$$EI_\omega \varphi'''' - GI_t \varphi'' = m = -\frac{dM}{dx}$$

where

φ = Angle of twist
m = Distributed torsion moment per unit length
E = Young's modulus
G = Shear modulus
It = Torsion constant
Iw = Warping constant

General solution to differential equation of twist angle:

$$\varphi(x) = C_1 + C_2 x + C_3 \sinh(kx) + C_4 \cosh(kx) + \varphi_0(x)$$

Use the general solution above to acquire following properties:

$$M_t = GI_t \varphi' = GI_t [C_2 + C_3 k \cosh(kx) + C_4 k \sinh(kx) + \varphi_0']$$

$$B = -EI_\omega \varphi'' = -GI_t \left[C_3 \sinh(kx) + C_4 \cosh(kx) + \frac{1}{k^2} \varphi_0'' \right]$$

$$M_\omega = B' = -EI_\omega \varphi''' = -GI_t \left[C_3 k \cosh(kx) + C_4 k \sinh(kx) + \frac{1}{k^2} \varphi_0''' \right]$$

where

Mt = St. Venants torsion
B = Bimoment
Mw = Warping torsion
φ_0 = Particular solution (from distributed torsion moment)
C = Integration constants

Total torsion:

$$M_x = M_t + M_\omega = GI_t\left[C_2 + \varphi_0' - \frac{1}{k^2}\varphi_0'''\right]$$

Common boundary conditions:

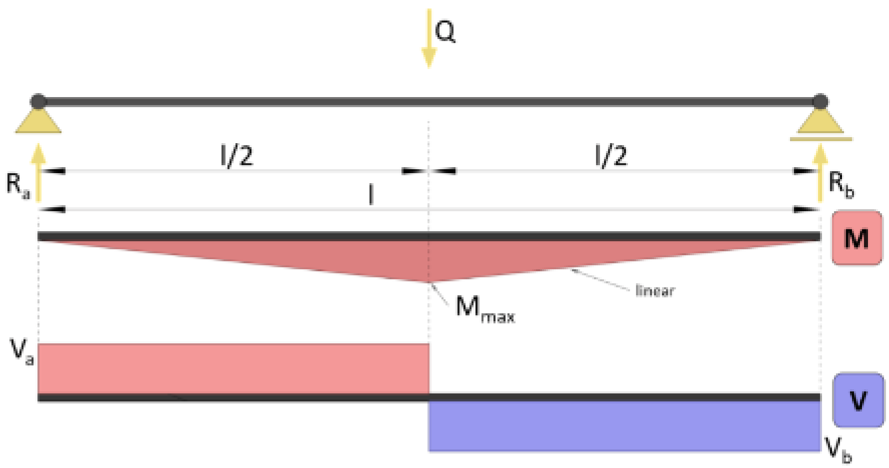
fixed end	$\phi = 0$	no twist	$\phi' = 0$	no slope	
pinned end	$\phi = 0$	no twist	$Bi = 0$	free warping	
free end	$Bi = 0$	free warping	$\phi''' = 0$	no warping shear	
continuous supports	$\phi = 0$	no twist	$\phi_{l'} = \phi_{r'}$	$Bi_l = Bi_r$	continuous
transition point within span	$\phi_l = \phi_r$		$\phi_{l'} = \phi_{r'}$	$Bi_l = Bi_r$	continuous
general	$\phi'' = 0$	free end from bending			

Usefull information:

- Distributed torsion moment is the negative derivative of the bending moment with respect to longitudinal coordinate. (example: next picture)
  You can find functions of bending moment for simply supported beams online

- You can find value of the constants such as It and Iw from Dlubal.com (cross-sectional properties) or from eurocodeapplied.com

4. Simply supported beam – Point load at midspan (formulas)



Bending moment and shear force diagram | Simply supported beam with point load at midspan.

Bending moment

$M(x) = 1/2 \cdot Q \cdot x$ if x < l/2

$M(x) = 1/2 \cdot Q \cdot (l - x)$ if x > l/2

Max bending moment

$M_{max} = 1/4 \cdot Q \cdot l$

Shear forces at supports

$V_a = 1/2 \cdot Q$

$V_b = -1/2 \cdot Q$

Reaction forces

$R_a = 1/2 \cdot Q$

$R_b = 1/2 \cdot Q$