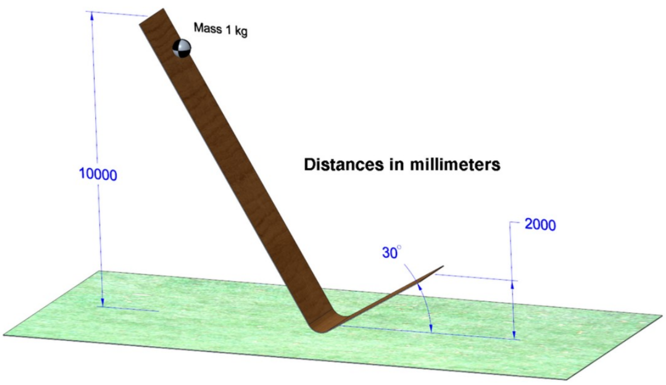
Mathcad Community Challenge May 2023

Mathcad Community Challenge May 2023 - Optimize Trajectory for Maximum Horizontal Distance

DaveMartin 16-Pearl May 01, 2023 04:13 PM

A ball with a mass of 1 kilogram is at the top of a frictionless ramp 10 meters above the ground. The ball rolls down the incline and launches from a height of 2 meters and an angle of 30 degrees above the ground.



(This picture was created in Creo.)

1. Create a function that calculates the horizontal distance as a function of initial height, launch height, and launch angle.
2. Calculate the horizontal distance the ball will land from the end of the ramp.
3. Solve for the angle that will optimize the horizontal distance.
4. How will the horizontal distance change if this were performed on the Moon instead of on the Earth's surface? Assume the acceleration due to gravity on the Moon's surface is 1/6 that of Earth.
5. Use the Chart Component to depict how the horizontal landing distance changes as a function of angle.
6. Use a 3D Plot to show how the horizontal landing distance changes as a function of ramp height and launch angle. Assume the ball starts at a height of 10 meters.

The worksheet should contain sufficient documentation to stand on its own; someone unfamiliar with the initial problem should be able to understand what is being calculated.

Find the Mathcad Community Challenge Guidelines here!

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Labels: Puzzles Games

Tags: Mathcad Community Challenge

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1. Input data

ball mass

$m_b := 1\ kg$

height of frictionless ramp

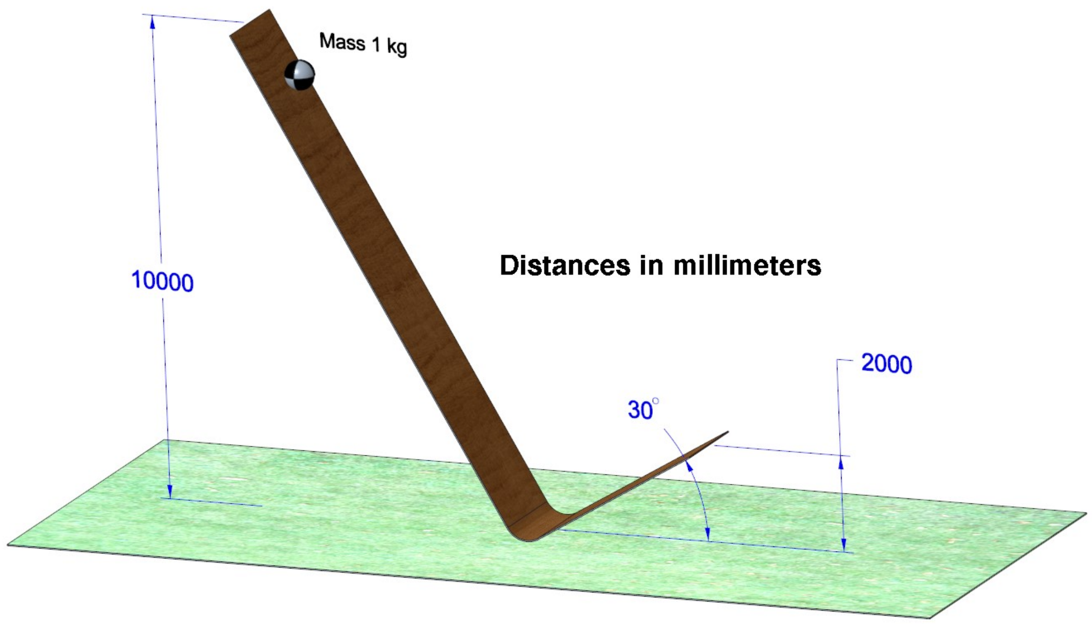
$h_1 := 10\ m$

launch height

$h_2 := 2\ m$

launch angle

$\theta := 30\ deg$



2. Calculation

2.1 Task 1

> Create a function that calculates the horizontal distance as a function of initial height, launch height, and launch angle

Application of the principle of conservation of energy between the start and the end of the ramp (E1=E2) gives the launch speed:

$$E_1 = m_b \cdot g \cdot h_1$$

$$E_2 = m_b \cdot g \cdot h_2 + \frac{1}{2} \cdot m_b \cdot v_2^{\,2}$$

$$v_2 := \sqrt{2 \cdot g \cdot (h_1 - h_2)} = 12.526 \ \frac{m}{s}$$

The classic formula for trajectory (https://en.wikipedia.org/wiki/Trajectory) gives the relation between the verticale coordinate y(x) and the horizontale coordinate x.

