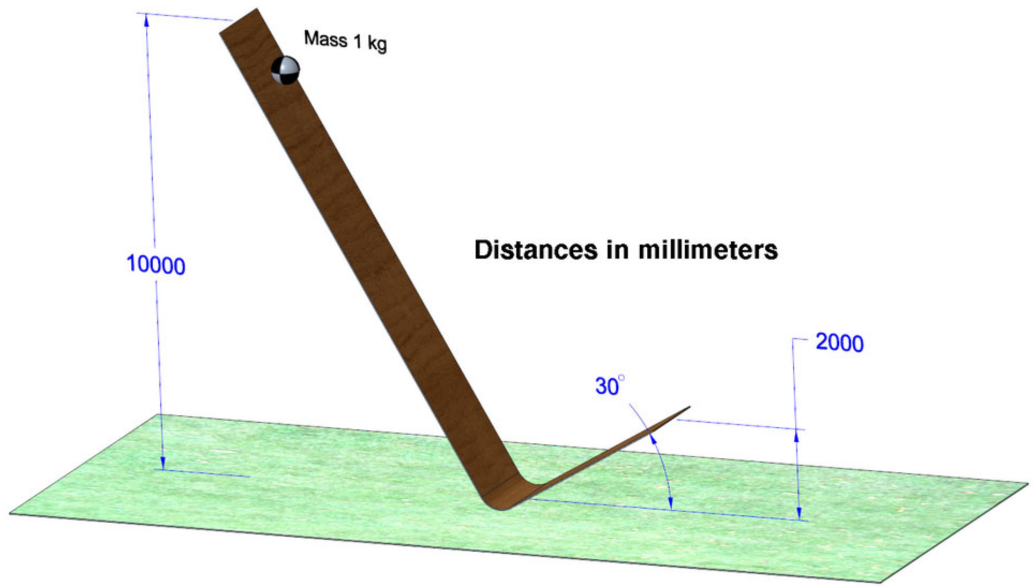
Mathcad May Challenge

Create a function that calculates the horizontal distance as a function of initial height, launch height, and launch angle.
Calculate the horizontal distance the ball will land from the end of the ramp.
Solve for the angle that will optimize the horizontal distance.

How will the horizontal distance change if this were performed on the Moon instead of on the Earth's surface? Assume the acceleration due to gravity on the Moon's surface is 1/6 that of Earth.
Use the Chart Component to depict how the horizontal landing distance changes as a function of angle.
Use a 3D Plot to show how the horizontal landing distance changes as a function



Input Values

$m_b := 1\ kg$

$h_1 := 10\ m$

$h_2 := 2\ m$

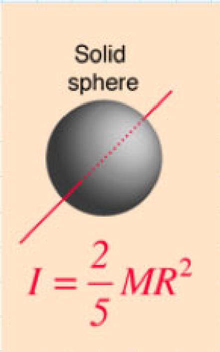
$\theta := 30\ deg$

$W_{pot}(h, m) := m \cdot g \cdot h$ Potential Energy $W_{pot}(h_1, m_b) = 98.067\ J$

Where A is 1 for a hoop, 1/2 for a cylinder or disk, 2/3 for a hollow sphere and 2/5 for a solid sphere. We have a ball so A = 2/5. From Wikipedia a Ball is a SOLID SPHERE !! So a soccer ball is apparently solid.

$$KE = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

From gsu.edu



$$\omega = \frac{v}{r}$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left[\frac{2}{5}mr^2\right]\left[\frac{v}{r}\right]^2$$

By combining the aforementioned information, rearranging the variables, and utilizing the drop and launch heights along with the launch angle, we can determine the horizontal distance covered by the ball, as demonstrated below. The process involves finding the velocity at the bottom of the ramp, subtracting the gain in potential energy as the ball ascends the second incline, calculating the new velocity immediately prior to launch, determining the X and Y components of this velocity, computing the time it takes for the object to reach a surface 2 meters below the launch height, and finally multiplying this time by the horizontal component of the velocity. All of these necessary calculations are encompassed within the following equations.
But if you want more read : https://www.whitman.edu/Documents/Academics/Mathematics/2016/Henelsmith.pdf

$A := \frac{2}{5}$

If Rotational KE is to be ignored then set A =0

Earth

Moon

Moment of Inertia

Velocity at Bottom of Incline

$$v_{earth} := \sqrt{\frac{2 \cdot g \cdot h_1}{1+A}} = 11.836 \ \frac{m}{s}$$

$$v_{moon} := \sqrt{\frac{2 \cdot \left(\frac{g}{6}\right) \cdot h_1}{1+A}} = 4.832 \ \frac{m}{s}$$

