ = 50$



$j2_{sa} := 0 .. \mathrm{rows}\left(VDA_0\right) - 1$ $\qquad \omega_{bde} = 1.571\cdot\frac{krads}{s}$





$$Bw_{sa} := VDA_3 \cdot Hz$$

$$Bw_{sa} = 4.5 \times 10^{-3} \cdot MHz$$

sampling frequency: $fpt_{so} := 2 \cdot Bw_{sa}$ $\quad fpt_{so} = 9 \times 10^{-3} \cdot MHz$

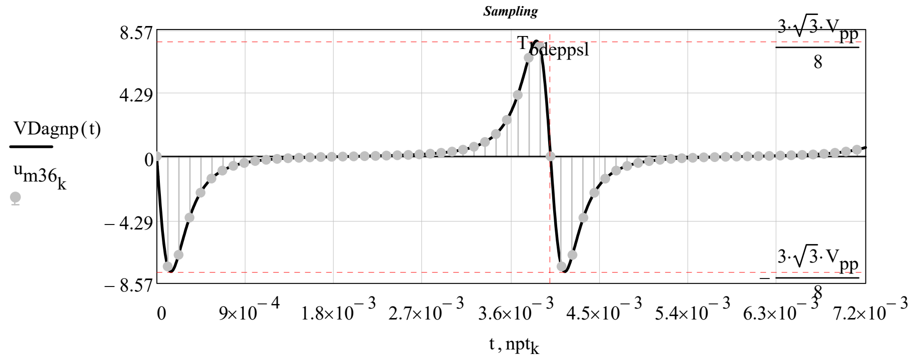
$$npt_k := \frac{k}{fpt_{so}}$$

Frequency resolution: $\frac{N0_{gd}}{fpt_{so}} \cdot \frac{1}{T_{bdeppsl}} = 7.111$

$$(u_{m36})_k := VDagnp(npt_k)$$

$u_{m36}^T =$

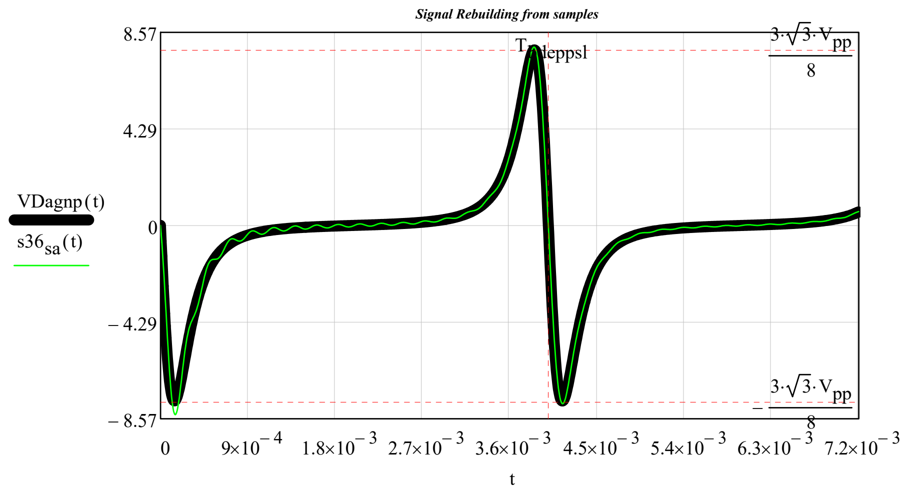
	0	1	2	3	4
0	$6.996 \cdot 10^{-3}$	-7.43	-6.649	-4.138	...



relerr = 10·% $\quad \omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa}$ $\quad \omega_{bwr} = 0.028 \cdot \frac{Mrads}{sec}$ $\quad n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula: $s36_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1} \left(u_{m36_n} \cdot \text{sinc}(\omega_{bwr} \cdot t - n \cdot \pi) \right) \right]$ $\quad N0_{gd} - 1 = 255$

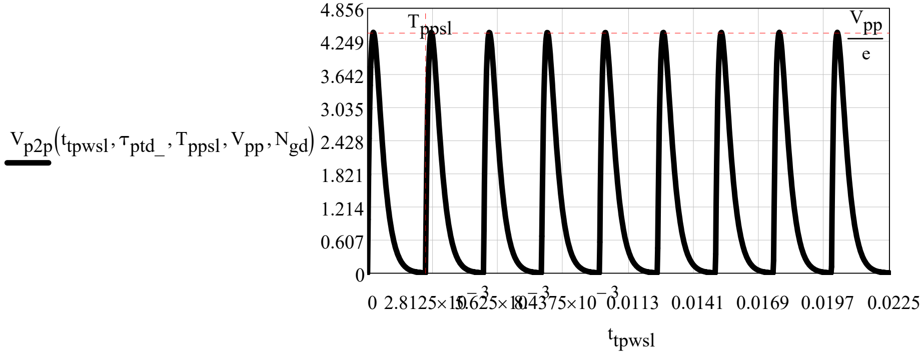


Periodic Waveforms

35 Poisson Profile Voltage Pulse Train

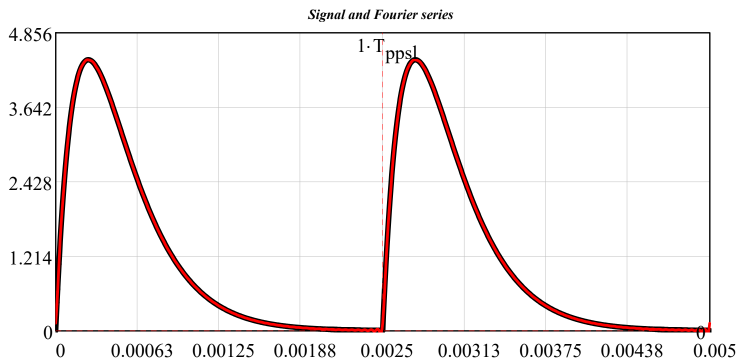
$$t_{tpw} := 0\cdot\tau_{ptd\_}, 0\cdot\tau_{ptd\_} + \frac{200\cdot\tau_{ptd\_}}{2000} .. 200\cdot\tau_{ptd\_}$$

$$T_{ppsl} := 10\cdot\tau_{ptd\_}$$

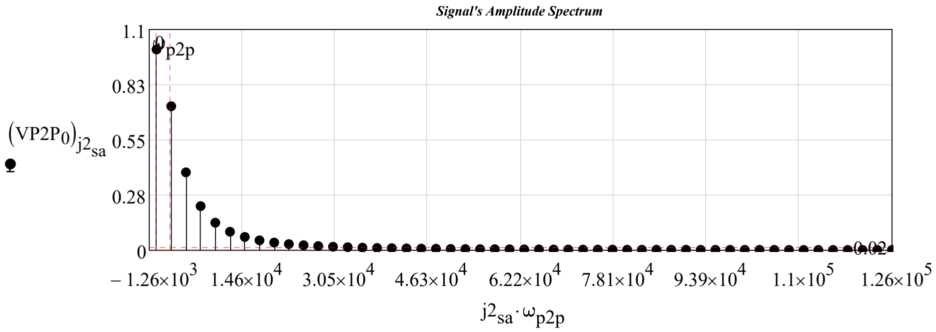


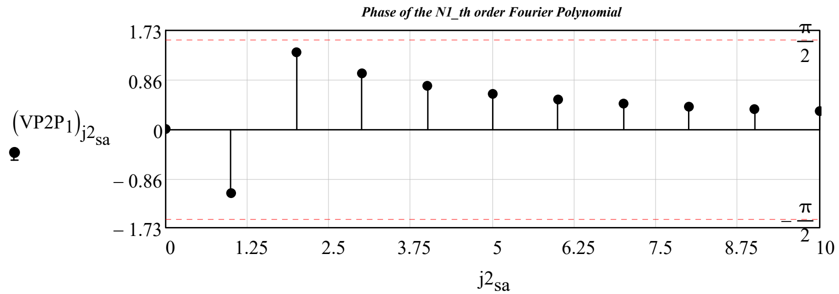
$$\omega_{p2p} := \frac{2\cdot\pi}{T_{ppsl}} \qquad V_{p2p}(t) := \frac{V_{p2p}\left(t, \tau_{ptd\_}, T_{ppsl}, V_{pp}, N_{gd}\right)}{V}$$

$$VP2P := SPCT\left(V_{p2p}, rt_{gd}, N1\_, 0\cdot s, T_{ppsl}\right) \qquad N1\_ = 50$$



$$j2_{sa} := 0 .. rows\left(VP2P_0\right) - 1 \qquad \omega_{p2p} = 2.513\cdot\frac{krads}{s}$$





$$Bw_{sa} := VP2P_3\cdot Hz$$

$$Bw_{sa} = 0.01\cdot MHz$$

sampling frequency: $fpt_{so} := 2\cdot Bw_{sa}$ $\qquad fpt_{so} = 0.02\cdot MHz$

