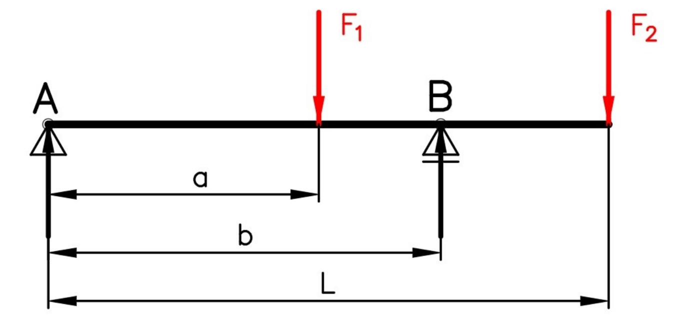
Beam calculations using singularity functions

$a := 6\ m$ $b := 8\ m$ $L := 12\ m$ $F1 := 5000\ N$ $F2 := 10000\ N$



Material: S275j0 (1.0143) $E := 210\ GPa$

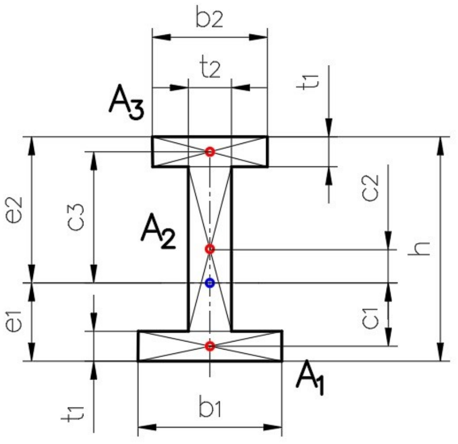
$t_1 := 10.7\ mm$ $t_2 := 7.1\ mm$ $b_1 := 150\ mm$ $b_2 := 150\ mm$

$h := 300\ mm$

$A_1 := b_1 \cdot t_1 = 1605\ mm^2$ $y_{01} := \frac{t_1}{2} = 5.35\ mm$

$A_2 := t_2 \cdot (h - 2 \cdot t_1) = 1978.06\ mm^2$ $y_{02} := \frac{h}{2} = 150\ mm$

$A_3 := b_2 \cdot t_1 = 1605\ mm^2$ $y_{03} := h - \frac{t_1}{2} = 294.65\ mm$



Cross-sectional area of the I-beam

$A := A_1 + A_2 + A_3$ $A = 5188.06\ mm^2$

$$e_1 = \frac{\sum A_i \cdot y_i}{\sum A_i}$$

$e_1 := \frac{A_1 \cdot y_{01} + A_2 \cdot y_{02} + A_3 \cdot y_{03}}{A_1 + A_2 + A_3} = 150\ mm$ $e_2 := h - e_1 = 150\ mm$

$c_1 := e_1 - \frac{t_1}{2} = 144.65\ mm$ $c_2 := \frac{h}{2} - e_1 = 0\ mm$ $c_3 := e_2 - \frac{t_1}{2} = 144.65\ mm$

Moment of Inertia

$I_1 := \frac{b_1 \cdot t_1^3}{12} = 15313.038\ mm^4$ $I_2 := \frac{t_2 \cdot (h - 2 \cdot t_1)^3}{12} = 12794415.163\ mm^4$ $I_3 := \frac{b_2 \cdot t_1^3}{12} = 15313.038\ mm^4$

$I_{zzi} = I_i + c_i^2 \cdot A_i$ $I_{zz1} := I_1 + c_1^2 \cdot A_1 = 33597727.15\ mm^4$ $I_{zz2} := I_2 + c_2^2 \cdot A_2 = 12794415.163\ mm^4$ $I_{zz3} := I_3 + c_3^2 \cdot A_3 = 33597727.15\ mm^4$

$I_{zz} := I_{zz1} + I_{zz2} + I_{zz3}$ $I_{zz} = 79989869.463\ mm^4$

Load function

$$q(x) = F_A \cdot x^{-1} - F1 \cdot (x-a)^{-1} + F_B \cdot (x-b)^{-1} - F2 \cdot (x-L)^{-1}$$

