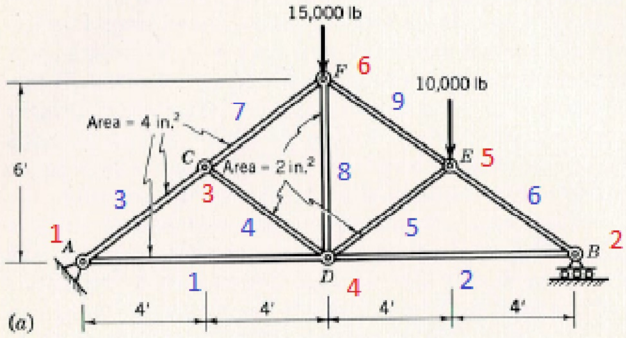
Analysis Of A Pin Jointed Triangulated Truss

1.0 Problem Definition

Example 2.5 Figure 2.8*a* shows the truss of Example 1.4 with exactly the same loads. The truss material is aluminum; all the outer members of the truss have a cross-sectional area of 4 in.^{2}, and each of the three inner members has an area of 2 in.^{2}. We wish to determine how much the length of each member changes due to the loads shown in Fig. 2.8*a*.



(a)

Methodology:

To make this sheet reusable all the entry is defined in the yellow boxes on this page. Except for selection of load case for plotting later in the sheet.

The matrix stiffness method is used.

This means a consistent unit system needs to be used. As I am not in the USA I prefer to work in Newtons and mm. The input values are made unitless but of values that match the N and mm system. Output member length changes are given in mm and inches. Prime takes care of the conversions.

Worksheet as I am not in the USA is A4, standard gridlines, narrow margins.

1.1 Coordinates Of Nodes

$$x := \begin{bmatrix} 0 \\ 16 \\ 4 \\ 8 \\ 12 \\ 8 \end{bmatrix} \cdot \frac{ft}{mm} \qquad \begin{bmatrix} A \\ B \\ C \\ D \\ E \\ F \end{bmatrix} \qquad y := \begin{bmatrix} 0 \\ 0 \\ 3 \\ 0 \\ 3 \\ 6 \end{bmatrix} \cdot \frac{ft}{mm}$$

X is horizontal left to right.
Y is vertical bottom to top.
Decide on an origin for the XY coordinates. Here A is used.
Enter the coordinates of each node point relative to the origin.

For a different problem resize x and y and worksheet adjusts.

1.2 Member Connectivity and Beam Areas

$$nc := \begin{bmatrix} 1 & 4 \\ 4 & 2 \\ 1 & 3 \\ 3 & 4 \\ 4 & 5 \\ 5 & 2 \\ 3 & 6 \\ 4 & 6 \\ 6 & 5 \end{bmatrix} \qquad \begin{bmatrix} AD \\ DB \\ AC \\ CD \\ DE \\ EB \\ CF \\ DF \\ FE \end{bmatrix} \qquad A := \begin{bmatrix} 4 \\ 4 \\ 4 \\ 2 \\ 2 \\ 4 \\ 4 \\ 2 \\ 4 \end{bmatrix} \cdot \frac{in^2}{mm^2}$$

Define the member connectivity matrix and matching area vector.

"nc" is a two column vector that declares the end points of a member on each row. In this type of analysis it does not matter what way round you define the endpoints.

"A" is a single column vector with each row being the value of the corresponding area.

For a different problem resize "nc" and "A" and worksheet adjusts. The same goes for "r" and "nf" below.

1.3 Restraint Information

$$r := \begin{bmatrix} 1 & 1 & 1 \\ 2 & 0 & 1 \end{bmatrix}$$

Create restraint matrix "r". First column is node that has the restraint. Second column is x axis restraint at that node. Third column is y axis restraint at that node. 0 indicates free and 1 indicates fixed.