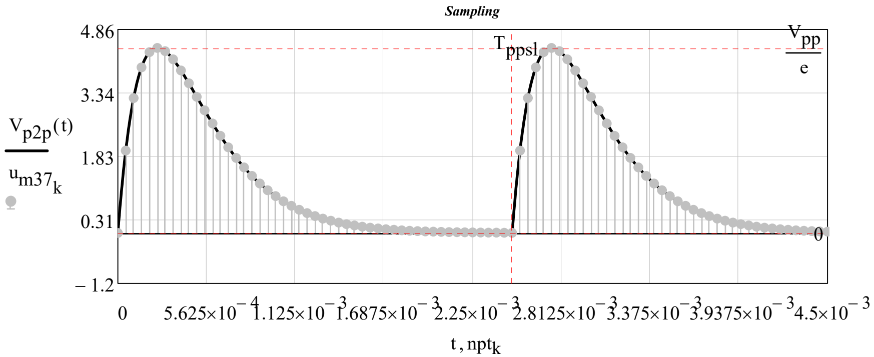
$$npt_k := \frac{k}{fpt_{so}}$$

Frequency resolution: $\frac{N0_{gd}}{fpt_{so}}\cdot\frac{1}{T_{ppsl}} = 5.12$

$$\left(u_{m37}\right)_k := V_{p2p}\left(npt_k\right)$$

$u_{m37}^{T} =$

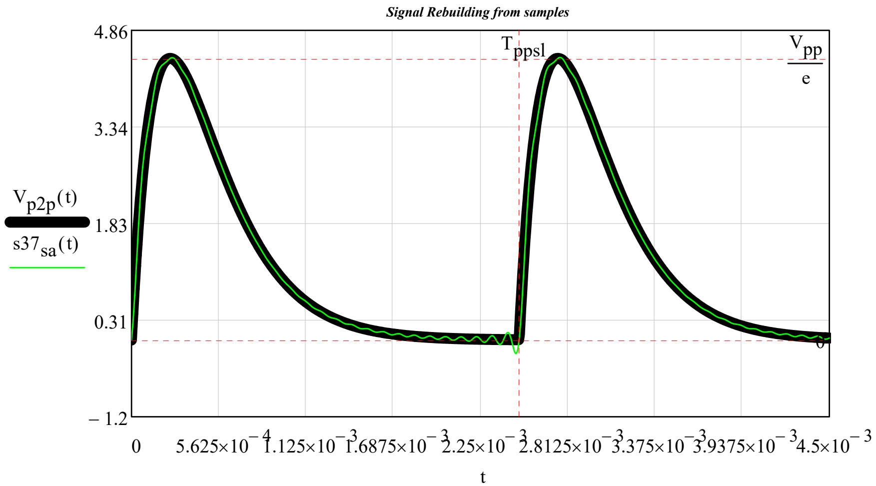
	0	1	2	3	4
0	0	1.965	3.218	3.951	...



$relerr = 10 \cdot \%$ $\qquad \omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa}$ $\qquad \omega_{bwr} = 0.063 \cdot \frac{Mrads}{sec}$ $\qquad n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

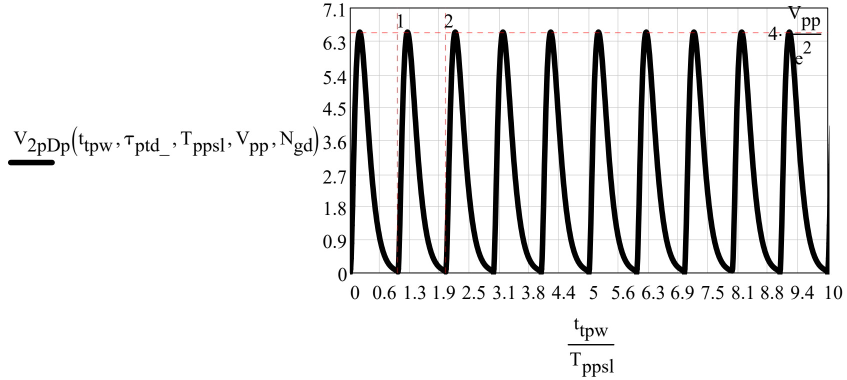
interpolation formula: $\quad s37_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1} \left(u_{m37_n} \cdot \mathrm{sinc}\left(\omega_{bwr} \cdot t - n \cdot \pi \right) \right) \right]$ $\qquad N0_{gd} - 1 = 255$ $\qquad relerr =$



TEST Waveforms

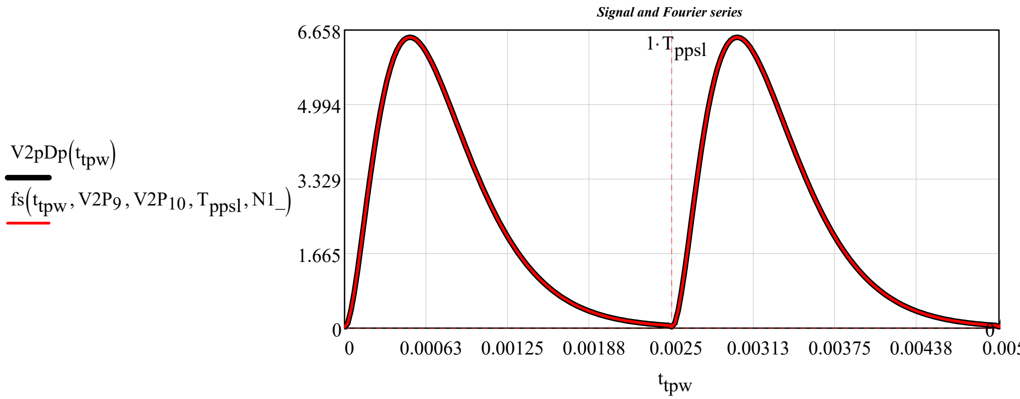
Periodic Waveforms

36 Poisson Derivative Profile Voltage Pulse Train



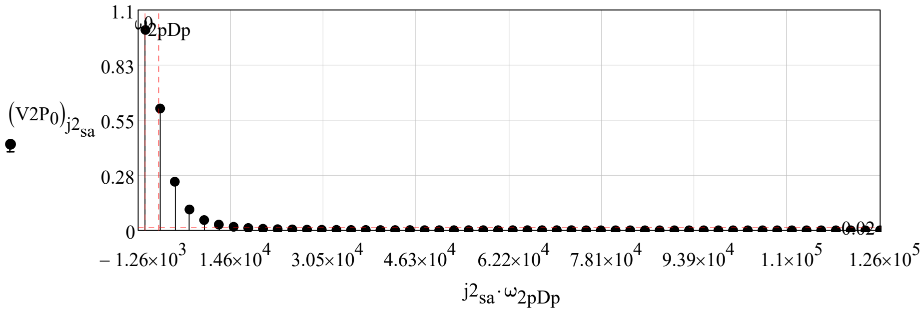
$\omega_{2pDp} := \frac{2 \cdot \pi}{T_{ppsl}}$ $\qquad V2pDp(t) := \frac{V_{2pDp}(t, \tau_{ptd\_}, T_{ppsl}, V_{pp}, N_{gd})}{V}$

$V2P := SPCT(V2pDp, rt_{gd}, N1\_, 0 \cdot s, T_{ppsl})$ $\qquad N1\_ = 50$

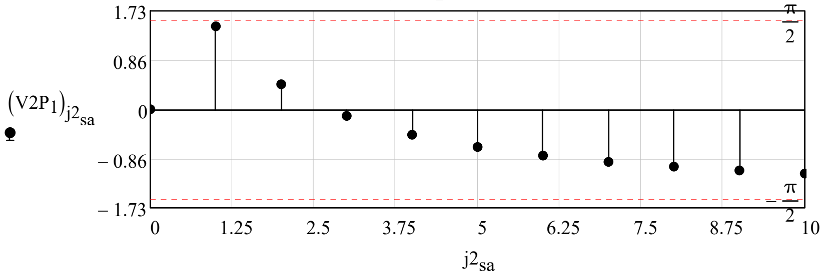


$j2_{sa} := 0 .. rows(V2P_0) - 1$ $\qquad \omega_{2pDp} = 2.513 \cdot \frac{krads}{s}$

Signal's Amplitude Spectrum



Phase of the N1_th order Fourier Polynomial



$Bw_{sa} := V2P_3 \cdot Hz$

$Bw_{sa} = 4.8 \times 10^{-3} \cdot MHz$

sampling frequency: $fpt_{so} := 2 \cdot Bw_{sa}$ $\quad fpt_{so} = 9.6 \times 10^{-3} \cdot MHz$

$npt_k := \frac{k}{fpt_{so}}$

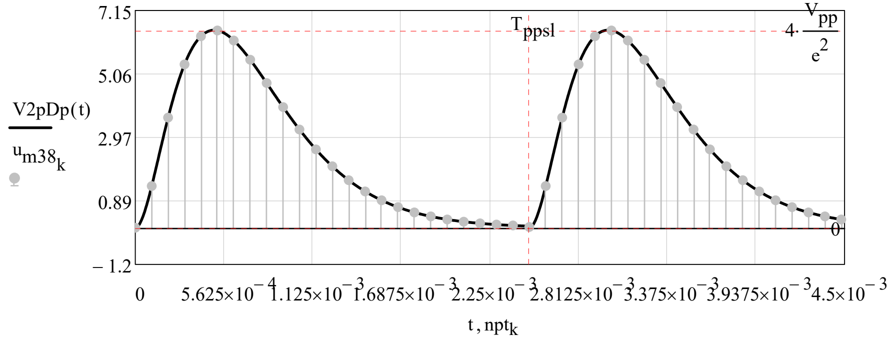
Frequency resolution: $\frac{N0_{gd}}{fpt_{so}} \cdot \frac{1}{T_{ppsl}} = 10.667$

$(u_{m38})_k := V2pDp(npt_k)$

$u_{m38}^T =$

	0	1	2	3	4	5	6	7	8
0	0	1.373	3.622	5.372	6.296	6.485	6.156	5.524	...