Shear function

$$T(x) = \int_{-\infty}^{x} q(x)\,\mathrm{d}x \qquad T(x) = F_A \cdot x^0 - F1 \cdot (x-a)^0 + F_B \cdot (x-b)^0 - F2 \cdot (x-L)^0$$

Moment function

$$M(x) = -\int_{-\infty}^{x} T(x)\,\mathrm{d}x \qquad M(x) = -\left(F_A \cdot x^1 - F1 \cdot (x-a)^1 + F_B \cdot (x-b)^1 - F2 \cdot (x-L)^1\right)$$

Slope function

$$\theta(x) = \int_{-\infty}^{x} M(x)\,\mathrm{d}x \qquad \theta(x) = \frac{\left(F_A \cdot \frac{x^2}{2} - F1 \cdot \frac{(x-a)^2}{2} + F_B \cdot \frac{(x-b)^2}{2} - F2 \cdot \frac{(x-L)^2}{2} + C_3\right)}{E \cdot I_{zz}}$$

Deflection function

$$y(x) = \int_{-\infty}^{x} \theta(x)\,\mathrm{d}x \qquad y(x) = \frac{\left(F_A \cdot \frac{x^3}{6} - F1 \cdot \frac{(x-a)^3}{6} + F_B \cdot \frac{(x-b)^3}{6} - F2 \cdot \frac{(x-L)^3}{6} + C_3 \cdot x + C_4\right)}{E \cdot I_{zz}}$$

Solution of the system of equations

at x=0 --> y=0, it follows that **C_4=0**
at x=b --> y=0
at x=L --> M=0

Guess Values

$$F_A := 1\ N \qquad F_B := 1\ N \qquad C_3 := 1\ N \cdot m^2$$

Constraints

$$F_A + F_B - F1 - F2 = 0$$

$$F_A \cdot \frac{b^3}{6} - F1 \cdot \frac{(b-a)^3}{6} + C_3 \cdot b = 0$$

$$F_A \cdot L - F1 \cdot (L-a) + F_B \cdot (L-b) = 0$$

Solver

$$\begin{bmatrix} F_A \\ F_B \\ C_3 \end{bmatrix} := \mathrm{find}\left(F_A, F_B, C_3\right)$$

$$F_A = -3750\ N \qquad F_B = 18750\ N \qquad C_3 = 40833.333\ N \cdot m^2$$

$S(x,z) := \text{if}(x \ge z, 1, 0)$

To display the solution, we use the function S, which takes the value 0 when x<z and the value 1 when x is equal to or greater than z.

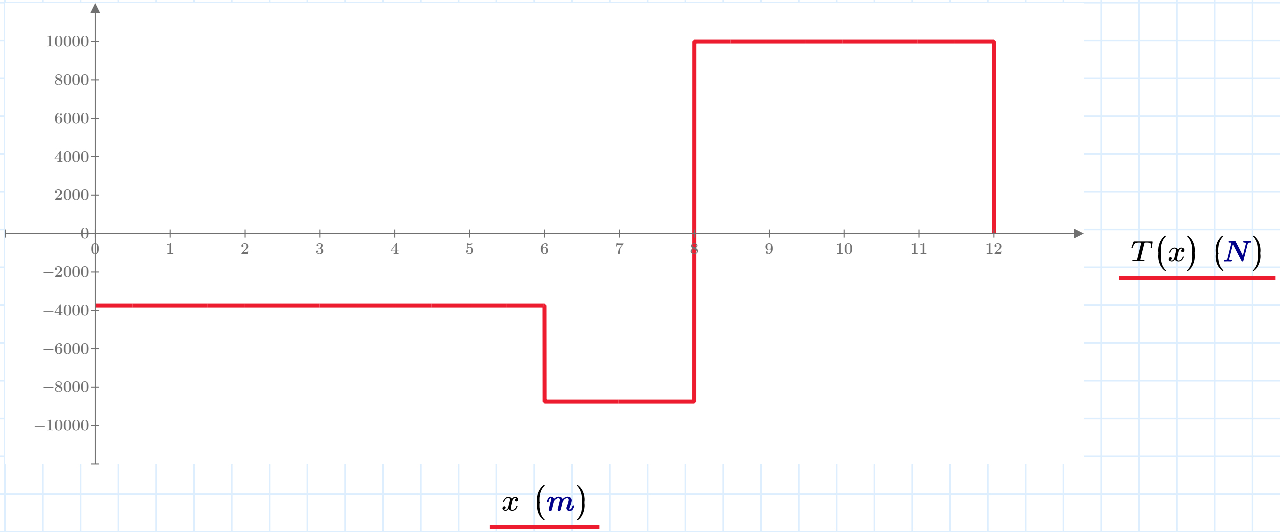
$$T(x) := F_A \cdot x^0 \cdot S(x,0) - F1 \cdot (x-a)^0 \cdot S(x,a) + F_B \cdot (x-b)^0 \cdot S(x,b) - F2 \cdot (x-L)^0 \cdot S(x,L)$$