1.4 Load Information

$$nf := \begin{bmatrix} 5 & 1 & 0 & -10000 \cdot \frac{lb \cdot g}{N} \\ 6 & 1 & 0 & -15000 \cdot \frac{lb \cdot g}{N} \end{bmatrix}$$

Add node loads to the "nf" matrix. First column is the node number that loads are applied too. Second column is the load case number. Third column is X force at node in this load case. Fourth column is Y force at node, remember that Y is positive bottom to top.

1.5 Material Information

$$E := 7.0 \cdot 10^{10} \cdot \frac{N}{m^2} \cdot \frac{mm^2}{N}$$

Enter the material Youngs Modulus. In this case general aluminium.

2.0 Problem Solution

2.1 Member Lengths And Angle In XY Plane

$$j := 0 .. \text{rows}(nc) - 1$$

$$ml_j := \sqrt{\left(y_{\left(nc_{j,1}-1\right)} - y_{\left(nc_{j,0}-1\right)}\right)^2 + \left(x_{\left(nc_{j,1}-1\right)} - x_{\left(nc_{j,0}-1\right)}\right)^2}$$

$$\beta_j := \text{angle}\left(x_{\left(nc_{j,1}-1\right)} - x_{\left(nc_{j,0}-1\right)}, y_{\left(nc_{j,1}-1\right)} - y_{\left(nc_{j,0}-1\right)}\right)$$

2.2 Establish Functions for Member Stiffness And Rotation Matrix

$$R(\theta) := \begin{bmatrix} \cos(\theta) & \sin(\theta) & 0 & 0 \\ -\sin(\theta) & \cos(\theta) & 0 & 0 \\ 0 & 0 & \cos(\theta) & \sin(\theta) \\ 0 & 0 & -\sin(\theta) & \cos(\theta) \end{bmatrix}$$

$$S(L,\theta,A,E) := \begin{bmatrix} \frac{E \cdot A}{L} & 0 & -\frac{E \cdot A}{L} & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{E \cdot A}{L} & 0 & \frac{E \cdot A}{L} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cdot R(\theta)$$

$$K_b(L,\theta,A,E) := R(\theta)^{\mathrm{T}} \cdot S(L,\theta,A,E)$$

2.3 Apply Function To Create Each Individual Member Stiffness

$$K_{bg_j} := K_b\left(ml_j, \beta_j, A_j, E\right)$$

2.4 Create Structure Global Matrix And Assemble Using Individual Member Stiffness

$$K_{sg_{\text{rows}(x)\cdot 2-1,\ \text{rows}(x)\cdot 2-1}} := 0 \qquad ii := 0 .. 1 \qquad ij := 0 .. 1$$

$$K_{sg_{\left(nc_{j,0}-1\right)\cdot 2+ii,\ \left(nc_{j,0}-1\right)\cdot 2+ij}} := K_{sg_{\left(nc_{j,0}-1\right)\cdot 2+ii,\ \left(nc_{j,0}-1\right)\cdot 2+ij}} + \left(K_{bg_j}\right)_{ii,ij}$$

$$K_{sg_{\left(nc_{j,0}-1\right)\cdot 2+ii,\ \left(nc_{j,1}-1\right)\cdot 2+ij}} := K_{sg_{\left(nc_{j,0}-1\right)\cdot 2+ii,\ \left(nc_{j,1}-1\right)\cdot 2+ij}} + \left(K_{bg_j}\right)_{ii,ij+2}$$

$$K_{sg_{\left(nc_{j,1}-1\right)\cdot 2+ii,\ \left(nc_{j,0}-1\right)\cdot 2+ij}} := K_{sg_{\left(nc_{j,1}-1\right)\cdot 2+ii,\ \left(nc_{j,0}-1\right)\cdot 2+ij}} + \left(K_{bg_j}\right)_{ii+2,ij}$$

$$K_{sg_{\left(nc_{j,1}-1\right)\cdot 2+ii,\ \left(nc_{j,1}-1\right)\cdot 2+ij}} := K_{sg_{\left(nc_{j,1}-1\right)\cdot 2+ii,\ \left(nc_{j,1}-1\right)\cdot 2+ij}} + \left(K_{bg_j}\right)_{ii+2,ij+2}$$

