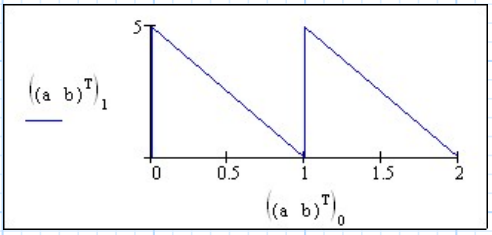
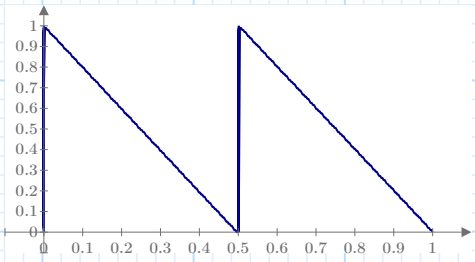
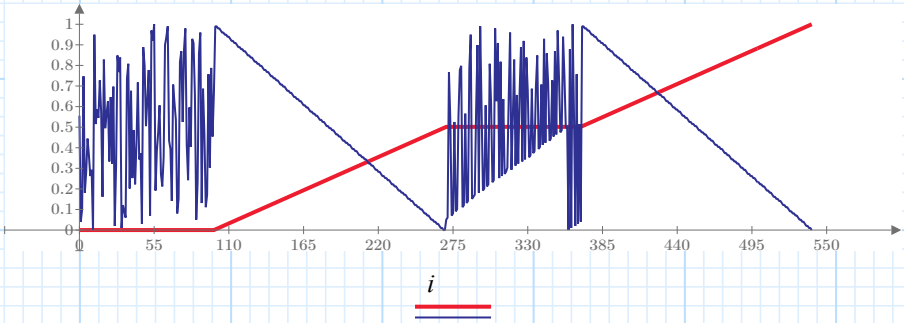
$$XY := \begin{bmatrix} 0 & 0.55556 \\ 0 & 0.0404 \\ 0 & 0.10101 \\ 0 & 0.74747 \\ 0 & 0.18182 \\ 0 & 0.28283 \\ \vdots & \end{bmatrix} \qquad i := 0 \,..\, 539$$





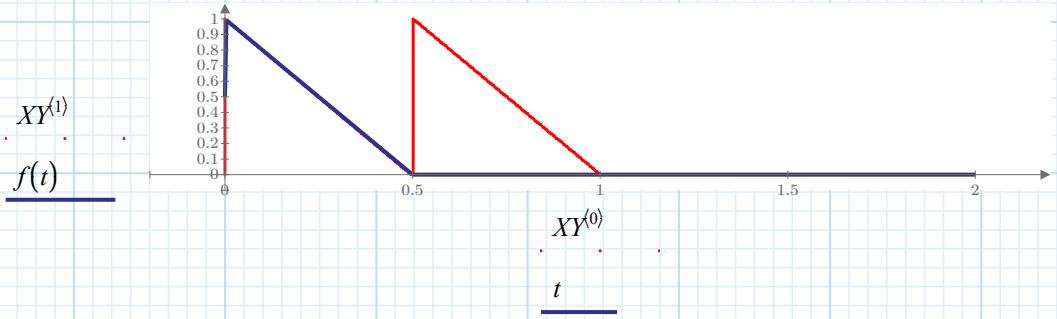


Fourier or else ... what else ?

To answer the question: is it possible to get differently valued points at the same ordinates ?
Answer = YES , simply from Mathcad [blue graph]
The blue graph contains more than needed but for the demo it does not matter. The blue graph is digitized, not too badly reproduced. We extract the spectral and try to recover the function. Is the spectral valid ? Answer = Don't know
Then can't recover the function ? Answer = can't

1. Make one cycle of above data.

$T := \frac{1}{2}$ $f(t) := (1 - 2 \cdot t) \cdot (\Phi(t) - \Phi(t - T))$



2. Digitize the data with 256 points for one cycle data.

$i := 0 .. 255$ $f256_i := f\left(\frac{i}{256} \cdot T\right)$ $f256 = \begin{bmatrix} 0.5 \\ 1 \\ 1 \\ 1 \\ 1 \\ \vdots \end{bmatrix}$

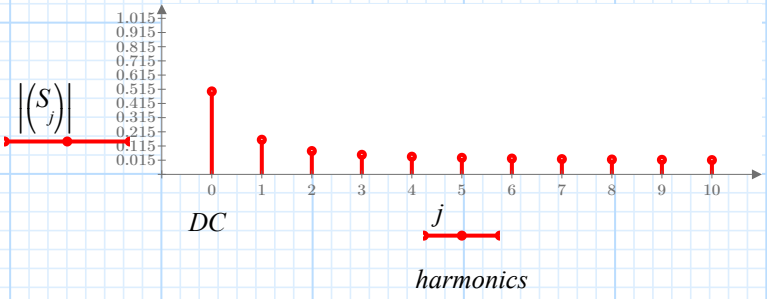
3. FFT the data with 256 points.