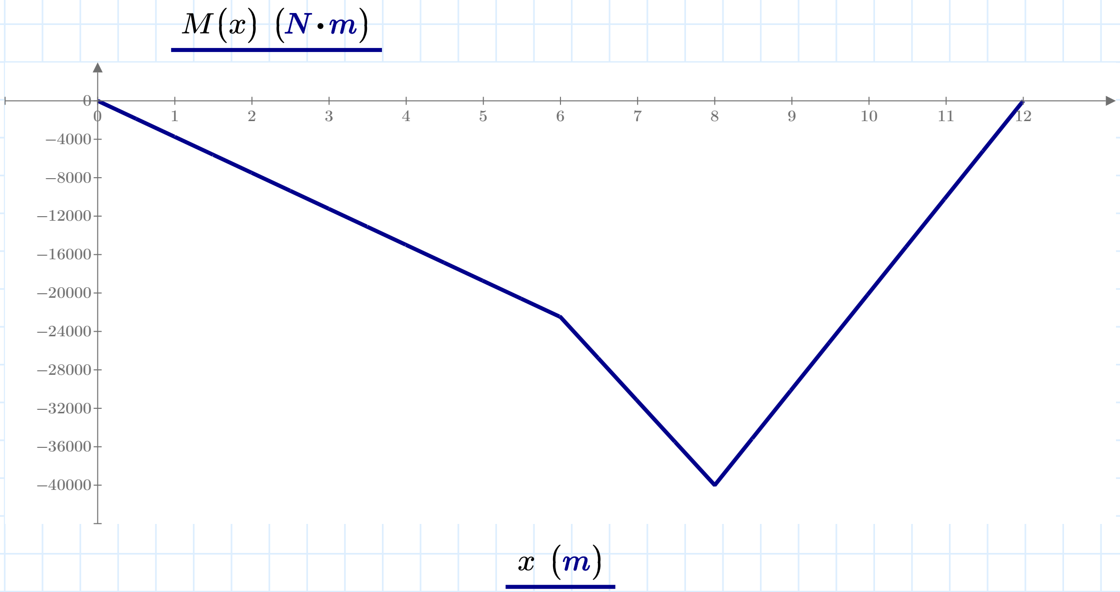
$$M(x) := F_A \cdot x^1 \cdot S(x,0) - F1 \cdot (x-a)^1 \cdot S(x,a) + F_B \cdot (x-b)^1 \cdot S(x,b) - F2 \cdot (x-L)^1 \cdot S(x,L)$$