2.5 Use Restraint Matrix To Add Large Value On Global Stiffness Diagonal At Fixed Restraints

$$Re := \max\left(K_{sg}\right) \cdot 10^{10} \qquad il := 0 .. \text{rows}(r) - 1$$

$$K_{sg_{\left(r_{il,0}-1\right)\cdot 2+ii,\ \left(r_{il,0}-1\right)\cdot 2+ii}} := Re \cdot r_{il,ii+1} + K_{sg_{\left(r_{il,0}-1\right)\cdot 2+ii,\ \left(r_{il,0}-1\right)\cdot 2+ii}}$$

2.6 Expand Input Force Matrix Into Global Force Matrix for Structure

$$lc := \max\left(nf^{\langle 1\rangle}\right) \qquad n := 0 .. \text{rows}(nf) - 1 \qquad zero(x,y) := 0.0 \qquad f := \text{matrix}\left(\text{rows}\left(K_{sg}\right), lc, zero\right)$$

$$f_{\left(nf_{n,0}-1\right)\cdot 2+ii,\ nf_{n,1}-1} := nf_{n,ii+2}$$

2.7 Solve Resulting Global Stiffness Equation for Deflections of Structure

$$\delta i := K_{sg}^{-1} \cdot f$$

3.0 Plotting Structure And Deformed Shape

3.1 Global Deflection Result

$$\delta i \cdot mm = \begin{bmatrix} 0.000 \\ 0.000 \\ 0.079 \\ 0.000 \\ 0.061 \\ -0.123 \\ 0.032 \\ -0.163 \\ 0.002 \\ -0.164 \\ 0.047 \\ -0.145 \end{bmatrix} in$$

3.2 Plot Function

$$plot(x,y) := \left| \begin{array}{l} j \leftarrow 0 \\ \text{for } i \in 0 .. \text{rows}(nc)-1 \\ \quad \left| \begin{array}{l} plot_{j,0} \leftarrow x_{nc_{i,0}-1} \\ plot_{j,1} \leftarrow y_{nc_{i,0}-1} \\ j \leftarrow j+1 \\ plot_{j,0} \leftarrow x_{nc_{i,1}-1} \\ plot_{j,1} \leftarrow y_{nc_{i,1}-1} \\ j \leftarrow j+1 \\ \text{if } i < \text{rows}(nc)-1 \\ \quad \left| \begin{array}{l} plot_{j,0} \leftarrow 1j \\ plot_{j,1} \leftarrow 1j \\ j \leftarrow j+1 \end{array} \right. \end{array} \right. \\ plot \end{array} \right.$$

3.3 Select Load Case “δp” And Scaling Factor "δs" For Plot

$\delta p := 1$ $\qquad io := 0 .. \text{rows}(x)-1$

$$diag := \sqrt{(\max(x)-min(x))^2 + (\max(y)-min(y))^2}$$

$$\delta s := \frac{diag}{\max\left(\left|\delta i^{\langle \delta p - 1 \rangle}\right|\right)} \cdot \frac{2}{100} \qquad \delta s = 104.168$$