Sampling

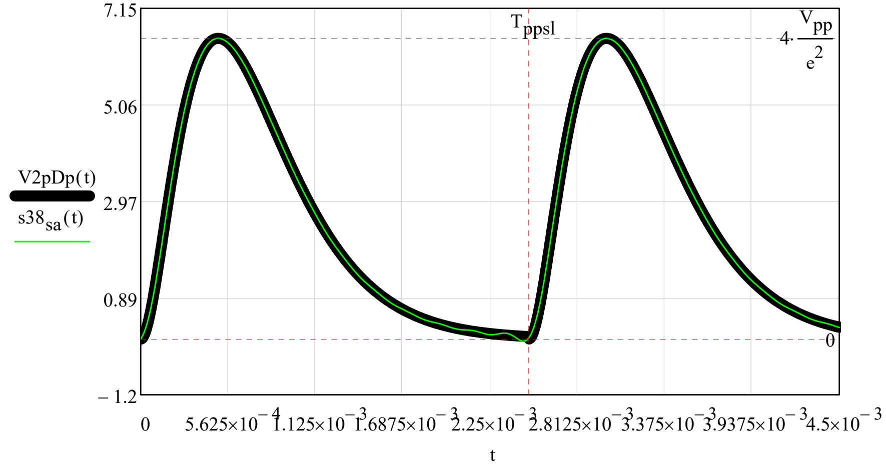


$relerr = 10 \cdot \%$ $\quad \omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa}$ $\quad \omega_{bwr} = 0.03 \cdot \frac{Mrads}{sec}$ $\quad n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

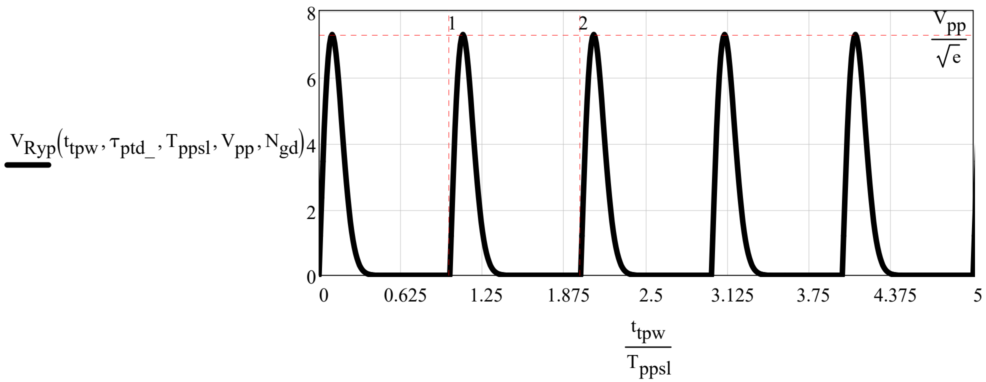
interpolation formula: $s38_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1} \left(u_{m38_n} \cdot \mathrm{sinc}(\omega_{bwr} \cdot t - n \cdot \pi)\right)\right]$ $\quad N0_{gd} - 1 = 255$ $\quad relerr =$

Signal Rebuilding from samples



Periodic Waveforms

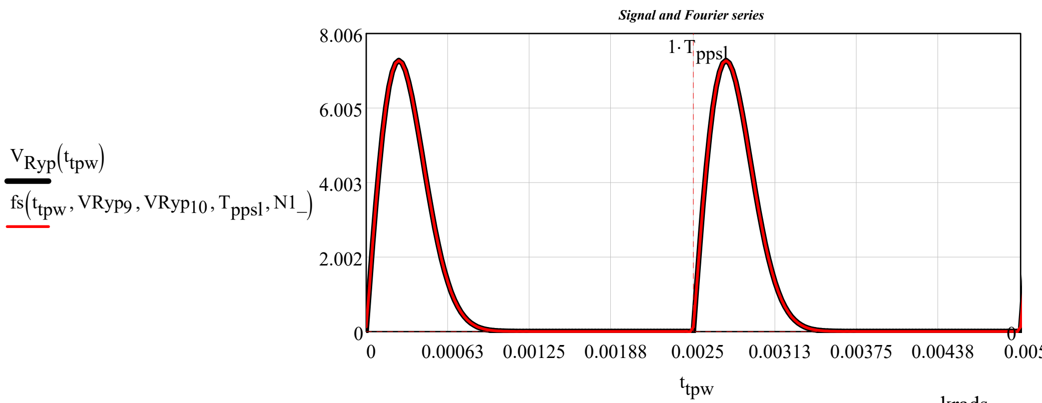
37 Rayleigh Profile Voltage Pulse Train



$$\omega_{ryp} := \frac{2\cdot\pi}{T_{ppsl}} \qquad V_{Ryp}(t) := \frac{V_{Ryp}(t, \tau_{ptd\_}, T_{ppsl}, V_{pp}, N_{gd})}{V}$$

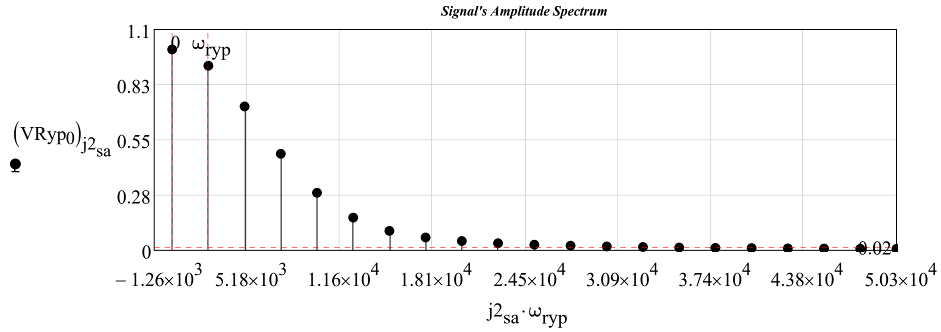
$$VRyp := SPCT(V_{Ryp}, rt_{gd}, N1\_, 0\cdot s, T_{ppsl}) \qquad N1\_ = 50$$

Signal and Fourier series

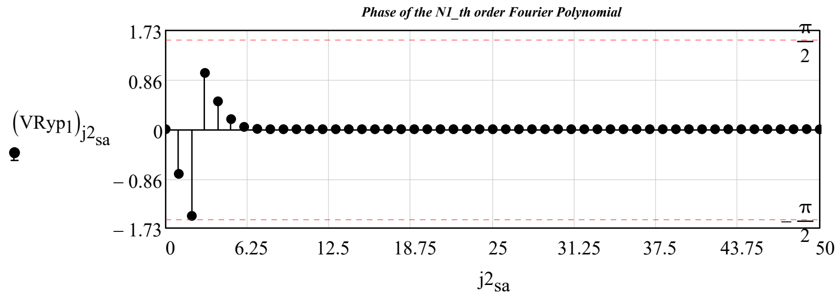


$$j2_{sa} := 0 .. rows(VRyp_0) - 1 \qquad \omega_{ryp} = 2.513\cdot\frac{krads}{s}$$

Signal's Amplitude Spectrum



Phase of the N1_th order Fourier Polynomial



$$Bw_{sa} := VRyp_3\cdot Hz$$

$$Bw_{sa} = 8\times10^{-3}\cdot MHz$$

sampling frequency: $fpt_{so} := 2\cdot Bw_{sa}$ $\qquad fpt_{so} = 0.016\cdot MHz$

