Utilities

$$\text{MinMax}(f,a,b) := \left|\begin{array}{l} n \leftarrow 100 \\ \text{fn}(x) \leftarrow \text{NaN} \quad \text{on error } f(x) \\ \Delta x \leftarrow \dfrac{b-a}{n} \\ \left[\begin{pmatrix} xmin \\ ymin \end{pmatrix} \ \begin{pmatrix} xmax \\ ymax \end{pmatrix}\right] \leftarrow \left[\begin{pmatrix} a \\ \text{fn}(a) \end{pmatrix} \ \begin{pmatrix} a \\ \text{fn}(a) \end{pmatrix}\right] \\ \text{for } i \in 1..n \\ \quad \left|\begin{array}{l} x \leftarrow a + i\cdot\Delta x \\ y \leftarrow f(x) \\ \begin{pmatrix} xmax \\ ymax \end{pmatrix} \leftarrow \begin{pmatrix} x \\ y \end{pmatrix} \quad \text{if } y > ymax \\ \begin{pmatrix} xmin \\ ymin \end{pmatrix} \leftarrow \begin{pmatrix} x \\ y \end{pmatrix} \quad \text{if } y < ymin \quad \text{otherwise} \end{array}\right. \\ \text{return } \left[\begin{pmatrix} xmin \\ ymin \end{pmatrix} \ \begin{pmatrix} xmax \\ ymax \end{pmatrix}\right]^T \end{array}\right.$$

$$\text{y\_values}(f,a,b,n,type) := \left|\begin{array}{l} \Delta x \leftarrow \dfrac{b-a}{n} \\ \text{for } i \in 0..n-1 \\ \quad \left|\begin{array}{l} y_i \leftarrow f(a + i\cdot\Delta x) \quad \text{if } type = \text{"left"} \\ y_i \leftarrow f[a + (i+1)\cdot\Delta x] \quad \text{if } type = \text{"right"} \\ y_i \leftarrow \max[f(a + i\cdot\Delta x), f[a + (i+1)\cdot\Delta x]] \quad \text{if } type = \text{"maximum"} \\ y_i \leftarrow \min[f(a + i\cdot\Delta x), f[a + (i+1)\cdot\Delta x]] \quad \text{if } type = \text{"minimum"} \\ y_i \leftarrow f[a + (i+0.5)\cdot\Delta x] \quad \text{otherwise} \end{array}\right. \\ \text{return } y \end{array}\right.$$

$$\text{mk\_rectangles}(x1,x2,y1,y2) := \left|\begin{array}{l} (P \ \ N) \leftarrow [(\text{NaN} \ \ \text{NaN}) \ \ (\text{NaN} \ \ \text{NaN})] \\ \text{for } i \in 0..\text{last}(x1) \\ \quad \left|\begin{array}{l} P \leftarrow \text{stack}\left[P, \left(x1_i \ \ y1_i\right), \left(x2_i \ \ y1_i\right), \left(x2_i \ \ y2_i\right), \left(x1_i \ \ y2_i\right), \left(x1_i \ \ y1_i\right), (\text{NaN} \ \ \text{NaN} \right. \\ N \leftarrow \text{stack}\left[N, \left(x1_i \ \ y1_i\right), \left(x2_i \ \ y1_i\right), \left(x2_i \ \ y2_i\right), \left(x1_i \ \ y2_i\right), \left(x1_i \ \ y1_i\right), (\text{NaN} \ \ \text{NaN} \right. \end{array}\right. \\ \text{return } (P \ \ N)^T \end{array}\right.$$

Utilities

Function definitions: $f_1(x) := 0.3\cdot x^4 - 3.x^2 + 8$

$f_2(x) := 0.6\cdot(x-1)^2 + 1.5$ $\qquad f_2(x) := 0$ enable for "classic" 1 function view

Upper and lower limit: $a := -1.8$ $\qquad b := 2.865$

Number of intervals: $n := 30$ $n := round(4 + FRAME)$ for animation

$$n := \begin{cases} (FRAME + 4) & \text{if } FRAME < 56 \\ 7 \cdot FRAME - 332 & \text{otherwise} \end{cases}$$

Chose where to sample the first and the second function:

f_1: left / right / **midpoint** / maximum / minimum

f_2: left / right / **midpoint** / maximum / minimum

Calculations

Calculating limits for plotting

$ov := 15\%$

$x_{left} := a - (b - a) \cdot ov$

$x_{right} := b + (b - a) \cdot ov$

$y_{max} := max\left[0, \left(MinMax(f_1, a, b)_1\right)_1, \left(MinMax(f_2, a, b)_1\right)_1\right]$