3.4 Extract Node Deflections δx and δy From δi

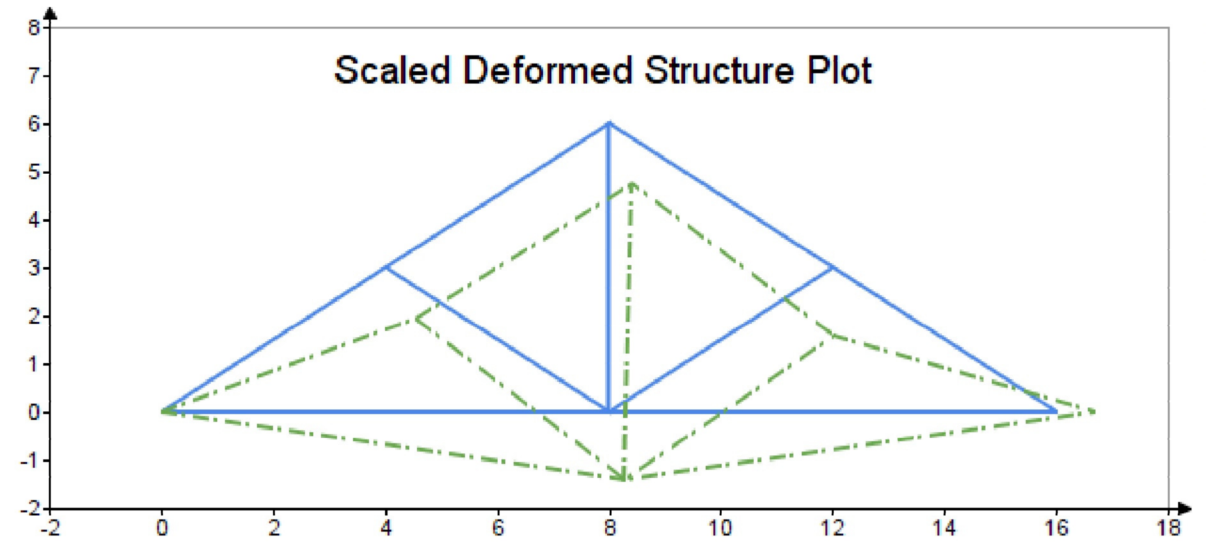
$$\delta x_{io} := \left(\delta i^{\langle \delta p - 1 \rangle}\right)_{io \cdot 2} \qquad \delta y_{io} := \left(\delta i^{\langle \delta p - 1 \rangle}\right)_{io \cdot 2 + 1}$$

3.5 Plot The Result

Inputs

$$X_1 := plot(x,y)^{\langle 0 \rangle} \cdot \frac{mm}{ft} \qquad Y_1 := plot(x,y)^{\langle 1 \rangle} \cdot \frac{mm}{ft}$$

$$X_2 := plot(x + \delta x \cdot \delta s, y + \delta y \cdot \delta s)^{\langle 0 \rangle} \cdot \frac{mm}{ft} \qquad Y_2 := plot(x + \delta x \cdot \delta s, y + \delta y \cdot \delta s)^{\langle 1 \rangle} \cdot \frac{mm}{ft}$$