This formula can be easily derived by making a distinction between the horizontal and the vertical trajectory: both trajectories don't influence each other!
Only the vertical trajectory is subject to gravity.

$$x(t) = v_2 \cdot cos(\alpha) \cdot t$$

$$y(t) = -\frac{1}{2} \cdot g \cdot t^2 + v_2 \cdot sin(\alpha) \cdot t$$

$$y(x) := -\frac{g \cdot (sec(\theta))^2}{2 \cdot v_2^{\,2}} \cdot x^2 + x \cdot \tan(\theta) + h_2$$

By stating that y=0, we get the required formula

$$\Delta_0(y_1, y_2, \alpha) := -\frac{(sec(\alpha))^2}{4 \cdot (y_1 - y_2)} \cdot x^2 + x \cdot \tan(\alpha) + y_2 = 0 \xrightarrow[explicit]{solve, x} \begin{bmatrix} \sqrt{(y_2^{\,2} - 2 \cdot y_1 \cdot y_2 + \ldots} \\ -\sqrt{(y_2^{\,2} - 2 \cdot y_1 \cdot y_2} \end{bmatrix}$$

$$\Delta(y_1, y_2, \alpha) := \sqrt{\left((y_2 - y_1)^2 \cdot \sin(2 \cdot \alpha)^2 + 4 \cdot y_2 \cdot (y_1 - y_2) \cdot \cos(\alpha)^2\right)} + (y_1 - y_2) \cdot \sin(2 \cdot \alpha)$$

2.2 Task 2

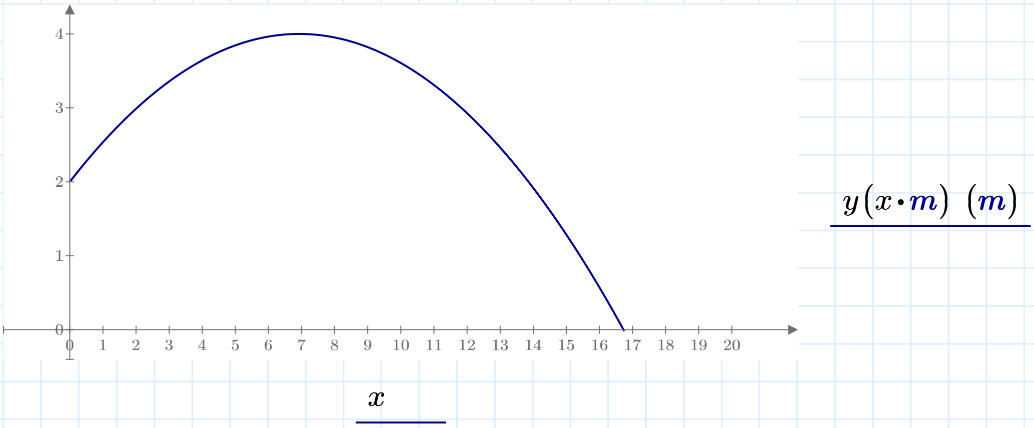
> Calculate the horizontal distance the ball will land from the end of the ramp

Application of the input data in the distance formula of §2.1 delivers the answer

$\Delta(h_1, h_2, \theta) = 16.726\ m$

We can verify this value by applying the root function on y(x)

$\mathbf{root}(y(x), x, 1\ m, 20\ m) = 16.726\ m$



2.3 Task 3

> Solve for the angle that will optimize the horizontal distance

The optimal launch angle is calculated starting from a reworked distance formula (§2.1) with only the launch angle as variable, for optimization a solve block has been used

$$\Delta_2(\alpha) := \sqrt{\left((h_2-h_1)^2 \cdot \sin(2\cdot\alpha)^2 + 4\cdot h_2 \cdot (h_1-h_2)\cdot \cos(\alpha)^2\right)} + (h_1-h_2)\cdot\sin(2\cdot\alpha)$$

Guess Values

$$\beta := 40\ deg$$

Constraints

$$\beta > 0$$

Solver

$$\beta_{opt} := \mathbf{maximize}\left(\Delta_2, \beta\right)$$

$$\beta_{opt} = 41.81\ deg$$

$$\Delta_2\left(\beta_{opt}\right) = 17.889\ m$$

2.4 Task 4

> How will the horizontal distance change if this were performed on the Moon instead of on the Earth's surface?
Assume the acceleration due to gravity on the Moon's surface is 1/6 that of Earth.

