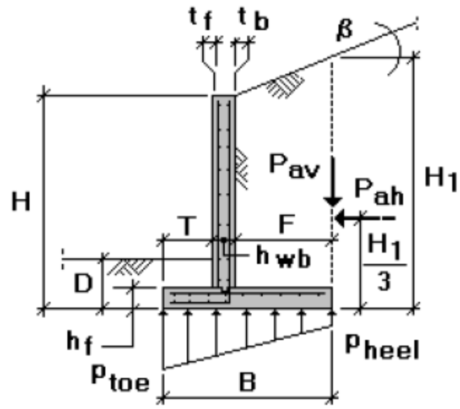
Notation



Input Variables:

$d_b := [0\ \ 0\ \ 0\ \ 0.375\ \ .5\ \ .625\ \ .75\ \ .875\ \ 1\ \ 1.128\ \ 1.27\ \ 1.41\ \ \ldots]^T \cdot in$

Total height from top of wall to bottom of footing:	$H := 22\ ft$
Depth from lower grade level to bottom of footing:	$D := 0\ ft$
Estimated footing thickness:	$h_f := 18\ in$
Estimated wall thickness at top of footing:	$h_{wb} := 18\ in$
Front face wall taper:	$t_f := 0\ in$
Back face wall taper:	$t_b := 6\ in$
Unit weight of soil:	$\gamma := 100\ pcf$
Coefficient of friction between footing and soil:	$c_f := 0.55$
Allowable soil bearing pressure:	$p_s := 5\ ksf$
Wall reinforcing bar diameter:	$d_{bw} := d_{b_7}$
Toe reinforcing bar diameter:	$d_{bt} := d_{b_6}$
Heel reinforcing bar diameter:	$d_{bh} := d_{b_8}$

Angle of Internal Friction
(Enter "0" if kah and kav are to be entered) $\phi := 33.67\ deg$

Backfill Slope Angle $\beta := \text{atan}\left(\frac{1}{2}\right) = 26.565\ deg$

$$k_a := \cos(\beta) \cdot \left(\frac{\cos(\beta) - \sqrt{\left(\cos(\beta)^2 - \cos(\phi)^2\right)}}{\cos(\beta) + \sqrt{\left(\cos(\beta)^2 - \cos(\phi)^2\right)}} \right) \cdot \gamma = 41.48\ \frac{psf}{ft}$$

$$k_{ah} := k_a \cdot \cos(\beta) = 37.1\ \frac{psf}{ft}$$

$$k_{av} := k_a \cdot \sin(\beta) = 18.55\ \frac{psf}{ft}$$

Material Properties and Constants

Specified compressive strength of concrete:	$f`_c = 4\ ksi$
Specified yield strength of reinforcement (f_y may not exceed 60 ksi, ACI 318 11.5.2):	$f_y := 60\ ksi$
Unit weight of concrete:	$w_c := 145\ pcf$
Weight of reinforced concrete:	$w_{rc} := 150\ pcf$
Shear strength reduction factor for lightweight concrete k_v = 1 for normal weight, 0.75 for all-lightweight and 0.85 for sand-lightweight concrete (ACI 318, 11.2.1.2.):	$k_v := 1$
Weight factor for increasing development and splice lengths k_w =1 for normal weight and 1.3 for lightweight aggregate concrete (ACI 318, 12.2.4.2):	$k_w := 1$
Modulus of elasticity of reinforcement (ACI 318, 8.5.2):	$E_s := 29000\ ksi$
Strain in concrete at compression failure (ACI 318, 10.3.2):	$\varepsilon_c := .003$
Strength reduction factor for flexure (ACI 318, 9.3.2.1):	$\phi_f := 0.9$
Strength reduction factor for shear (ACI 318, 9.3.2.3):	$\phi_v := 0.85$
Sizing factor for rounding wall and footing thicknesses:	$SzF := 2\ in$
Sizing factor for rounding base width:	$SzB := 3\ in$
Concrete cover of bottom toe reinforcement:	$cl_t := 3\ in$
Concrete cover of top heel reinforcement:	$cl_h := 1.5\ in$
Concrete cover of wall reinforcement:	$cl_w := 1.5\ in$
Sliding factor of safety:	$SF := 1.5$