4.0 Determine Member Forces From Global Deflections Matrix & Get Extension/Contraction Of Members

4.1 Member Forces All Load Cases

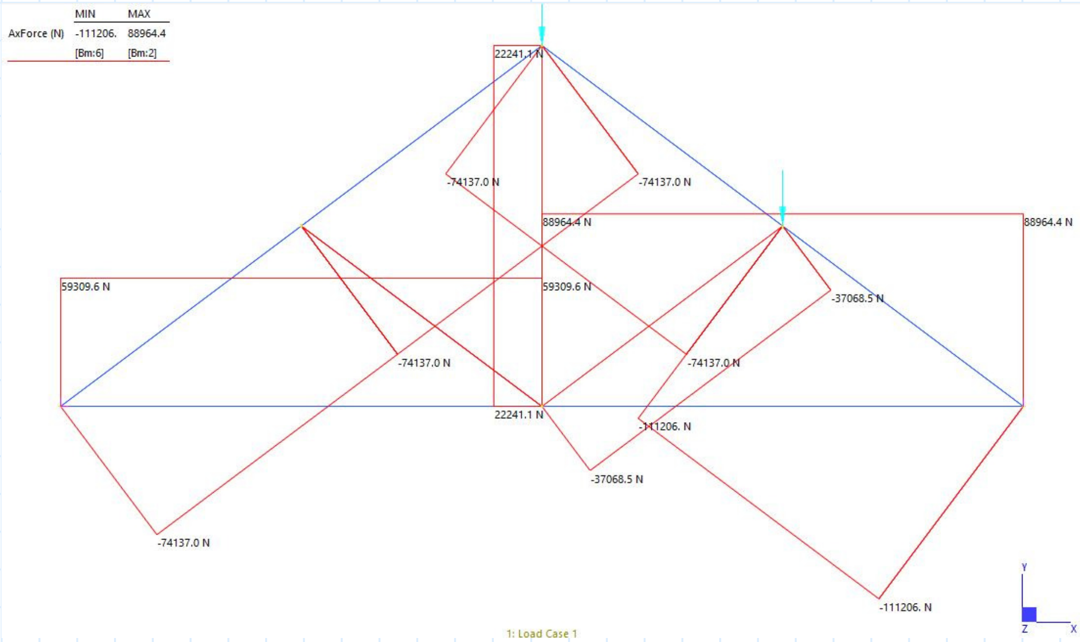
$$ki := 0 \,..\, \text{cols}(f) - 1$$

$$Am_{j,ki} := S\left(ml_j, \beta_j, A_j, E\right) \cdot \begin{bmatrix} \delta i_{\left(nc_{j,0}-1\right)\cdot 2+0,\, ki} \\ \delta i_{\left(nc_{j,0}-1\right)\cdot 2+1,\, ki} \\ \delta i_{\left(nc_{j,1}-1\right)\cdot 2+0,\, ki} \\ \delta i_{\left(nc_{j,1}-1\right)\cdot 2+1,\, ki} \end{bmatrix}$$

4.2 Member Forces For Chosen Load Case

$$P_j := -\left(Am_j\right)_{0,\,\delta p-1} \cdot \boldsymbol{N} = \begin{bmatrix} 59310 \\ 88964 \\ -74137 \\ 0 \\ -37069 \\ -111206 \\ -74137 \\ 22241 \\ -74137 \end{bmatrix} \boldsymbol{N} \qquad \begin{bmatrix} AD \\ DB \\ AC \\ CD \\ DE \\ EB \\ CF \\ DF \\ FE \end{bmatrix} \qquad P = \begin{bmatrix} 13333 \\ 20000 \\ -16667 \\ 0 \\ -8333 \\ -25000 \\ -16667 \\ 5000 \\ -16667 \end{bmatrix} \boldsymbol{lbf}$$

4.3 Verification Against Commercial Software



4.4 Answer to Question - Local Member Length Extension "+", Contraction "-"

$$\delta m_j := \frac{-\left(Am_j\right)_{0,\,\delta p-1} \boldsymbol{N} \cdot ml_j \cdot \boldsymbol{mm}}{A_j \cdot \boldsymbol{mm}^2 \cdot E \cdot \boldsymbol{MPa}} = \begin{bmatrix} 0.801 \\ 1.201 \\ -0.625 \\ 0.000 \\ -0.625 \\ -0.938 \\ -0.625 \\ 0.450 \\ -0.625 \end{bmatrix} \boldsymbol{mm} \qquad \begin{bmatrix} AD \\ DB \\ AC \\ CD \\ DE \\ EB \\ CF \\ DF \\ FE \end{bmatrix} \qquad \delta m = \begin{bmatrix} 0.0315 \\ 0.0473 \\ -0.0246 \\ 0.0000 \\ -0.0246 \\ -0.0369 \\ -0.0246 \\ 0.0177 \\ -0.0246 \end{bmatrix} \boldsymbol{in}$$

5.0 Sanity Check Yield Of Aluminium Approximately 80 MPa

$$\sigma_j := \frac{-\left(Am_j\right)_{0,\delta p-1} N}{A_j \cdot mm^2} = \begin{bmatrix} 22.983 \\ 34.474 \\ -28.728 \\ 0.000 \\ -28.728 \\ -43.092 \\ -28.728 \\ 17.237 \\ -28.728 \end{bmatrix} MPa \qquad \begin{bmatrix} AD \\ DB \\ AC \\ CD \\ DE \\ EB \\ CF \\ DF \\ FE \end{bmatrix} \qquad \sigma = \begin{bmatrix} 3333 \\ 5000 \\ -4167 \\ 0 \\ -4167 \\ -6250 \\ -4167 \\ 2500 \\ -4167 \end{bmatrix} psi$$