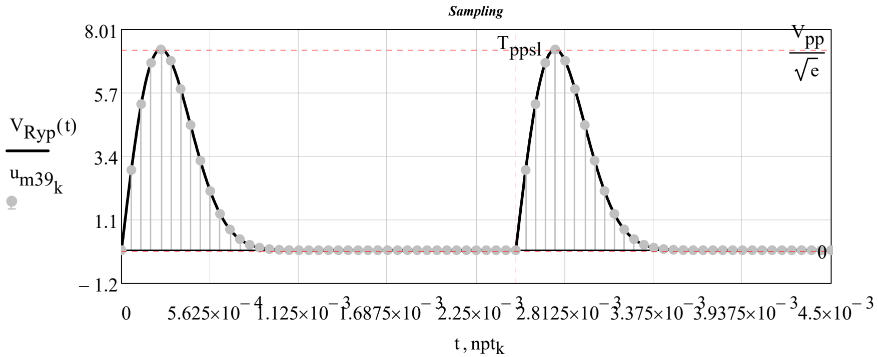
$$npt_k := \frac{k}{fpt_{so}}$$

Frequency resolution: $\frac{N0_{gd}}{fpt_{so}}\cdot\frac{1}{T_{ppsl}} = 6.4$

$$(u_{m39})_k := V_{Ryp}(npt_k)$$

$u_{m39}^{T} =$

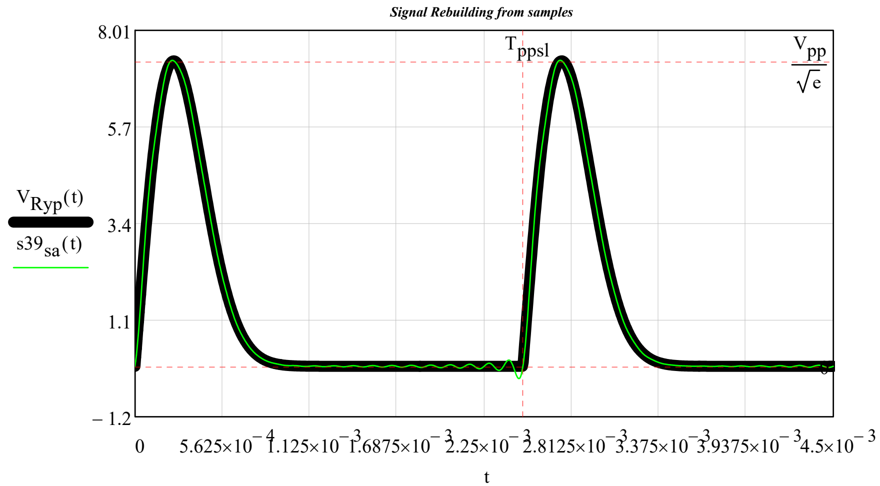
	0	1	2	3	4
0	0	2.908	5.295	6.794	...



$relerr = 10 \cdot \%$ $\qquad \omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa} \qquad \omega_{bwr} = 0.05 \cdot \frac{Mrads}{sec} \qquad n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula: $\quad s39_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1} \left(u_{m39_n} \cdot sinc\left(\omega_{bwr} \cdot t - n \cdot \pi\right)\right)\right] \qquad N0_{gd} - 1 = 255 \quad relerr = 10 \cdot \%$



TEST Waveforms

Periodic Waveforms

38 Cap. Charge and Discharge Pulse Train

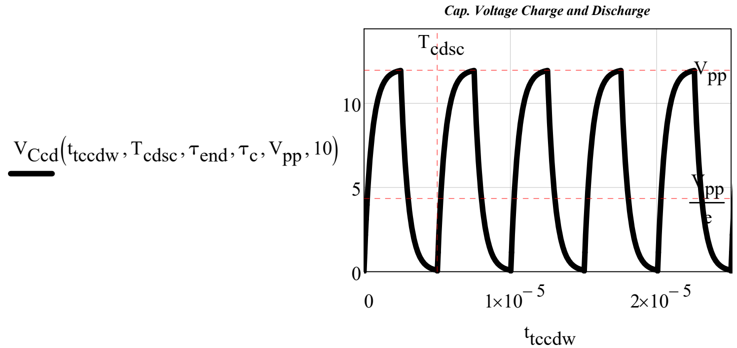
$\tau_{end} = 2.5 \cdot \mu s \qquad V_{pp} = 12\,V$

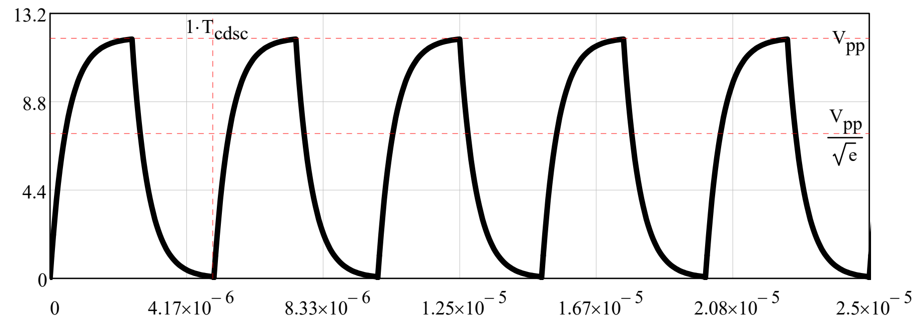
pulse width: $\quad 2 \cdot \tau_{end}$

time constant $\quad \tau_c = 0.5 \cdot \mu s$

Period: $\quad T_{cdsc} := 2 \cdot \tau_{end} \qquad \omega_{cdsc} := \frac{2 \cdot \pi}{T_{cdsc}}$

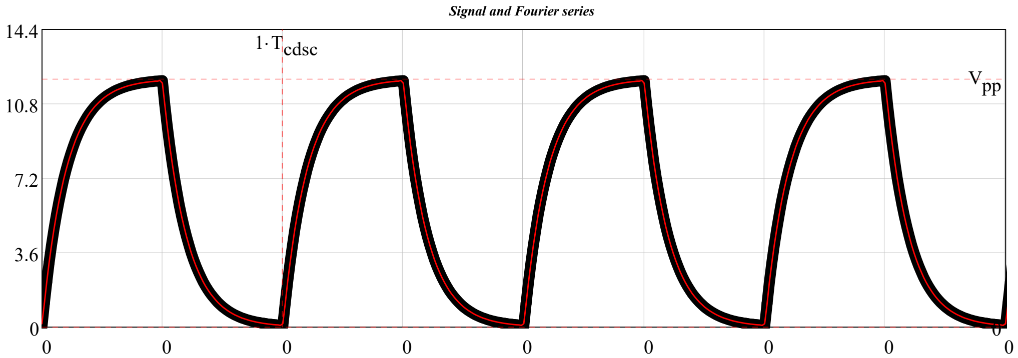
$t_{tccdw} := 0 \cdot T_{cdsc}, 0 \cdot T_{cdsc} + \frac{100 \cdot T_{cdsc}}{10000} .. 100 \cdot T_{cdsc}$



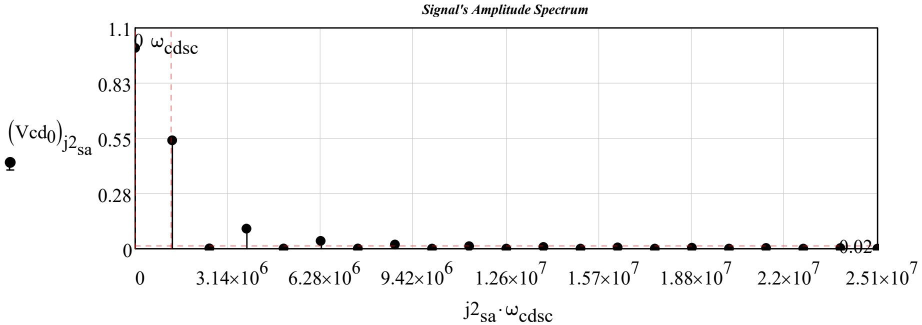


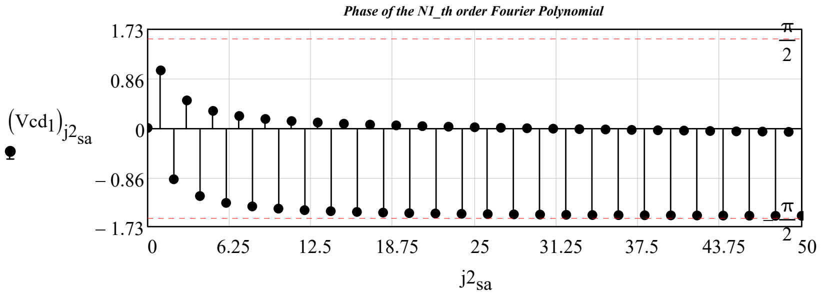
$VCcd(t) := \frac{V_{Ccd}\left(t, T_{cdsc}, \tau_{end}, \tau_c, V_{pp}, N1\_\right)}{V}$

$Vcd := SPCT\left(VCcd, rt_{gd}, N1\_, 0 \cdot s, T_{cdsc}\right) \qquad N1\_ = 50$



$$j2_{sa} := 0..\mathrm{rows}(Vcd_0) - 1 \qquad \omega_{cdsc} = 1.257\cdot\frac{Mrads}{s}$$





$$Bw_{sa} := Vcd_3\cdot Hz$$

$$Bw_{sa} = 5.2\cdot MHz$$

sampling frequency: $fpt_{so} := 2\cdot Bw_{sa}$ $\qquad fpt_{so} = 10.4\cdot MHz$