Gravity has no influence on the horizontal distance covered.
As a result, the horizontal distance is the same on earth as on the Moon.
This is clear in §2.1: the equation for x(t) contains no gravitational acceleration.

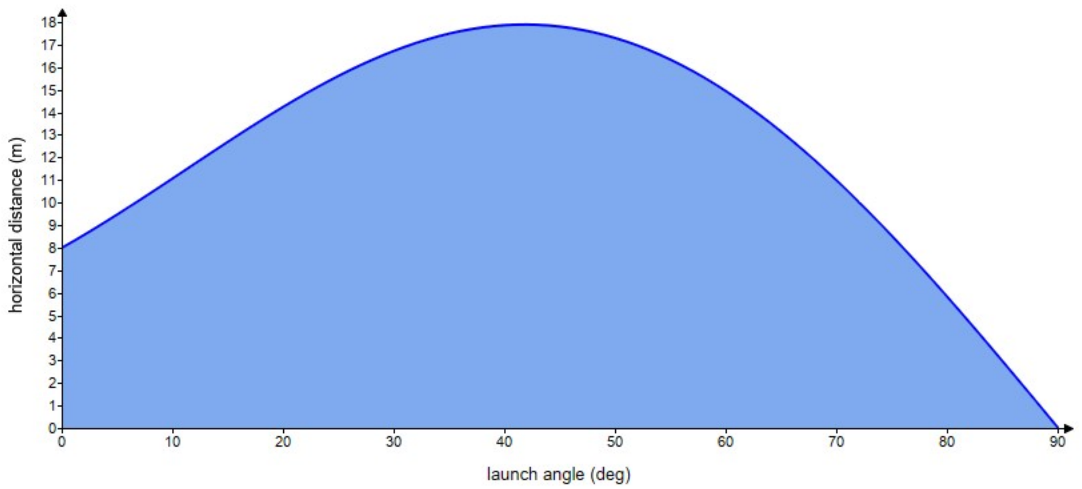
2.5 Task 5

> Use the Chart Component to depict how the horizontal landing distance changes as a function of angle

$$range := 0 \cdot deg, 1\ deg .. 90\ deg$$

Inputs

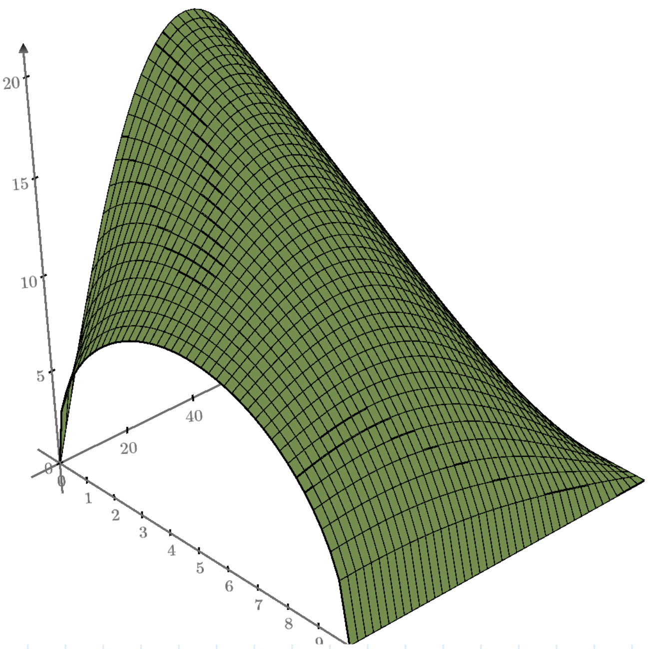
$$X_1 := \frac{range}{deg} \qquad Y_1 := \Delta_2(range)$$



2.6 Task 6

> Use a 3D Plot to show how the horizontal landing distance changes as a function of ramp height and launch angle.
Assume the ball starts at a height of 10 meters.

$$\Delta_3\left(y_2, \alpha\right) := \sqrt{\left(\left(y_2 - h_1\right)^2 \cdot \sin\left(2 \cdot \alpha\right)^2 + 4 \cdot y_2 \cdot \left(h_1 - y_2\right) \cdot \cos\left(\alpha\right)^2\right)} + \left(h_1 - y_2\right) \cdot \sin\left(2 \cdot \alpha\right)$$



$\Delta_3\left(ramp\_height \cdot m, launch\_angle \cdot deg\right)$ (m)

3. References

https://en.wikipedia.org/wiki/Trajectory
https://www.hhofstede.nl/modules/projectielbanen.htm
https://nl.wikipedia.org/wiki/Lijst_van_goniometrische_gelijkheden