MECHANICAL ENGINEERING PROBLEMS

Machine Design: Fatigue Failure

Introduction

Machine members that are subjected to repeated or fluctuating stresses often fail even though the stresses are below the ultimate strength of the part's material or even below the yield strength. The distinguishing feature of these failures is that the part has been subjected to the fluctuating stresses a very large number of times. These types of failures are called **fatigue failure**.

Statement

A rotating-beam specimen made of ductile steel, and having known material properties for its yield and ultimate strengths, is subjected to an alternating stress, σa. Determine if the part will have finite or infinite life. If finite, determine the number of cycles that it can endure. If infinite, determine the part's factor of safety.

Parameters

Define the units MPa and cycles: $MPa := 10^6 \cdot Pa$ $\quad$ $cycles := Hz \cdot sec$

Alternating stress: $\sigma_a := 220 \cdot MPa$

Material yield strength: $S_y := 300 \cdot MPa$

Material ultimate strength: $S_{ut} := 385 \cdot MPa$

Solution

In the absence of the endurance limit for the rotating-beam specimen the following estimate is made:

$S'_e := \textbf{if}\left(S_{ut} \le 1400 \cdot MPa, 0.5 \cdot S_{ut}, 700 \cdot MPa\right)$ $\quad$ $S'_e = 192.5\ MPa$

Factor of Safety: $FS := \frac{S'_e}{\sigma_a}$ $FS = 0.875$

$FS \geq 1$ If the value of the factor of safety, FS, is equal to or greater than one, the part will endure at least a million or more cycles at the applied stress level, σa.

$FS < 1$ To estimate the number of finite cycles that the part will endure when the FS is less then one, the following calculations are made:

$$m := \frac{1}{3} \cdot \log\left(\frac{0.9 \cdot S_{ut}}{S'_e}\right) \qquad m = 0.085$$

$$b := \log\left(\frac{(0.9 \cdot S_{ut})^2}{S'_e \cdot MPa}\right) \qquad b = 2.795$$

The estimated number of cycles, N, the part will endure is then:

$$N := \frac{10^{\frac{b}{m}}}{\left(\frac{\sigma_a}{MPa}\right)^{\frac{1}{m}}} \qquad N = \left(2.082 \cdot 10^5\right) \; \textit{cycles}$$

If N is less than 1000, the above estimate is not correct and the estimate of the part's life should be made using a failure analysis based on low cycle fatigue.

If N is greater than 106, the part is considered to have infinite life.