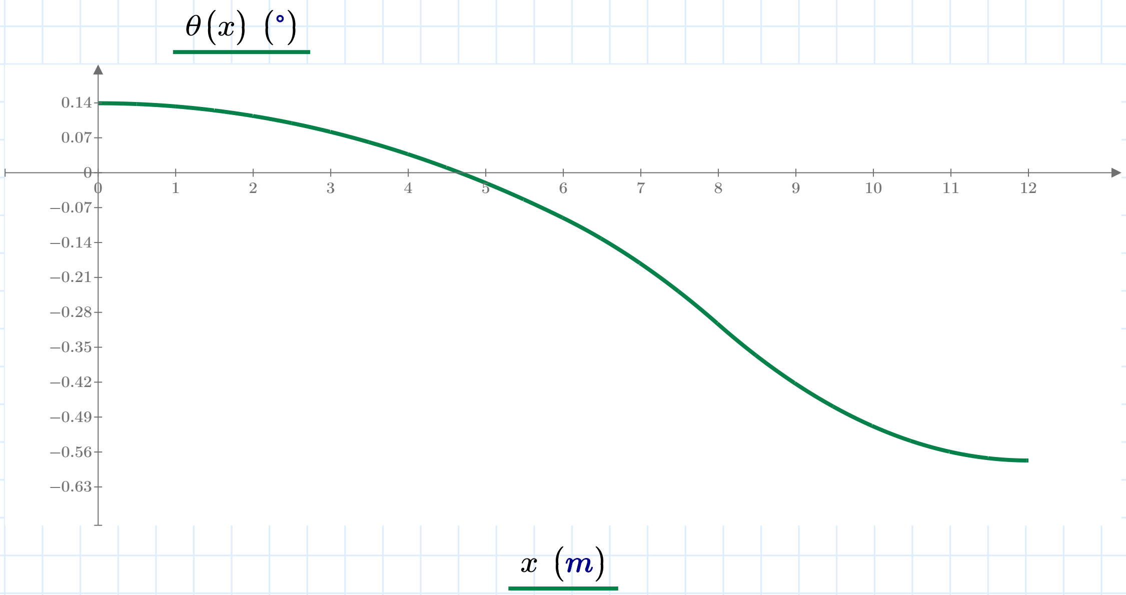
$$\theta(x) := \frac{\left(F_A \cdot \frac{x^2}{2} \cdot S(x,0) - F1 \cdot \frac{(x-a)^2}{2} \cdot S(x,a) + F_B \cdot \frac{(x-b)^2}{2} \cdot S(x,b) - F2 \cdot \frac{(x-L)^2}{2} \cdot S(x,L) + C_3\right)}{E \cdot I_{zz}}$$

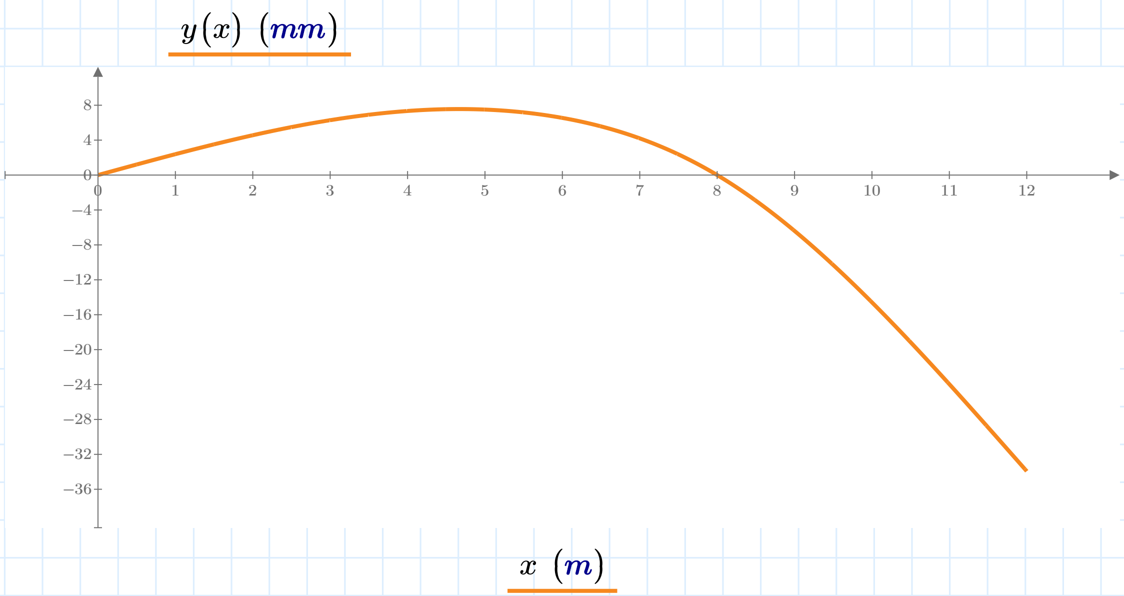
$$y(x) := \frac{\left(F_A \cdot \frac{x^3}{6} \cdot S(x,0) - F1 \cdot \frac{(x-a)^3}{6} \cdot S(x,a) + F_B \cdot \frac{(x-b)^3}{6} \cdot S(x,b) - F2 \cdot \frac{(x-L)^3}{5} \cdot S(x,L) + C_3 \cdot x\right)}{E \cdot I_{zz}}$$