Velocity at launch

$$v_{l\_earth} := \sqrt{\frac{2 \cdot g \cdot (h_1 - h_2)}{1+A}} = 10.587 \ \frac{m}{s}$$

$$v_{l\_moon} := \sqrt{\frac{2 \cdot \left(\frac{g}{6}\right) \cdot (h_1 - h_2)}{1+A}} = 4.322 \ \frac{m}{s}$$

$$S_E(t) := \frac{1}{2} \cdot g \cdot t^2 - v_{l\_earth} \cdot \sin(\theta) \cdot t - h_2$$

Basic kinematic equation for Earth and Moon

$$S_M(t) := \frac{1}{2} \cdot \left(g \cdot 6^{-1}\right) \cdot t^2 - v_{l\_moon} \cdot \sin(\theta) \cdot t - h_2$$

Use Prime's Polyroot function to find the time to reach the surface

$$D_{horiz}\left(h_1, h_2, \theta\right) := v_{l\_earth} \cdot \cos(\theta) \cdot \text{polyroots}\left(\begin{bmatrix} -h_2 \\ -v_{l\_earth} \cdot \sin(\theta) \\ \frac{g}{2} \end{bmatrix}\right)^{\langle 1 \rangle}$$

$$D_{horiz}\left(h_1, h_2, \theta\right) = [12.615]\ m$$

$$\text{polyroots}\left(\begin{bmatrix} -h_2 \\ -v_{l\_earth} \cdot \sin(\theta) \\ \frac{g}{2} \end{bmatrix}\right)^{\langle 1 \rangle} = [1.376]\ s$$

or

$$\textbf{root}\left(S_E(t), t, 0\ s, 10\ s\right) = 1.376\ s$$

Or use the ubiquitous

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Select the correct sign and use the simplify function

$$\left(\frac{-v_{l\_earth} \cdot \sin(\theta) - \sqrt{\left(\left(-v_{l\_earth} \cdot \sin(\theta)\right)^2 + 4 \cdot \left(\frac{g}{2}\right) \cdot h_2\right)}}{-2 \cdot \left(\frac{g}{2}\right)}\right) = 1.376\ s$$

Or We can solve for time using

$$v_f^2 = v_i^2 + 2*a*d$$

$$d = \frac{v_i + v_f}{2} * t$$

$v_{l\_earth} = 10.587\ \frac{m}{s}$ Launch Velocity on Earth

$v_y := -v_{l\_earth} \cdot \sin(\theta) = -5.293\ \frac{m}{s}$ Vertical Component of Velocity

$v_x := v_{l\_earth} \cdot \cos(\theta) = 9.168\ \frac{m}{s}$ Horizontal Component of velocity

$v_{impact\_y} := \sqrt{{v_y}^2 + 2 \cdot g \cdot h_2} = 8.2\ \frac{m}{s}$ Vertical Impact Velocity

$\frac{h_2 \cdot 2}{(v_{impact\_y} + v_y)} = 1.376\ s$ Time of Travel

$\frac{h_2 \cdot 2}{(v_{impact\_y} + v_y)} \cdot v_x = 12.615\ m$ Horizontal Reach

$$Distance_{x\_earth}(h_1, h_2, \theta) := \left(\frac{-v_{l\_earth} \cdot \sin(\theta) - \sqrt{\left(\left(-v_{l\_earth} \cdot \sin(\theta)\right)^2 + 4 \cdot \left(\frac{g}{2}\right) \cdot h_2 \right)}}{-2 \cdot \left(\frac{g}{2}\right)} \right) \cdot v_{l\_earth} \cdot \cos(\theta)$$

$$Horizontal_{reach}(\theta) := Distance_{x\_earth}(h_1, h_2, \theta)$$

$$Distance_{x\_earth}(h_1, h_2, \theta) = 12.615\ m$$

Optimal Launch Angle

Optimal launch angle using max function

$$Ang_{opt} := \mathbf{maximize}\left(Horizontal_{reach}, \theta\right) = 40.717\ deg$$