$\omega := T = \frac{2 \cdot \pi}{\omega} \xrightarrow{solve, \omega} 4 \cdot \pi$

$S := \mathrm{FFT}(f256)$ $S = \begin{bmatrix} 0.5 \\ -0.159i \\ -0.08i \\ -0.053i \\ -0.04i \\ -0.032i \\ \vdots \end{bmatrix}$

DC	
fundamental_wave	ω
Second harmonic wave	$2 \cdot \omega$
	$3 \cdot \omega$

$j := 0 .. 10$



4. Make data from DC to n harmonic spectrums.

clear (t)

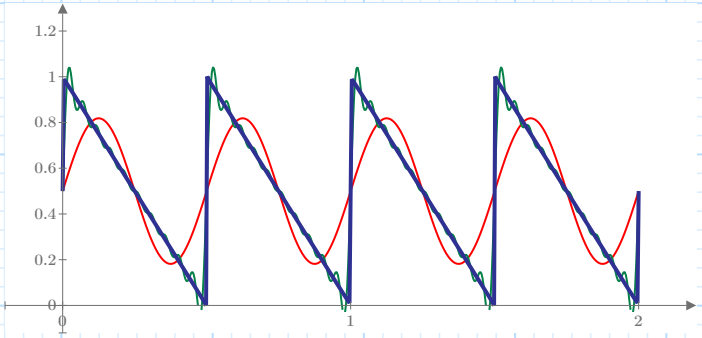
$S_{cos} := \mathrm{Re}(S)$ $S_{sin} := \mathrm{Im}(S)$

$$f_{FFT}(n,t) := S_{cos_0} + \sum_{i=1}^{n} \left(-2 \cdot S_{sin_i} \cdot \sin(i \cdot \omega \cdot t)\right)$$

$f_{FFT}(1,t)$

$f_{FFT}(10,t)$

$f_{FFT}(127,t)$



t

5. IFFT to make f(t) data from DC to n_th spectrum.

$T = 0.5$ $i := 0 .. 255$ $n := 10$

$k := n + 1 .. 128$ $S_k := 0$

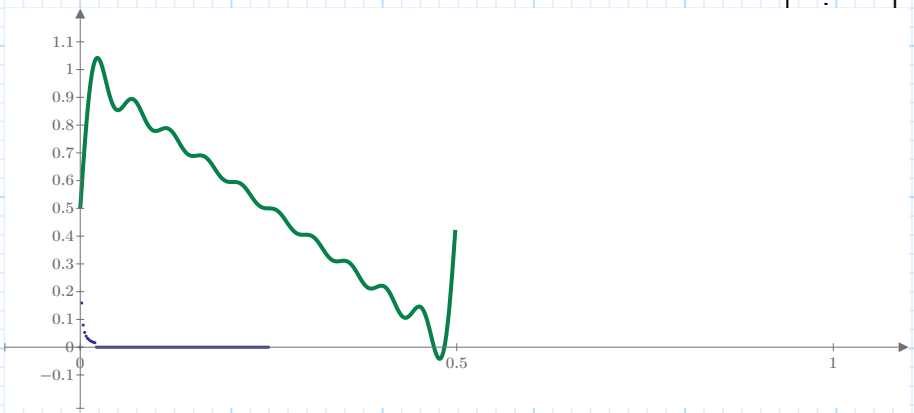
$f := \mathrm{IFFT}(S)$

$$f = \begin{bmatrix} 0.5 \\ 0.6 \\ 0.7 \\ 0.7 \\ \vdots \end{bmatrix}$$

$$S = \begin{bmatrix} 0.5 \\ -0.159\mathrm{i} \\ -0.08\mathrm{i} \\ -0.053\mathrm{i} \\ -0.04\mathrm{i} \\ -0.032\mathrm{i} \\ -0.026\mathrm{i} \\ -0.023\mathrm{i} \\ -0.02\mathrm{i} \\ -0.018\mathrm{i} \\ -0.016\mathrm{i} \\ 0 \\ 0 \\ \vdots \end{bmatrix}$$

$-\mathrm{Im}(S_i)$

f_i



$\frac{i}{256} \cdot T$

6. Digitize the data with 1000 (not 2^n) points for one cycle data.

$T := \frac{1}{2}$ $\quad f(t) := (1 - 2 \cdot t) \cdot (\Phi(t) - \Phi(t - T))$ $\quad$ **clear** (ω)

$i := 0 .. 999$ $\quad f_{1000_i} := f\left(\frac{i}{1000} \cdot T\right)$

7. dft the data with 1000 points.