Reinforcing bar number designations, diameters and areas:

$$No := [0\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ 13\ 14\ 15\ 16\ 17\ 18]^T$$

$$d_b^T = [0\ 0\ 0\ 0.375\ 0.5\ 0.625\ 0.75\ 0.875\ 1\ 1.128\ 1.27\ 1.41\ \ldots]\ in$$

$$A_b := [0\ 0\ 0\ .11\ .2\ .31\ .44\ .6\ .79\ 1\ 1.27\ 1.56\ 0\ 0\ 2.25\ 0\ 0\ 0\ 4]^T \cdot in^2$$

$$f`_{c\_max} := \mathbf{if}\left(f`_c > 10\ ksi, 10\ ksi, f`_c\right) = 4\ ksi$$

$$\upsilon_c := k_v \cdot 2 \cdot \sqrt{\frac{f`_{c\_max}}{psi}} \cdot psi = 126.491\ psi$$

$$\varepsilon_y := \frac{f_y}{E_s} = 0.002$$

$$\beta_1 := \mathbf{if}\left(\left(f`_c \ge 4\ ksi\right) \cdot \left(f`_c \le 8\ ksi\right), 0.85 - .05 \cdot \frac{f`_c - 4\ ksi}{ksi}, \mathbf{if}\left(\left(f`_c \le 4\ ksi\right), 0.85, 0.65\right)\right) = 0.85$$

$$\rho_b := \frac{\beta_1 \cdot 0.85 \cdot f`_c}{f_y} \cdot \frac{E_s \cdot \varepsilon_c}{E_s \cdot \varepsilon_c + f_y} = 0.029$$

$$\rho_{max} := \frac{3}{4} \cdot \rho_b = 0.021$$

$$\rho_{min} := \frac{200}{f_y} \cdot \frac{lbf}{in^2} = 0.003$$

$$\rho_{temp} := \begin{array}{l} \text{if } f_y \le 50\ ksi \\ \quad 0.002 \\ \text{else if } f_y \le 60\ ksi \\ \quad .002 - \frac{f_y}{60\ ksi} \cdot .0002 \\ \text{else if } \frac{.0018 \cdot 60\ ksi}{f_y} \ge .0014 \\ \quad .0018 \cdot \frac{60\ ksi}{f_y} \\ \text{else} \\ \quad .0014 \end{array} = 0.002$$

$F := 10\ ft$

$T := 0\ ft$

Preferred reinforcement ratio:

$$\rho_{pref} := \frac{1}{2} \cdot \rho_{max} = 0.011$$

$$K(\rho) := \phi_f \cdot \rho \cdot \left(1 - \frac{\rho \cdot f_y}{2 \cdot 0.85 \cdot f`_c}\right) \cdot f_y$$

Calculations

Thickness at top of wall: $h_{wt} := h_{wb} - t_f - t_b = 1\ ft$

Stem Wall height: $C := H - h_f = 20.5\ ft$

Depth of fill over toe: $E := \textbf{if}\left(\left(D - h_f\right) \ge 0\ ft, D - h_f, 0\ ft\right) = 0\ ft$

Footing width B as a function of F and T: $B(F, T) := T + h_{wb} + F$

Height H1 as a function of F: $H_1(F) := H + \left(F + t_b\right) \cdot \tan(\beta)$

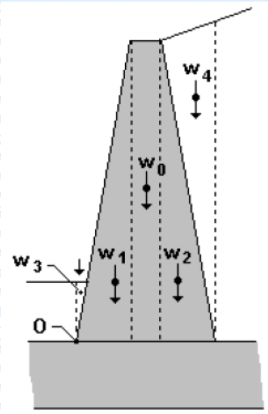
Soil pressure resultants Pah and Pav as a function of F:

$$P_{ah}(F) := k_{ah} \cdot \frac{H_1(F)^2}{2} \cdot ft$$

$$P_{av}(F) := \frac{H_1 \cdot (F)^2}{2} \cdot ft$$

Overturning moment: $M_{ot}(F) := P_{ah}(F) \cdot \frac{H_1(F)}{3}$

$$w := \begin{bmatrix} h_{wt} \cdot C \cdot w_{rc} \\ \frac{t_f}{2} \cdot C \cdot w_{rc} \\ t_b \cdot C \cdot w_{rc} \\ \frac{t_f \cdot E^2}{2 \cdot C} \cdot \gamma \\ \frac{t_b \cdot (C + t_b \cdot \tan(\beta))}{2} \cdot \gamma \end{bmatrix} = \begin{bmatrix} 3.075 \\ 0 \\ 1.538 \\ 0 \\ 0.519 \end{bmatrix} \frac{kip}{ft}$$