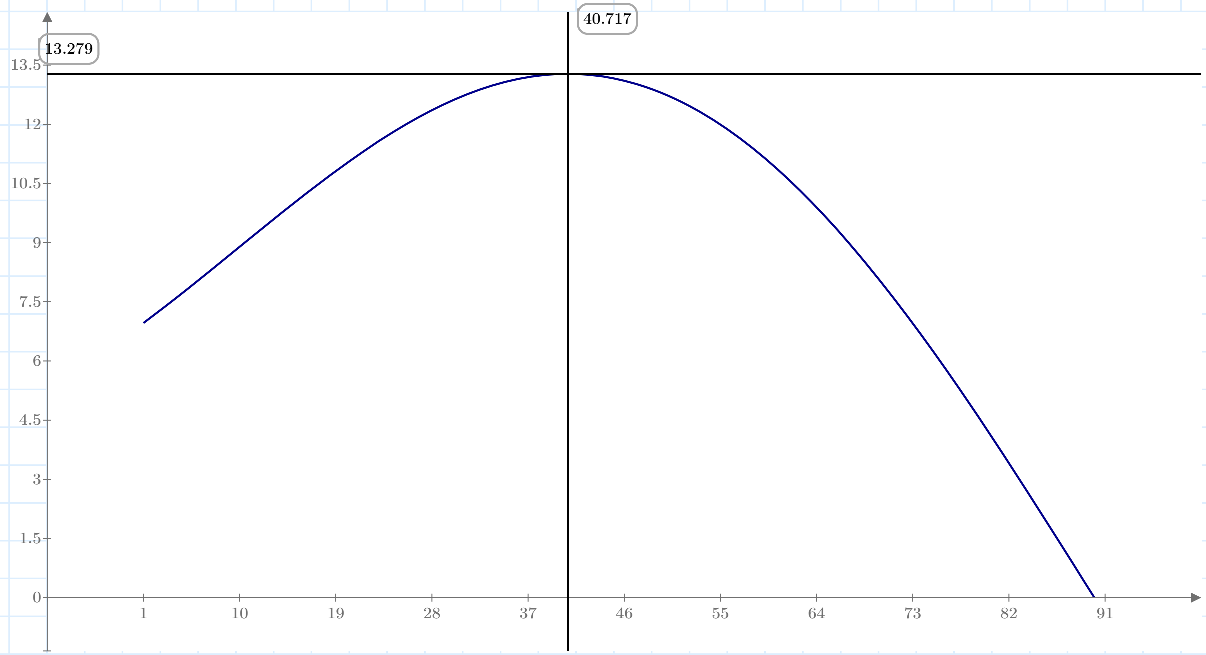
$$Reach_{opt} := Horizontal_{reach}\left(\mathbf{maximize}\left(Horizontal_{reach}, \theta\right)\right) = 13.279\ m$$

Optimal launch angle using root function function

$$\mathbf{root}\left(\frac{d}{d\theta} Distance_{x\_earth}(h_1, h_2, \theta), \theta, 0, \frac{\pi}{2}\right) = 40.716\ deg$$

Horizontal Reach Vs Launch angle
showing optimal angle for max Horizontal Reach

$\alpha := 1\ deg, 2\ deg..90\ deg$



$Distance_{x\_earth}(h_1, h_2, \alpha)\ (m)$

$\alpha\ (deg)$

Time to Reach Surface on Earth

$$t_{earth} := \text{polyroots}\left(\begin{bmatrix} -h_2 \\ -v_{l\_earth} \cdot \sin(\theta) \\ \frac{g}{2} \end{bmatrix}\right)^{\langle 1 \rangle} = [1.376]\ s$$

Time to Reach Surface on Moon

$$g_{moon} := \frac{g}{6}$$

$$v_{l\_moon} = 4.322\ \frac{m}{s}$$

$$v_y := -v_{l\_moon} \cdot \sin(\theta) = -2.161\ \frac{m}{s}$$

$$v_x := v_{l\_moon} \cdot \cos(\theta) = 3.743\ \frac{m}{s}$$

$$v_{impact\_y} := \sqrt{{v_y}^2 + 2 \cdot \frac{g}{6} \cdot h_2} = 3.348\ \frac{m}{s}$$

$$\frac{h_2 \cdot 2}{(v_{impact\_y} + v_y)} = 3.37\ s$$

$$t_{moon} := \text{polyroots}\left(\begin{bmatrix} -h_2 \\ -v_{l\_moon} \cdot \sin(\theta) \\ \left(\frac{g}{12}\right) \end{bmatrix}\right)^{\langle 1 \rangle} = [3.37]\ s$$

$$\textbf{root}\left(S_M(t), t, 0\ s, 10\ s\right) = 3.37\ s$$

When solving a quadratic equation, such as using the quadratic formula, the process typically yields two real solutions: one negative and one positive value. For our case select the positive value.