$x := 0 \cdot m, 0.001 \cdot m .. L$


10000
8000
6000
4000
2000
0
−2000
−4000
−6000
−8000
−10000
0 1 2 3 4 5 6 7 8 9 10 11 12
T(x) (N)
x (m)


M(x) (N·m)
0
−4000
−8000
−12000
−16000
−20000
−24000
−28000
−32000
−36000
−40000
0 1 2 3 4 5 6 7 8 9 10 11 12
x (m)


θ(x) (°)
0.14
0.07
0
−0.07
−0.14
−0.21
−0.28
−0.35
−0.42
−0.49
−0.56
−0.63
0 1 2 3 4 5 6 7 8 9 10 11 12
x (m)


y(x) (mm)
8
4
0
−4
−8
−12
−16
−20
−24
−28
−32
−36
0 1 2 3 4 5 6 7 8 9 10 11 12
x (m)

Maximum deflection of the beam

$x_{ymax} := \mathrm{root}\left(\theta(x), x, 2\ m, 6\ m\right)$ $x_{ymax} = 4.667\ m$

$y\left(x_{ymax}\right) = 7.563\ mm$

$y(12\ m) = -33.933\ mm$

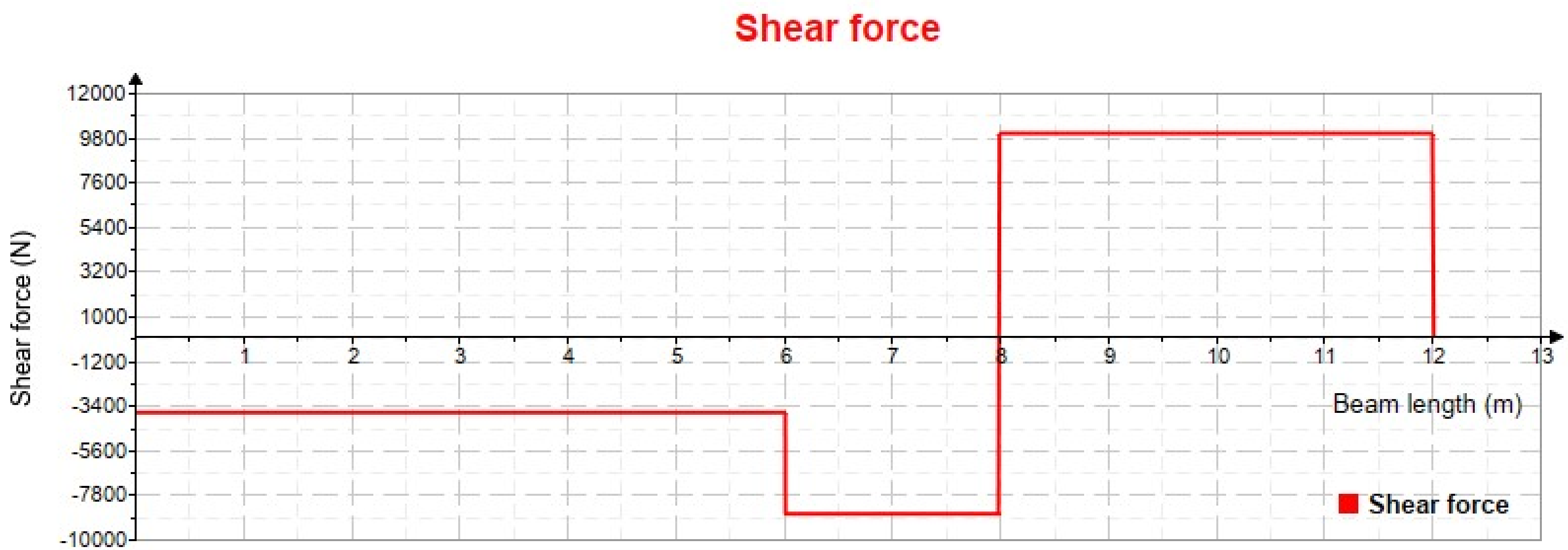
Maximum bending moment

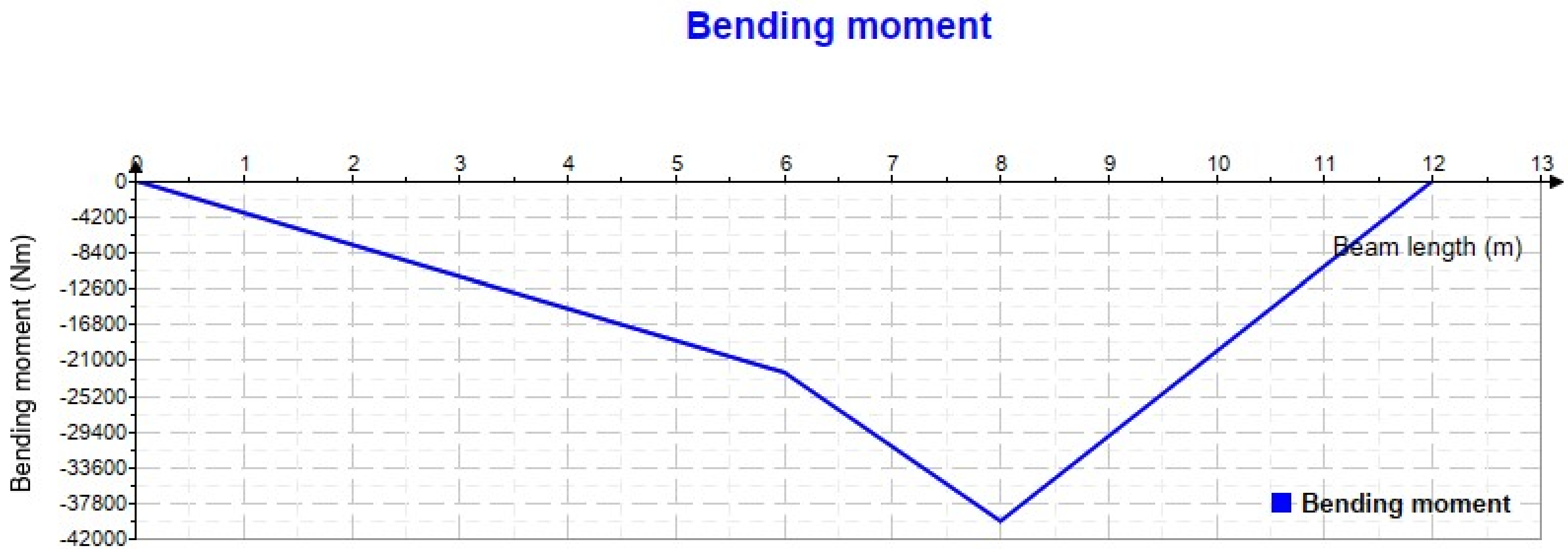
$M_{max} := M\left(x_{ymax}\right) = -17.5\ kN \cdot m$

