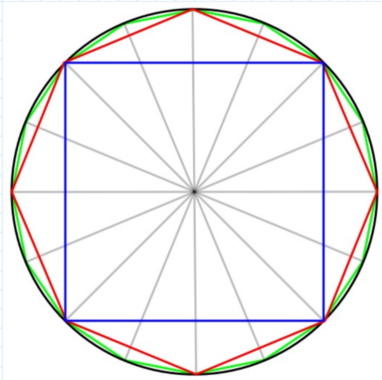
Calculate Perimeter Length of Ellipse



Method

Archimedes calculated the circumference of a circle (and pi) by dividing it into many small triangle and summing the known lengths of the straight sides. The more divisions used, the closer the estimate. This same approach is used on the ellipse.

Given:

$a := 30$ Major axis length

$b := 15$ Minor axis length

$N := 1 \cdot 10^3$ Number of divisions

Calculations

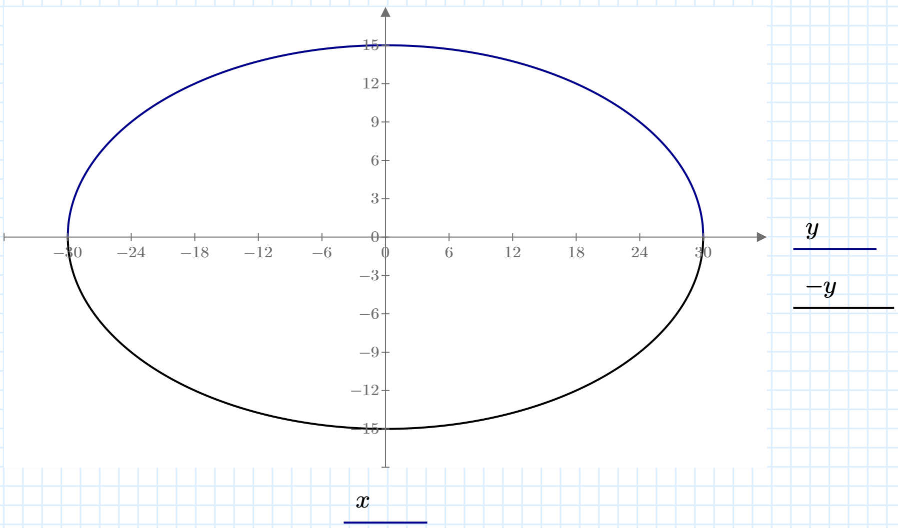
$dx := \frac{2 \cdot a}{N} = 0.06$ Length of division in x direction

Vector of x coordinates
(Note: using a program because range variable bug causes issues)

```
x := | x_0 <- -a
     | for i ∈ 1..N
     |   | x_i <- x_(i-1) + dx
     |   | "Need this to stop x>a"
     |   | if x_i > a
     |   |   | x_i <- a
     | return x
```

$y_1(x) := \frac{b \cdot \sqrt{a - x} \cdot \sqrt{a + x}}{a}$ Y equation of ellipse (top half)

$y := \overrightarrow{y_1(x)}$ Vector of y coordinates



$$Length := \left\| \begin{array}{l} L \leftarrow 0 \\ k \leftarrow 0 \\ \text{for } i \in 1 .. \text{last}(x) - 1 \\ \quad \left\| \begin{array}{l} L_{seg} \leftarrow \sqrt{\left(x_i - x_{i-1}\right)^2 + \left(y_i - y_{i-1}\right)^2} \\ L \leftarrow L + L_{seg} \end{array} \right. \\ \text{"Multiply by 2 for bottom half"} \\ L \leftarrow 2 \cdot L \\ \text{return } L \end{array} \right.$$

$$Length = 143.423485$$

Running several cases we see we start to converge on a solution

N_1	L_1
$1 \cdot 10^3$	143.423485
$1 \cdot 10^4$	144.726535
$1 \cdot 10^5$	145.136981
$1 \cdot 10^6$	145.266723
$5 \cdot 10^6$	145.299895
$9 \cdot 10^6$	145.306726

Approximate Perimeter Formula

Muir

$$P_{Muir} := 2 \cdot \pi \cdot \left(\frac{a^{1.5} + b^{1.5}}{2} \right)^{\frac{2}{3}} = 145.299701$$

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$$P := 2 \cdot \pi \cdot \sqrt{\frac{a^2 + b^2}{2}} = 149.018824$$