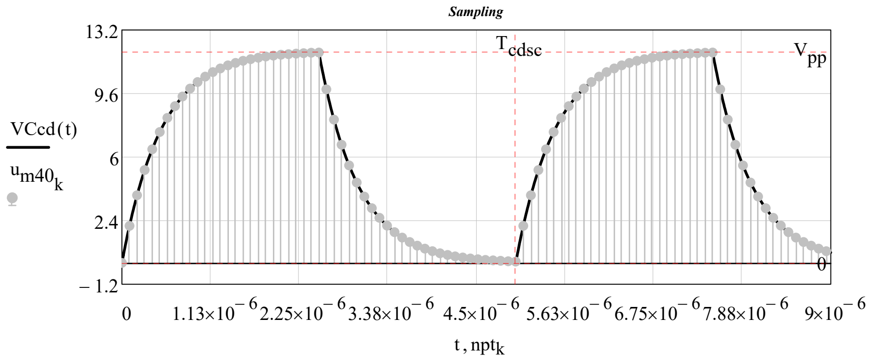
$$npt_k := \frac{k}{fpt_{so}}$$

Frequency resolution: $\frac{N0_{gd}}{fpt_{so}}\cdot\frac{1}{T_{cdsc}} = 4.923$

$$u_{m40_k} := VCcd(npt_k)$$

$u_{m40}^T =$

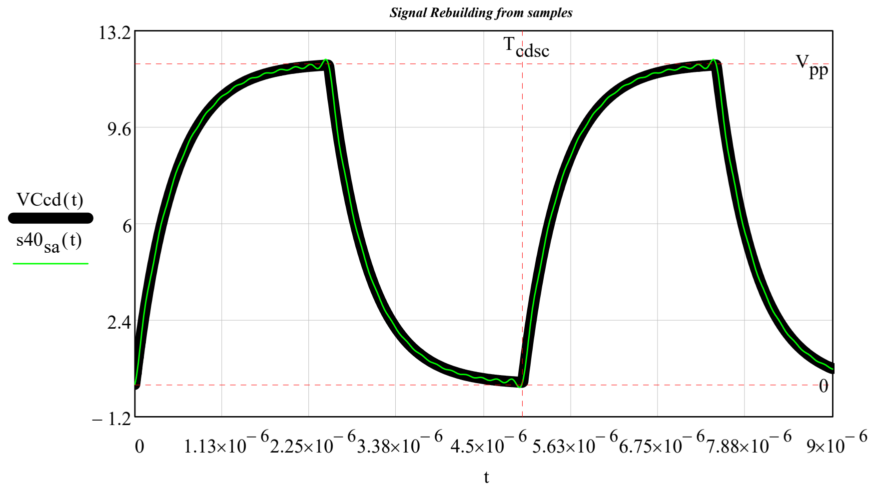
	0	1	2	3	4	5	6	7
0	0	2.099	3.831	5.261	6.44	7.412	8.215	...



relerr $= 10\cdot\%$ $\qquad \omega_{bwr} := 2\cdot\pi\cdot Bw_{sa}$ $\qquad \omega_{bwr} = 32.673\cdot\frac{Mrads}{sec}$ $\qquad n\cdot\frac{\pi}{\omega_{bwr}} = n\cdot\frac{1}{2\cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

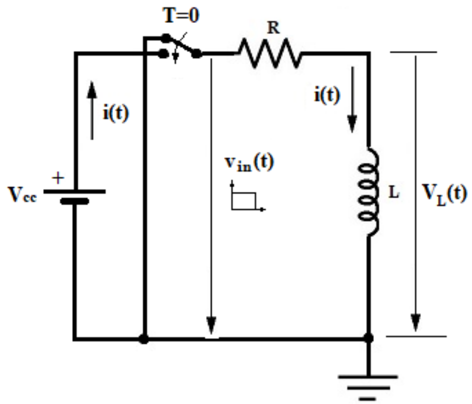
interpolation formula: $s40_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1}\left(u_{m40_n}\cdot\mathrm{sinc}(\omega_{bwr}\cdot t - n\cdot\pi)\right)\right]$ $\qquad N0_{gd} - 1 = 255$ $\qquad$ relerr =

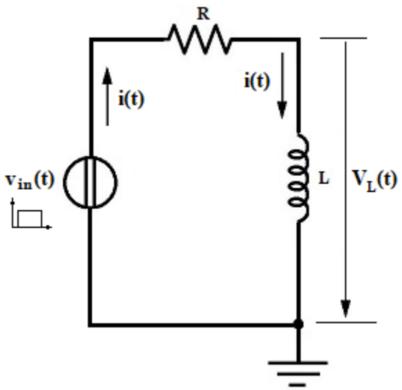


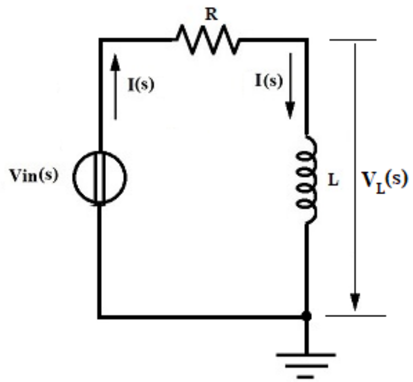
Periodic Waveforms

39 Induct Charge and Discharge Pulse Train

$R := 1 \cdot k\Omega \qquad L := 1 \cdot \mu H$







$$V_L(s) = \frac{s \cdot L}{R + s \cdot L} \cdot V_{in}(s) \qquad I_L(s) = \frac{V_L(s)}{s \cdot L}$$

$$V_{in}(s) = V_{pp} \cdot \mathcal{L}\{\Phi(t) - \Phi(t-\tau)\} = V_{pp} \cdot \left(\mathcal{L}\{\Phi(t)\} - \mathcal{L}\{\Phi(t-\tau)\}\right) = V_{pp} \cdot \left(\frac{1}{s} - \frac{1}{s} \cdot e^{-\tau \cdot s}\right)$$

$$\Phi(t) - \Phi(t-\tau) \left| \begin{array}{l} \text{assume}, \tau > 0 \\ \text{laplace} \end{array} \right. \rightarrow -\frac{e^{-\tau \cdot s} - 1}{s}$$

$$\omega_0 := \frac{R}{L} \qquad \tau_L := \frac{1}{\omega_0} \qquad \tau_L = 1\,ns \qquad \tau := \frac{5}{\omega_0} \qquad \omega_0 = 1 \frac{Grads}{s} \qquad \tau = 5\,ns$$

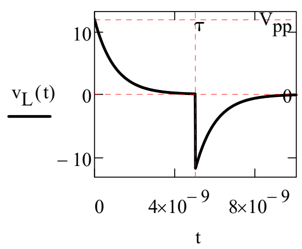
$$\omega_0 := \omega_0 \qquad \tau := \tau$$

$$\boxed{V_{in}(s) = V_{pp} \cdot \frac{1 - e^{-\tau \cdot s}}{s}}$$

$$V_L(s) = \frac{s \cdot L}{R + s \cdot L} \cdot \left(V_{pp} \cdot \frac{1 - e^{-\tau \cdot s}}{s}\right) = V_{pp} \cdot \frac{s}{\frac{R}{L} + s} \cdot \frac{1 - e^{-\tau \cdot s}}{s} = V_{pp} \cdot \frac{s}{\omega_0 + s} \cdot \frac{1 - e^{-\tau \cdot s}}{s}$$

$$\frac{s}{\omega_0 + s} \cdot \frac{1 - e^{-\tau \cdot s}}{s} \left| \begin{array}{l} \text{assume}, ALL = \text{real} \\ \text{assume}, \omega_0 > 0 \\ \text{assume}, \tau > 0 \\ \text{invlaplace}, s \\ \text{simplify} \end{array} \right. \rightarrow e^{-t \cdot \omega_0} \cdot \left(e^{\tau \cdot \omega_0} \cdot \Phi(\tau - t) - e^{\tau \cdot \omega_0} + 1\right)$$

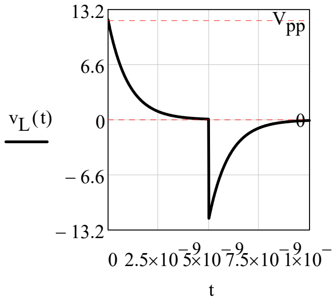
$$\boxed{v_L(t) := V_{pp} \cdot e^{\frac{-t}{\tau_L}} \cdot \left[e^{\frac{\tau}{\tau_L}} \cdot (\Phi(\tau - t) - 1) + 1\right]}$$

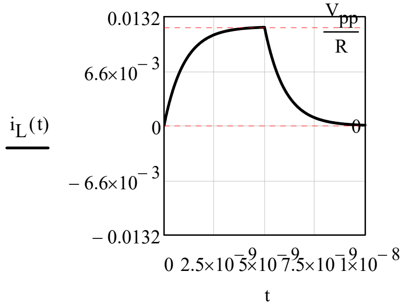


$$I_L(s) = \frac{V_L(s)}{s \cdot L} = \frac{V_{pp}}{L} \cdot \frac{1}{(\omega_0 + s)} \cdot \frac{1 - e^{-\tau \cdot s}}{s}$$