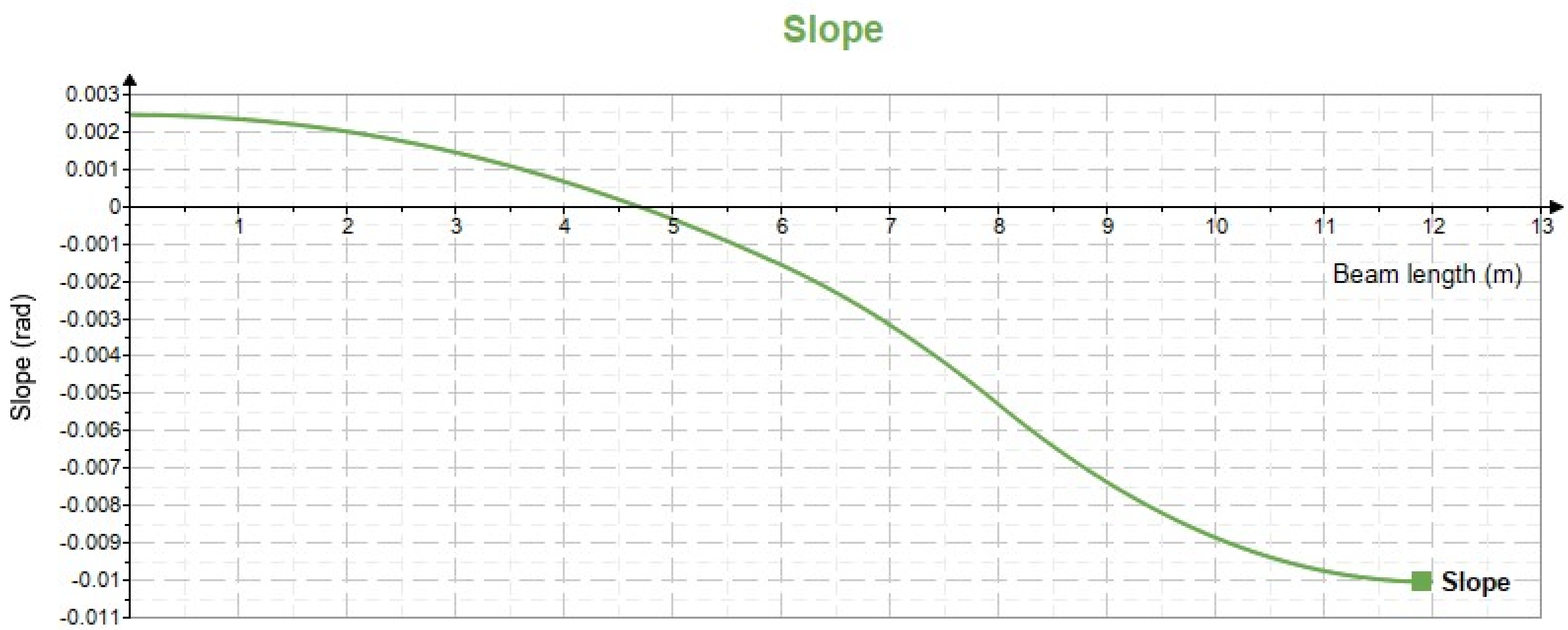
$M(6\ m) = -22.5\ kN \cdot m$

$M(8\ m) = -40\ kN \cdot m$

$M(12\ m) = 0\ kN \cdot m$


Shear force
12000
9800
7600
5400
3200
1000
-1200
-3400
-5600
-7800
-10000
Shear force (N)
1
2
3
4
5
6
7
8
9
10
11
12
13
Beam length (m)
Shear force


Bending moment
0
1
2
3
4
5
6
7
8
9
10
11
12
13
0
-4200
-8400
-12600
-16800
-21000
-25200
-29400
-33600
-37800
-42000
Bending moment (Nm)
Beam length (m)
Bending moment


Slope
0.003
0.002
0.001
0
-0.001
-0.002
-0.003
-0.004
-0.005
-0.006
-0.007
-0.008
-0.009
-0.01
-0.011
Slope (rad)
1
2
3
4
5
6
7
8
9
10
11
12
13
Beam length (m)
Slope


Deflection
0.01
0.005
0
-0.005
-0.01
-0.015
-0.02
-0.025
-0.03
-0.035
-0.04
Deflection (m)
1
2
3
4
5
6
7
8
9
10
11
12
13
Beam length (m)
Deflection