$s := \text{dft}(f_{1000})$ $\quad s = \begin{bmatrix} 500 \\ -159.154\text{i} \\ -79.576\text{i} \\ -53.05\text{i} \\ -39.787\text{i} \\ -31.828\text{i} \\ \vdots \end{bmatrix}$

$j := 0 .. 20$

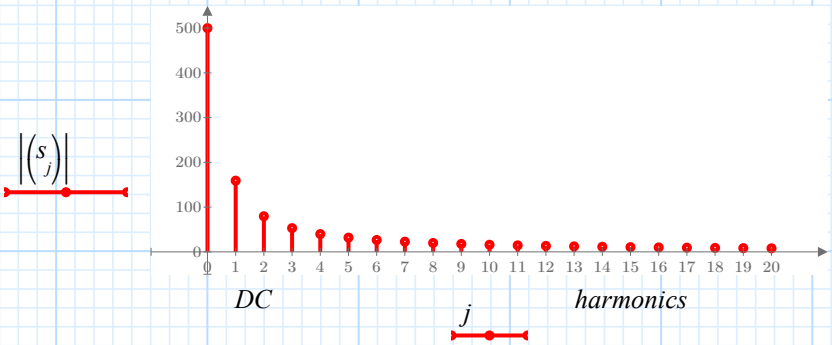
DC

fundamental_wave

Second harmonic wave $2 \cdot \omega$

$3 \cdot \omega$

$\omega := T = \frac{2 \cdot \pi}{\omega}$ *solve*, $\omega \rightarrow 4 \cdot \pi$



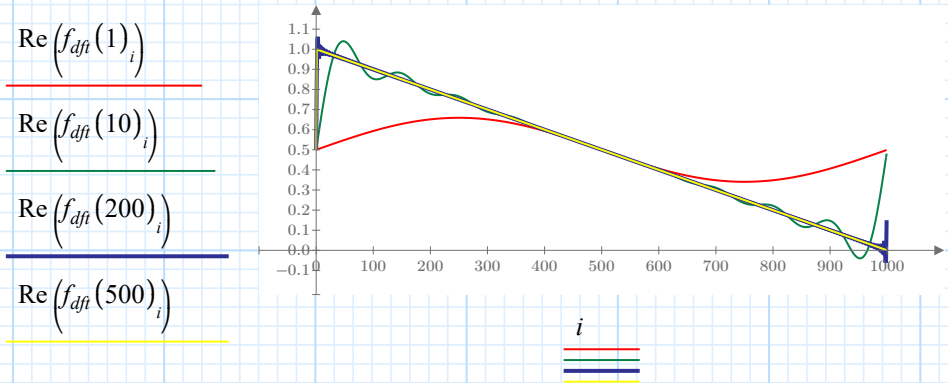
8. Make data from DC to n_th harmonic spectrums.

clear (t, n)

$f_{dft}(n) := \left\| \begin{array}{l} s1000 \leftarrow s \\ \text{for } k \in n+1 .. 1000-n \\ \quad s1000_k \leftarrow 0 \\ Ans \leftarrow \text{idft}(s1000) \end{array} \right\|$

$s = \begin{bmatrix} 500 \\ -1.4 \cdot 10^{-14} - 159.2\text{i} \\ \vdots \end{bmatrix}$

$i := 0 .. 999$



9 Saw wave by Fourier series.

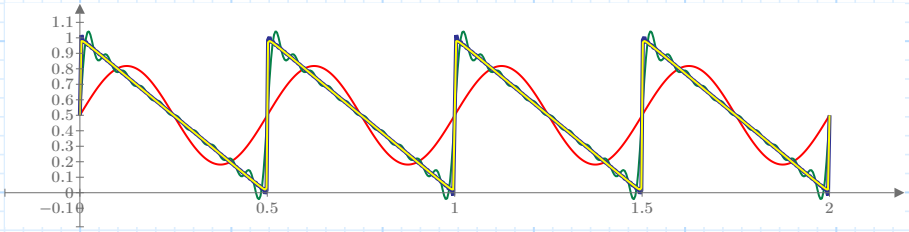
$$f(n,t) := \sum_{k=1}^{n} \left(\frac{1}{\pi} \cdot \frac{(-1)^k}{k} \cdot \sin\left(k \cdot (\omega \cdot t + \pi)\right) \right) + \frac{1}{2}$$

$f(1,t)$

$f(10,t)$

$f(200,t)$

$f(500,t)$



t