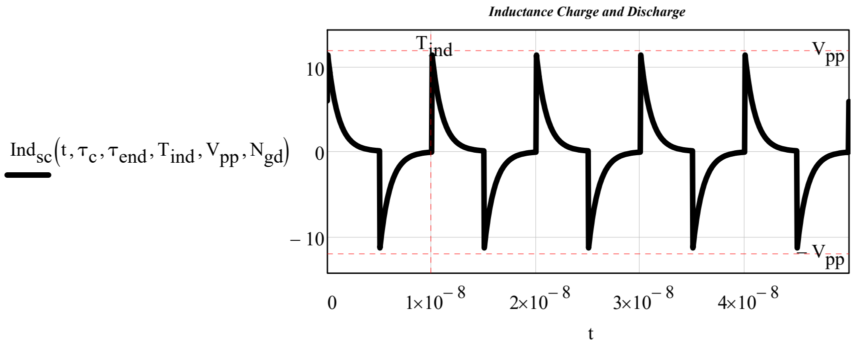
$$\frac{1}{\omega_0 + s} \cdot \frac{1 - e^{-\tau \cdot s}}{s} \left| \begin{array}{l} \text{assume}, ALL = \text{real} \\ \text{assume}, \omega_0 > 0 \\ \text{assume}, \tau > 0 \\ \text{invlaplace}, s \\ \text{simplify} \\ \text{collect}, e^{-t \cdot \omega_0} \end{array} \right. \rightarrow \left(-\frac{e^{\tau \cdot \omega_0} \cdot \Phi(\tau - t) - e^{\tau \cdot \omega_0} + 1}{\omega_0}\right) \cdot e^{-t \cdot \omega_0} + \frac{\Phi(\tau - t)}{\omega_0}$$

$$i_L(t) := \frac{V_{pp}}{R}\cdot\left[\left[1 - e^{\left(\frac{\tau - t}{\tau_L}\right)}\right]\cdot\Phi(\tau - t) + e^{\frac{-t}{\tau_L}}\cdot\left(e^{\frac{\tau}{\tau_L}} - 1\right)\right]$$



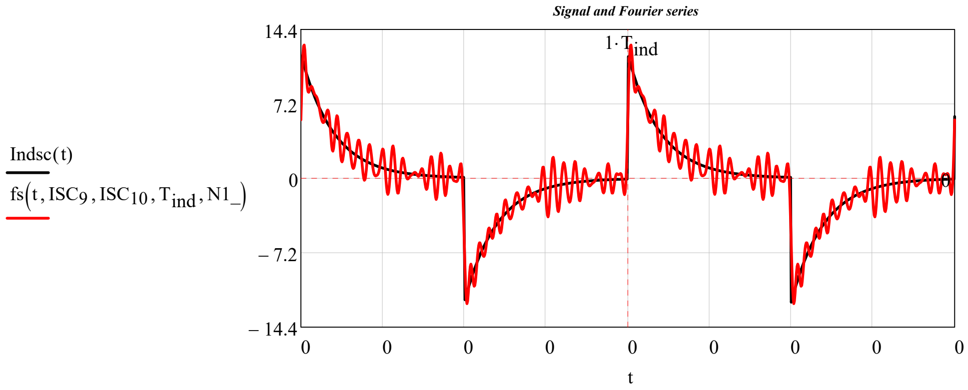


$T_{ind} := 2\cdot\tau$ $\quad \tau_{end} := \tau$ $\quad \tau_c = 1\,ns$

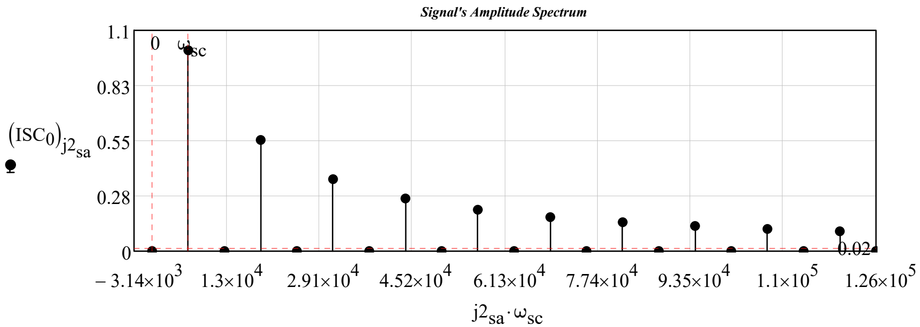


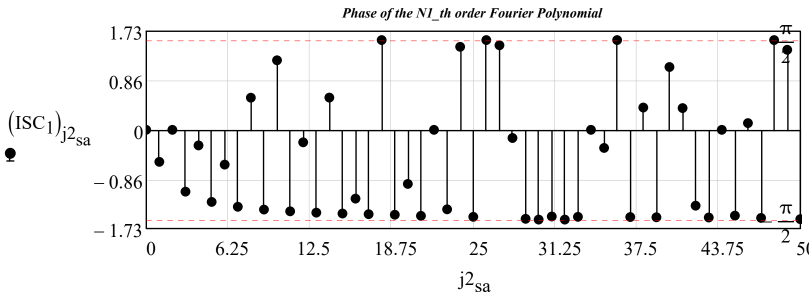
$$Indsc(t) := \frac{Ind_{sc}\left(t, \tau_c, \tau_{end}, T_{ind}, V_{pp}, N_{gd}\right)}{V}$$

$\omega_{sc} := \frac{2\cdot\pi}{T_{0gd}}$ $\quad ISC := SPCT\left(Indsc, rt_{gd}, N1\_, 0\cdot s, T_{ind}\right)$ $\quad N1\_ = 50$



$j2_{sa} := 0 .. rows\left(ISC_0\right) - 1$ $\quad \omega_{ptd\_} = 6.283\times 10^{-3}\cdot\frac{Mrads}{s}$





$Bw_{sa} := ISC_3\cdot Hz$

$Bw_{sa} = 4.8\times 10^3\cdot MHz$

sampling frequency: $\quad fpt_{so} := 2\cdot Bw_{sa}$ $\quad fpt_{so} = 9.6\times 10^3\cdot MHz$

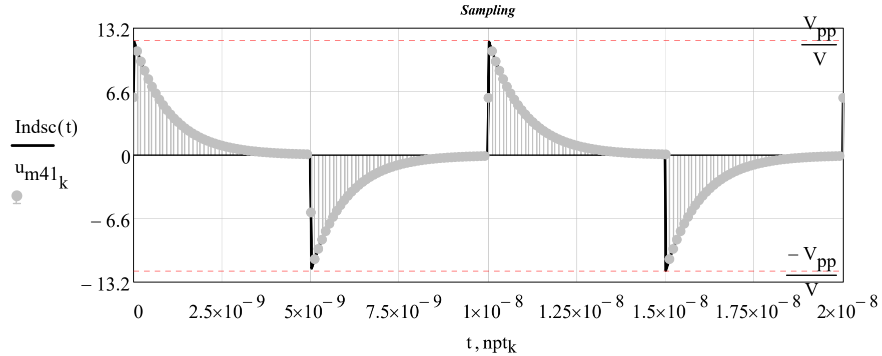
$$npt_k := \frac{k}{fpt_{so}}$$

Frequency resolution: $\quad \frac{N0_{gd}}{fpt_{so}}\cdot\frac{1}{T_{ind}} = 2.667$

$$u_{m41_k} := \mathrm{Indsc}(npt_k)$$

$u_{m41}^{T} =$

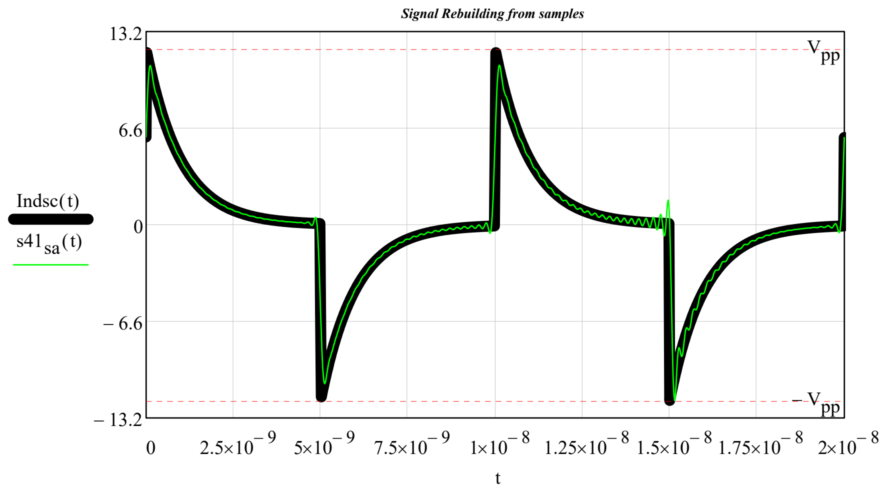
	0	1	2	3	4	5	6
0	6	10.813	9.743	8.779	7.911	7.128	...