$y_{min} := min\left[0, \left(MinMax(f_1, a, b)_0\right)_1, \left(MinMax(f_2, a, b)_0\right)_1\right]$

$y_{top} := y_{max} + (y_{max} - y_{min}) \cdot ov$

$y_{bot} := y_{min} - (y_{max} - y_{min}) \cdot ov$

Vectors with the left and right limits of the rectangles

$$\begin{pmatrix} x1 \\ x2 \end{pmatrix} := \left| \begin{array}{l} \Delta x \leftarrow \dfrac{b - a}{n} \\ \text{for } i \in 0..n - 1 \\ \quad \left| \begin{array}{l} x_{1_i} \leftarrow a + i \cdot \Delta x \\ x_{2_i} \leftarrow a + (i + 1)\Delta x \end{array} \right. \\ \text{return } (x_1 \;\; x_2)^T \end{array} \right.$$

Vectors with upper and lower limits of the rectangles

$y1 := y\_values(f_1, a, b, n, L1)$

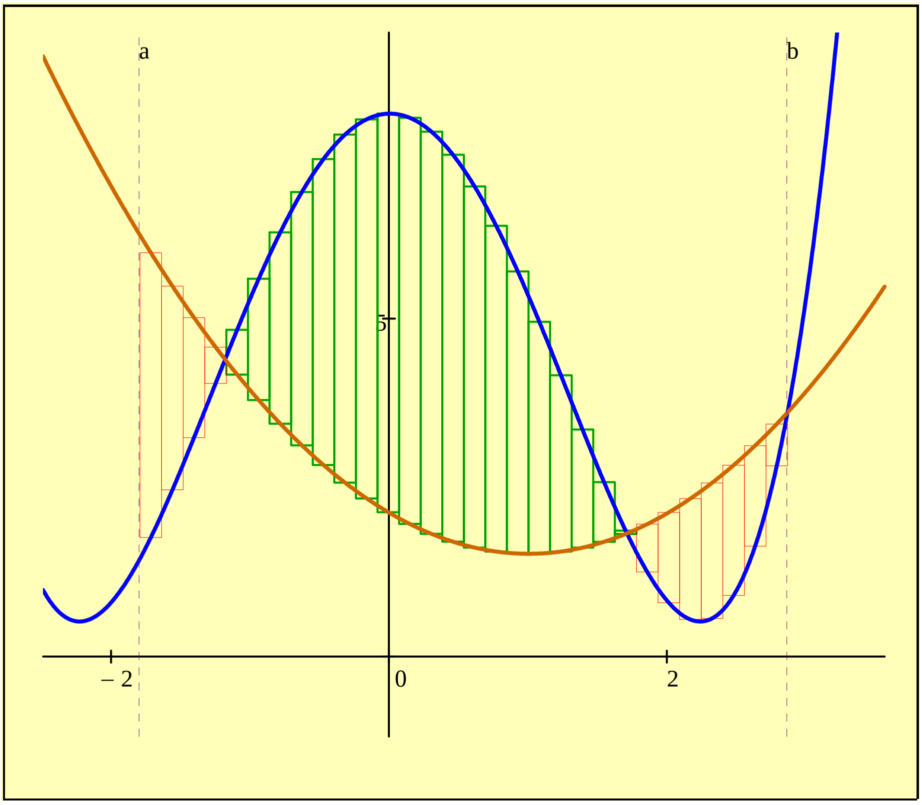
$y2 := y\_values(f_2, a, b, n, L2)$

$$S_{approx} := \sum \left[(y1 - y2) \cdot \frac{b - a}{n} \right] \qquad S_{exact} := \int_a^b (f_1(x) - f_2(x))\, dx$$

$$rel\_error := NaN \text{ on error } \frac{S_{approx} - S_{exact}}{S_{exact}} \qquad abs\_error := S_{approx} - S_{exact}$$

$$\begin{pmatrix} P \\ N \end{pmatrix} := mk\_rectangles(x1, x2, y1, y2)$$

Calculations



Intervals: $n = 30$

$S_{exact} = 8.00165347$

$S_{approx} = 8.00003481$

$rel\_error = -0.02022901 \cdot \%$

$abs\_error = -0.00161866$

$f_1(x) := 0.3 \cdot x^4 + -3.0 \cdot x^2 + 8$

$f_2(x) := 0.6 \cdot (x - 1)^2 + 1.5$

$a = -1.8$

$b = 2.865$

$$\int_a^b f_1(x) - f_2(x)\, dx = 8.00165347$$