Distance from the area centroids of segments to the face of the wall at the top of footing:

$$x := \begin{bmatrix} t_f + \frac{h_{wt}}{2} & \frac{2 \cdot t_f}{3} & t_f + h_{wt} + \frac{t_b}{3} & \frac{E \cdot t_f}{3 \cdot C} & h_{wb} - \frac{t_b}{3} \end{bmatrix}^{T} = \begin{bmatrix} 0.5 \\ 0 \\ 1.167 \\ 0 \\ 1.333 \end{bmatrix} ft$$

Total dead load as a function of F and T:

$$w_R(F,T) := P_{av}(F) + \left(\sum w + \left(B(F,T) \cdot h_f \cdot w_{rc} + \left(\left(T \cdot E + F \cdot \left(C + \left(\frac{F}{2} + t_b \right) \cdot \tan(\beta) \right) \right) \right) \cdot \gamma \right) \right) \cdot ft$$

Distance from top of wall to top of sloping backfill at the back face of wall at the top of footing:

$$X1 := t_b \cdot \tan(\beta) = 0.25\ ft$$

Distance from top of backfill at back face of wall at top of footing to top of backfill at edge of heel as a function of F:

$$X2(F) := \frac{F \cdot \tan(\beta)}{2} = 0.25\ F$$

Dead load resisting moment as a function of F and T:

$$M_R(F,T) := \left\| \begin{array}{l} B \leftarrow B(F,T) \\ P_{av}(F) \cdot B + \left(\sum \left(\overrightarrow{w \cdot (x+T)} \right) + \left(\frac{B^2 \cdot h_f}{2} \cdot w_{rc} + \left(\frac{T^2}{2} \cdot E + F \cdot \left((C + X1) \cdot \left(\left(B - \frac{F}{2} \right) + X2(F) \right) \cdot B - \frac{F}{3} \right) \right) \cdot \gamma \right) \right) \end{array} \right.$$

Location of dead load resultant from the edge of the toe as a function of F an T:

$$x_R(F,T) := \frac{M_R(F,T) - M_{ot}(F)}{w_R(F,T)}$$

Eccentricity of the dead load resultant from the footing midpoint as a function of F and T:

$$e(F,T) := \frac{B(F,T)}{2} - \frac{M_R(F,T) - M_{ot}(F)}{w_R(F,T)}$$

Base contact bearing length as a function of F and T:

$$L_B(F,T) := \mathbf{if}\left(e(F,T) \le \frac{B(F,T)}{6}, B(F,T), 3 \cdot x_R(F,T) \right)$$

Concrete volume of the wall per unit length as a function of F and T:

$$Vol(F,T) := \left(\left(h_{wt} + \frac{t_f}{2} + \frac{t_b}{2}\right) \cdot C + B(F,T) \cdot h_f\right) \cdot ft$$

Soil bearing pressure at toe of base as a function of F and T:

$$p_{toe}(F,T) := \textbf{if}\left(e(F,T) \geq \frac{B(F,T)}{6}, \frac{2 \cdot w_R(F,T)}{L_B(F,T)}, \frac{w_R(F,T)}{B(F,T)} + \frac{6 \cdot e(F,T) \cdot (w_R(F,T))}{B(F,T)^2}\right) \cdot \frac{1}{ft}$$

Soil bearing pressure at heel of base as a function of F and T:

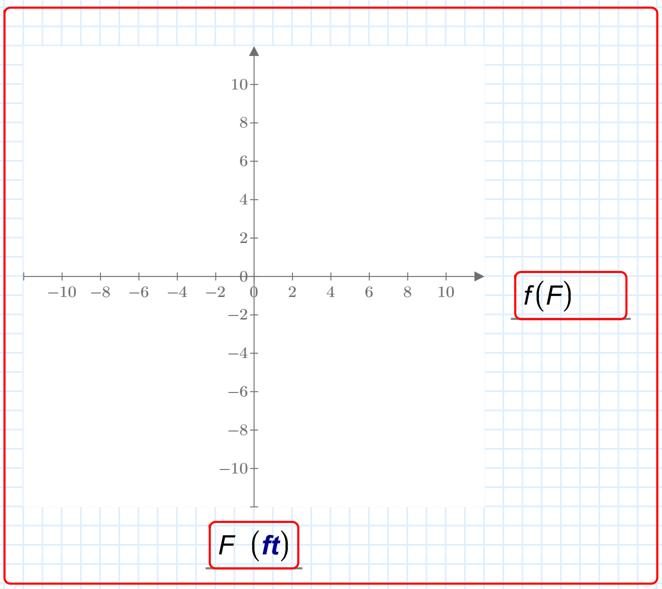
$$p_{heel}(F,T) := \textbf{if}\left(e(F,T) \geq \frac{B(F,T)}{6}, 0\ \frac{kip}{ft}, \frac{w_R(F,T)}{B(F,T)} - \frac{6 \cdot e(F,T) \cdot w_R(F,T)}{B(F,T)^2}\right) \cdot \frac{1}{ft}$$

Check feasibility of proposed design

For some walls with a steep sloping backfill and a low sliding resistance, there may be no feasible base length. A plot of the factor of safety against sliding with T = F/2, versus heel length F will show if a practical design is feasible:

$F := 0\ ft, 2\ ft .. 1.5 \cdot H$

$$f(F) := \frac{c_f \cdot w_R\left(F, \frac{F}{2}\right)}{P_{ah}(F)}$$



If the required factor of safety for sliding cannot be provided with a practical heel length the backfill slope must be reduced, or the sliding resistance must be increased. If necessary, a base shear lug can be used to raise the coefficient of sliding from the value for concrete on soil to the value for soil on soil (approximately 25%). The placement of base shear lugs requires inspection. If the permissible soil bearing pressure is very low it is possible that there is no base width that meets all of the criteria. In this case the Mathcad solve block will give the message "did not find solution".

Calculations to determine required toe and heel dimensions T and F

Guess values of F and T: $F := H$ $T := 0\ ft$

Estimated value of heel projection F calculated assuming the toe projection T is equal to zero, and that the weight of the wall and footing, and the soil over the heel, is just sufficient to provide the required safety factor against sliding:

$F := \mathbf{root}\left(c_f \cdot w_R(F,T) - SF \cdot \left(P_{ah}(F)\right), F\right)$

$\frac{M_R(F,T)}{M_{ot}(F)} = ?$ $\frac{c_f \cdot \left(w_R(F,T)\right)}{P_{ah}(F)} = ?$

$p_{toe}(F,T) = ?$ $P_{heel}(F,T) = ?$

$L_B(F,T) = ?$ $B(F,T) = ?$

$F := 10\ ft$

F and T with w_R equal to SF x Pah(F), the resisting moment greater than 2 times the overturning moment, full contact bearing on the soil, and toe and heel pressures less than or equal to the allowable p_s:

$F = 10\ ft$

$T = 0\ ft$

$\frac{c_f \cdot w_R(F,T)}{P_{ah}(F)} = SF$ $L_B(F,T) = B(F,T)$

$p_{toe}(F,T) \le p_s$ $p_{heel}(F,T) \le p_s$

$\begin{bmatrix} F \\ T \end{bmatrix} := \mathbf{find}(F,T)$

$F = ?$

$\frac{M_R(F,T)}{M_{ot}(F)} = ?$ $\frac{c_f \cdot w_R(F,T)}{P_{ah}(F)} = ?$

$P_{toe}(F,T) = ?$ $p_{heel}(F,T) = ?$

$L_B(F,T) = ?$ $B(F,T) = ?$

$F = ?$ $T = ?$

$B_1 := F + T + h_{wb} = ?$