$relerr = 10\cdot\%$ $\quad \omega_{bwr} := 2\cdot\pi\cdot Bw_{sa}$ $\quad \omega_{bwr} = 3.016\times10^{4}\cdot\frac{Mrads}{sec}$ $\quad n\cdot\frac{\pi}{\omega_{bwr}} = n\cdot\frac{1}{2\cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula: $\quad s41_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1}\left(u_{m41_n}\cdot \mathrm{sinc}(\omega_{bwr}\cdot t - n\cdot\pi)\right)\right]$ $\quad N0_{gd} - 1 = 255$ $\quad relerr = 10\cdot\%$



Periodic Waveforms

40 Parabolic Cusps Pulse Train

Signal amplitude: $\quad V_{pp} = 12\cdot V$

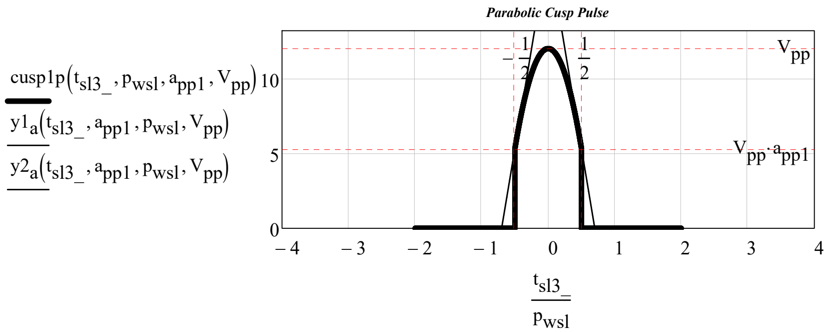
Pulse width: $\quad p_{wsl} = 250\cdot\mu s$

Duty cycle: $\quad \delta_{cysl} := \gamma$

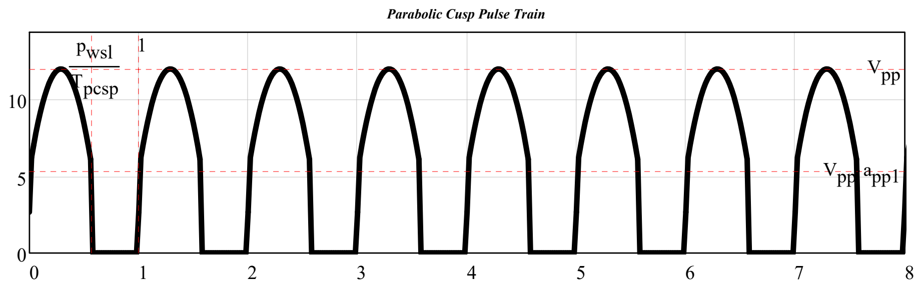
Period: $\quad T_{pcsp} := \frac{p_{wsl}}{\delta_{cysl}} \quad \omega_{pcsp} := \frac{2\cdot\pi}{T_{pcsp}}$

Max pulse amplitude and cusp ratio: $\quad a_{pp1} := \frac{4}{9}$

$$t_{sl3\_} := -2\cdot p_{wsl}, -2\cdot p_{wsl} + \frac{(2\cdot p_{wsl} + 2\cdot p_{wsl})}{10000} .. 2\cdot p_{wsl}$$



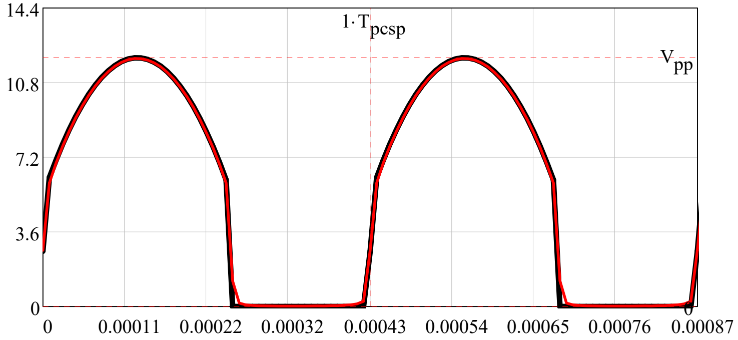
$$t_{t11w} := 0\cdot T_{pcsp}, 0\cdot T_{pcsp} + \frac{10\cdot T_{pcsp} - 0\cdot T_{pcsp}}{500} .. 10\cdot T_{pcsp}$$



$$Csp11(t) := \frac{csp11(t, p_{wsl}, a_{pp1}, T_{pcsp}, V_{pp}, N0_{gd})}{V}$$

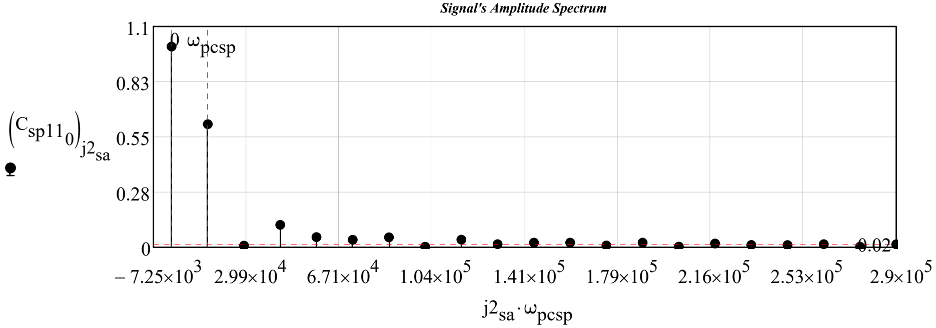
$C_{sp11} := \mathrm{SPCT}(Csp11, rt_{gd}, N1\_, 0\cdot s, T_{pcsp}) \qquad N1\_ = 50$

Signal and Fourier series

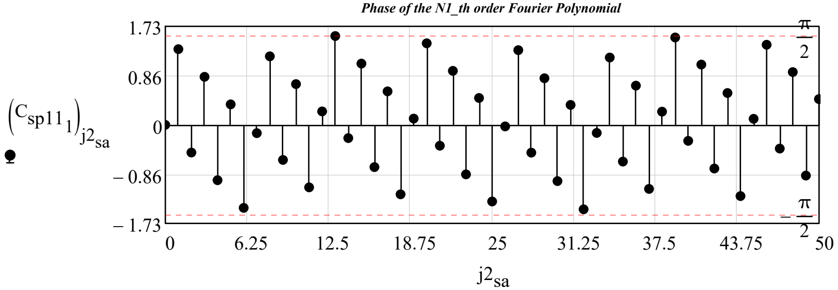


$j2_{sa} := 0 .. \mathrm{rows}(C_{sp11_0}) - 1 \qquad \omega_{ptd\_} = 6.283 \times 10^{-3} \cdot \frac{Mrads}{s}$

Signal's Amplitude Spectrum



Phase of the N1_th order Fourier Polynomial



$Bw_{sa} := C_{sp11_3} \cdot Hz$

$Bw_{sa} = 0.111 \cdot MHz$

sampling frequency: $fpt_{so} := 2 \cdot Bw_{sa} \qquad fpt_{so} = 0.222 \cdot MHz$

$npt_k := \frac{k}{fpt_{so}}$

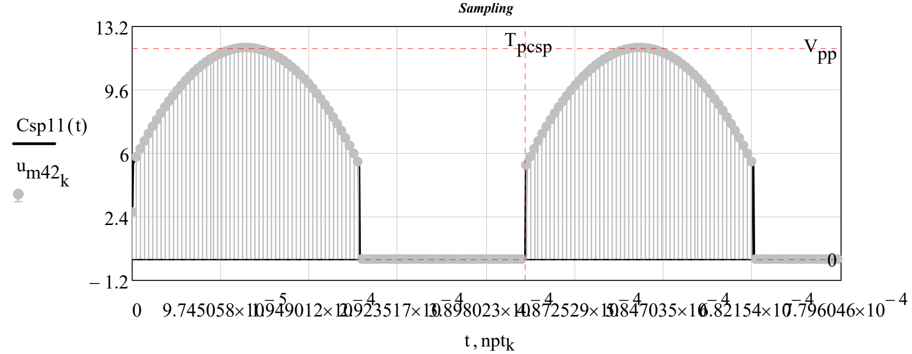
Frequency resolution: $\frac{N0_{gd}}{fpt_{so}} \cdot \frac{1}{T_{pcsp}} = 2.667$

$u_{m42_k} := Csp11(npt_k)$

$u_{m42}^T =$

	0	1	2	3	4	5	6	7
0	2.667	5.806	6.261	6.699	7.119	7.522	7.908	...

Sampling

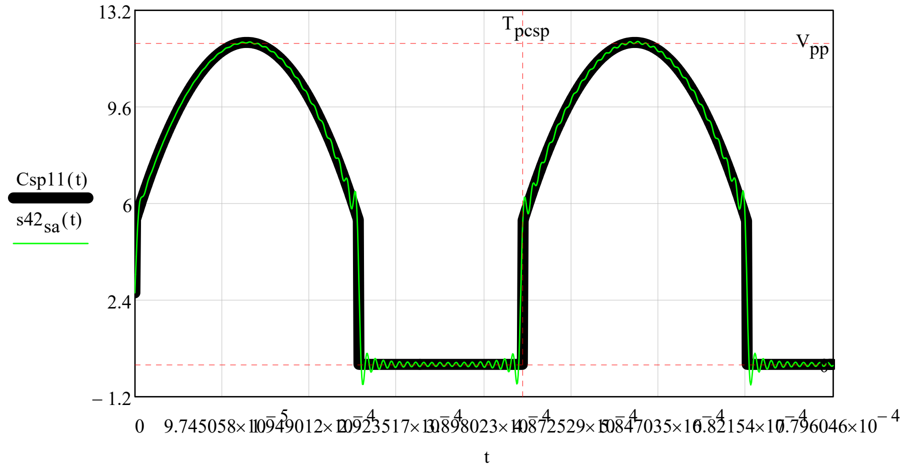


relerr = 10·% $\qquad \omega_{bwr} := 2 \cdot \pi \cdot Bw_{sa} \qquad \omega_{bwr} = 0.696 \cdot \frac{Mrads}{sec} \qquad n \cdot \frac{\pi}{\omega_{bwr}} = n \cdot \frac{1}{2 \cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula: $s42_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1} \left(u_{m42_n} \cdot \mathrm{sinc}(\omega_{bwr} \cdot t - n \cdot \pi) \right) \right] \qquad N0_{gd} - 1 = 255$