Time to Reach Surface on Moon Vs Time to Reach Surface on Earth

$$\frac{t_{\mathrm{moon}}}{t_{earth}} \rightarrow [2.44948974278317740818] \qquad \sqrt{6} = 2.449$$

$$Distance_{x\_moon}(h_1, h_2, \theta) := \left(\frac{-v_{l\_moon} \cdot \sin(\theta) - \sqrt{\left(\left(-v_{l\_moon} \cdot \sin(\theta)\right)^2 + 4 \cdot \left(\frac{g}{12}\right) \cdot h_2 \right)}}{-2 \cdot \left(\frac{g}{12}\right)} \right) \cdot v_{l\_moon} \cdot \cos(\theta)$$

$$Distance_{x\_moon}(h_1, h_2, \theta) = 12.615\ m$$

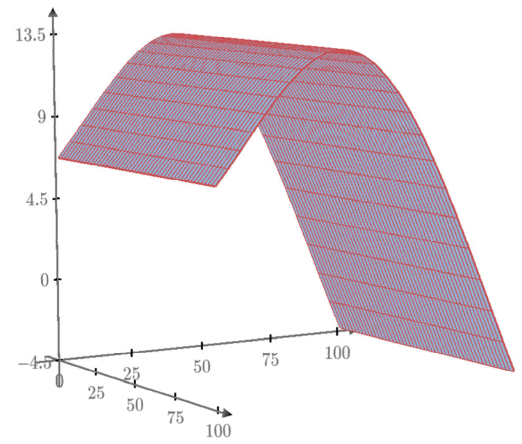
On the Moon

The horizontal reach on the moon remains unaffected . However, the time it takes to reach the same point is increases by a factor of $\sqrt{6}$. This calculation assumes that the acceleration due to gravity on the moon is one-sixth that of Earth's.

Ball Distance with height and angle as variables

$h_1 := 2, 4 .. 100$ Variable Drop Height
Fixed launch height of 2m

$\theta := 0, 4 .. 100$

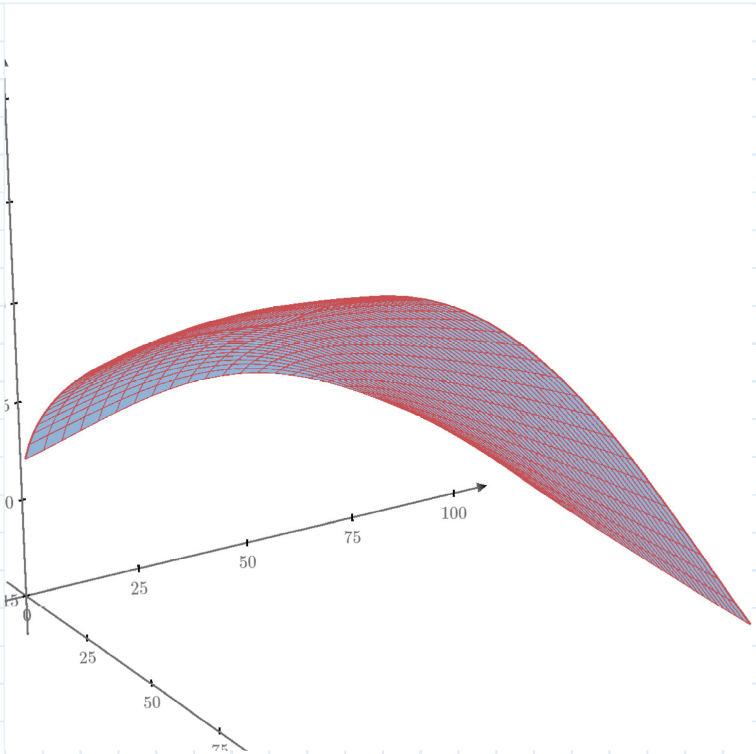


X Axis is height
Y -Axis is angle
Z -Axis Reach

$Distance_{x\_earth}(h_1 \cdot m, 2\ m, \theta \cdot deg)\ (m)$

Ball Distance with height and angle as variables

$h_2 := 2, 4 .. 100$ Variable Launch Height Fixed launch drop of 10m



X Axis is height
Y -Axis is angle
Z -Axis Reach

$Distance_{x\_earth}(10 \cdot m, h_2\ m, \theta \cdot deg)\ (m)$