Signal Rebuilding from samples



Periodic Waveforms

41 Elliptic Cusps Pulse Train

Signal amplitude: $V_{pp} = 12 \cdot V$

Pulse width: $p_{wsl} = 250 \cdot \mu s$

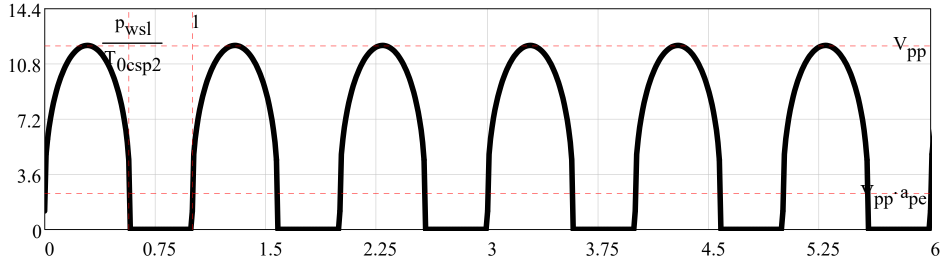
Duty cycle: $\delta_{cysl} := \gamma$

Period: $T_{0csp2} := \dfrac{p_{wsl}}{\delta_{cysl}}$

Max pulse amplitude and cusp ratio: $a_{pe} := \dfrac{2}{10}$

$$t_{22sl} := 0 \cdot T_{0csp2}, 0 \cdot T_{0csp2} + \frac{10 \cdot T_{0csp2} - 0 \cdot T_{0csp2}}{1000} .. 10 \cdot T_{0csp2}$$

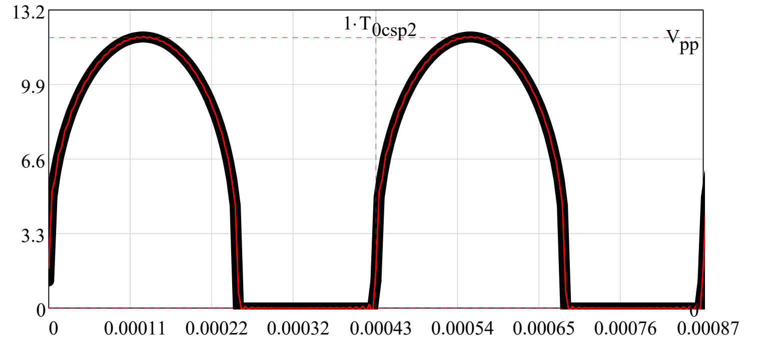
Elliptic Cusp Pulses Train



$$Csp22(t) := \frac{csp22\left(t, p_{wsl}, a_{pe}, T_{0csp2}, V_{pp}, N0_{gd}\right)}{V}$$

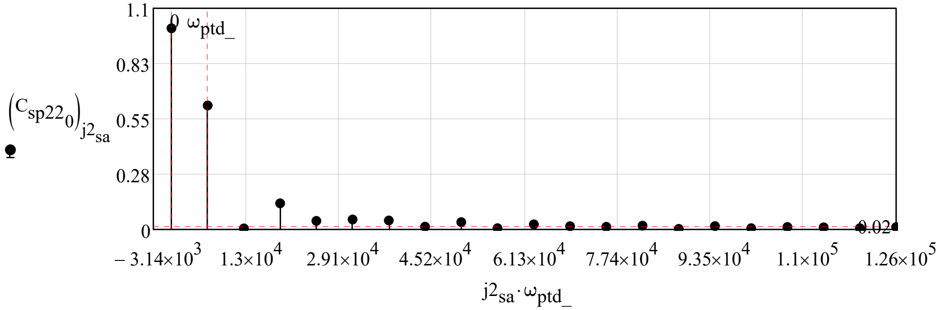
$$C_{sp22} := SPCT\left(Csp22, rt_{gd}, N1\_, 0 \cdot s, T_{0csp2}\right) \qquad N1\_ = 50$$

Signal and Fourier series

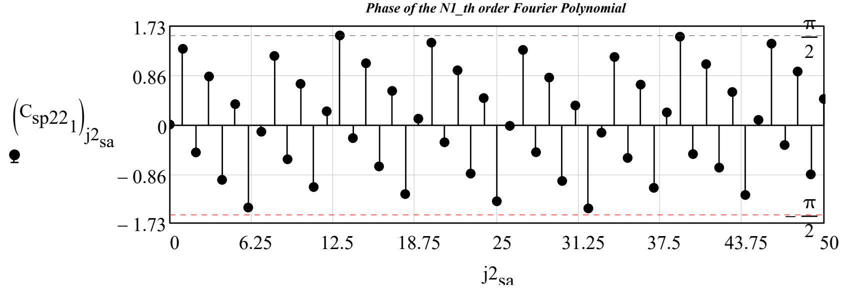


$$j2_{sa} := 0 .. rows\left(C_{sp22_0}\right) - 1 \qquad \omega_{ptd\_} = 6.283 \times 10^{-3} \cdot \frac{Mrads}{s}$$

Signal's Amplitude Spectrum



Phase of the N1_th order Fourier Polynomial



$Bw_{sa} := C_{sp22_3} \cdot Hz$

$Bw_{sa} = 0.111 \cdot MHz$

sampling frequency: $fpt_{so} := 2 \cdot Bw_{sa}$ $\qquad fpt_{so} = 0.222 \cdot MHz$

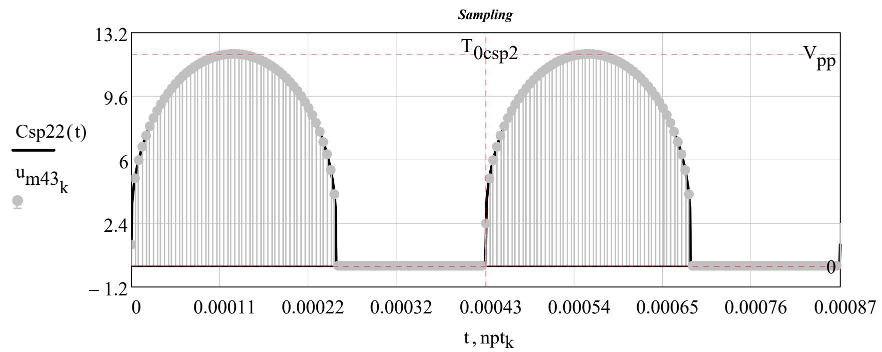
$npt_k := \dfrac{k}{fpt_{so}}$

Frequency resolution: $\dfrac{N0_{gd}}{fpt_{so}} \cdot \dfrac{1}{T_{0csp2}} = 2.667$

$$u_{m43_k} := Csp22(npt_k)$$

$u_{m43}^T =$

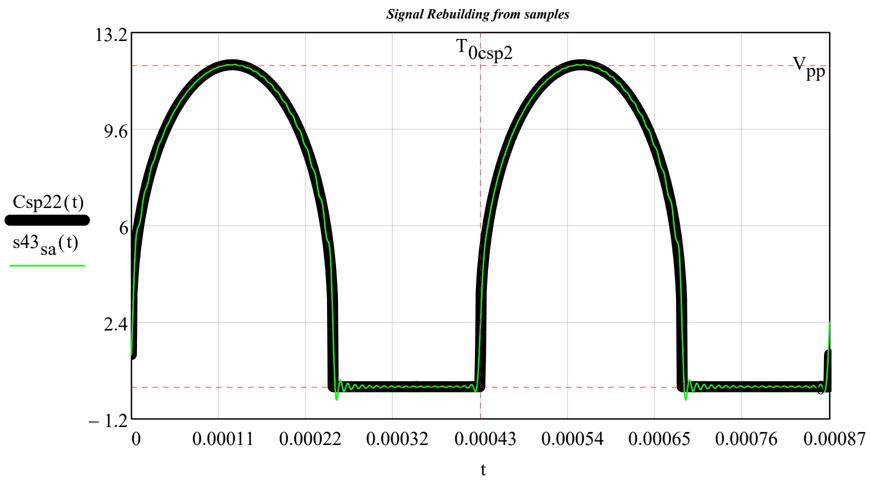
	0	1	2	3	4	5	6	7
0	1.2	4.956	5.981	6.745	7.369	7.901	8.366	...



$relerr = 10\cdot\%$ $\qquad \omega_{bwr} := 2\cdot\pi\cdot Bw_{sa} \qquad \omega_{bwr} = 0.696\cdot\frac{Mrads}{sec} \qquad n\cdot\frac{\pi}{\omega_{bwr}} = n\cdot\frac{1}{2\cdot Bw_{sa}}$

Signal reconstruction according to the Shannon sampling theorem:

interpolation formula: $$s43_{sa}(t) := \left[\sum_{n=0}^{N0_{gd}-1}\left(u_{m43_n}\cdot sinc(\omega_{bwr}\cdot t - n\cdot\pi)\right)\right] \qquad N0_{gd} - 1 = 255 \qquad relerr = 10\cdot\%$$



$t1\_ := 0 \qquad \tau_{end\_} := time(t1\_)$

$$\tau_{end\_} = 1.618\times 10^{9}$$

$$\frac{\tau_{end\_} - \tau_{init\_}}{3600} = 0.654$$

Fine